ASTRONOMY 8400 – SPRING 2024
Homework Set 3 – Answers

1.a) Note:

$\mu_B = -2.5 \log(F_B/\Omega) + k1$

$m_B = -2.5 \log(F_B) + k2$

At $\Omega = 1$ arcsec^2, $\mu_B = m_B$ (by definition), so k1=k2

Thus: $\mu_B - m_B = 2.5 \log(\Omega)$ (as long as Ω is in arcsec^2)

$M_{B\odot} = +5.47$ (Allen's AQ)

At 10 pc: B (apparent) = $M_B = 5.47$

$\Omega = (1\text{ pc}/10\text{ pc} \times 206265''/\text{radian})^2 = 4.25 \times 10^8$ arcsec^2

$\mu_{B1} - B = 2.5{*}\log(4.25 \times 10^8) = 21.57$ mag

$\mu_{B1} = 21.57 + 5.47 = 27.04$ mag arcsec^{-2}

b) Intensity doesn't change with distance - the combined intensity is twice as large:

$\mu_{Btot} = \mu_{B1} - 2.5 \log(2) = 27.04 - 0.75 = 26.29$ mag arcsec^{-2}

c) $\mu_{B2} = \mu_{B1} - 2.5 \log [1/\cos(60^o)] = 26.31$ mag arcsec^{-2} (for galaxy 2)

The combined intensity is 3 times as large at that of galaxy 1:

$\mu_{Btot} = \mu_{B1} - 2.5 \log(3) = 27.04 - 1.19 = 25.85$ mag arcsec^{-2}

d) At 10 Mpc, the angular radius is $\theta = [(15\text{ kpc}/10\text{ Mpc})(206{,}265''/\text{radian}) = 309''$

$\Omega_1 = \pi\theta^2 = 3.0 \times 10^5$ arcsec^2

$B_1 = \mu_{B1} - 2.5 \log (3.0 \times 10^5) = 27.04 - 13.69 = 13.35$ mag

At 50 Mpc, $\Omega_2 = \Omega_1/5^2/2 = 6.0 \times 10^3$ arcsec^2

(The division by 2 is due to the reduced area of the inclined disk.)

$B_2 = \mu_{B2} - 2.5 \log (6.0 \times 10^3) = 26.29 - 9.44 = 16.85$ mag

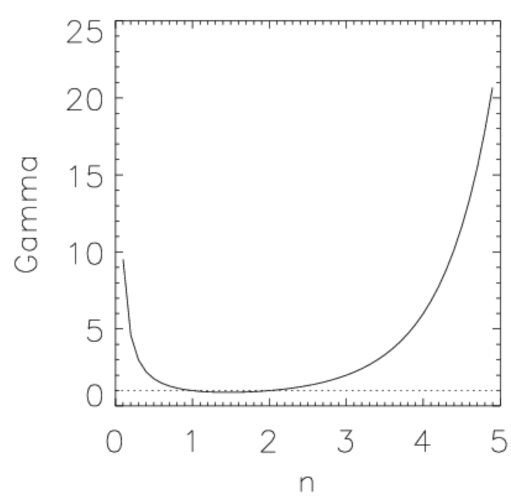
$F_2/F_1 = 10^{-0.4*(B2 - B1)} = 0.040$

$B_{tot} - B_1 = -2.5 \log [(F_1+F_2)/F_1]$

$B_{tot} = B_1 - 2.5 \log(1.040) = 13.35 - 0.04 = 13.31$ mag

2. Seems like this homework turned into a Γ- function-fest. Note that:

$$\int_0^\infty t^n e^{-t} dt = \Gamma(n+1) \quad = n! \text{ (for integers)}$$



2.

$$L_{tot} = \int_0^\infty L\Phi(L)\, dL$$

$$L_{tot} = \int_0^\infty L \frac{n*}{L*}\left(\frac{L}{L*}\right)^{\alpha} \exp\left(\frac{-L}{L*}\right) dL$$

$$L_{tot} = n*L*\int_0^\infty \left(\frac{L}{L*}\right)^{\alpha+1} \exp\left(\frac{-L}{L*}\right) \frac{dL}{L*}$$

Let t = L/L* dt = dL/L*

$$L_{tot} = n*L*\int_0^\infty (t)^{\alpha+1} \exp(-t)\, dt$$

$$L_{tot} = n*L*\Gamma(\alpha+2)$$

$n* = 0.02\ h^3\ Mpc^{-3},\ L* = 9.0 \times 10^9\ h^{-2}\ Mpc^{-3}, \alpha = -0.4$

From the IDL procedure GAMMA: $\Gamma(1.6) = 0.89$

$L_{tot} = 1.6 \times 10^8\ h\ L_\odot Mpc^{-3} = 1.1 \times 10^8\ L_\odot Mpc^{-3}$ (for h =0.71)

$M / L = 1750\ h\ M_\odot / L_\odot \approx 1240\ M_\odot / L_\odot$ (for h =0.71)

Note: $n* \neq$ total # galaxies/Mpc^{-3}

$$\int_0^\infty \Phi(L)\, dL = \int_0^\infty \frac{n*}{L*}\left(\frac{L}{L*}\right)^{\alpha} \exp\left(\frac{-L}{L*}\right) dL$$

$$\int_0^\infty \Phi(L)\, dL = n*\int_0^\infty \left(\frac{L}{L*}\right)^{\alpha} \exp\left(\frac{-L}{L*}\right) \frac{dL}{L*}$$

$$\int_0^\infty \Phi(L)\, dL = n*\int_0^\infty (t)^{\alpha} \exp(-t)\, dt = n*\Gamma(\alpha+1)$$

total # galaxies/Mpc^{-3} $= n*\Gamma(\alpha+1)$

For $\alpha = -0.4,\ \Gamma(\alpha+1) = \Gamma(0.6) = 1.49$

So total # galaxies/Mpc^{-3} $= 1.49n* = 0.03\ h^3\ Mpc^{-3}$

3. a)The de Vaucouleurs law holds for intensity in the form of flux/pc^2:

$$L=\int_0^\infty I(R)\ 2\pi R\ dR = I(R) = 2\pi I_e \int_0^\infty R\exp\left\{-7.67\ \left[(R/R_e)^{1/4}-1\right]\right\}\ dR$$

$$L=2\pi I_e\ e^{7.67}\int_0^\infty R\exp\left[-7.67\ (R/R_e)^{1/4}\right]\ dR$$

Let $t=-7.67\ (R/R_e)^{1/4}\ \rightarrow\ t^4=(7.67)^4\ (R/R_e)$

$R=R_e(7.67)^{-4}t^4,\ \ dR=4R_e(7.67)^4t^3dt$

$$L=8\pi I_e R_e^2\ e^{7.67}(7.67)^{-8}\int_0^\infty t^7\ e^{-t}\,dt,\ \ \text{Note}: \int_0^\infty t^7\ e^{-t}\,dt=\Gamma(8)=7!=5040$$

$$L=8\pi I_e R_e^2\ e^{7.67}(7.67)^{-8}(5040)$$

$$L=7.21\pi R_e^2 I_e$$

b) Integrate to R_e:

For $R=R_e$, $t=7.67\ (R_e/R_e)^{1/4}=7.67$

$$L_e=8\pi I_e R_e^2\ e^{7.67}(7.67)^{-8}\int_0^{7.67} t^7\ e^{-t}\,dt$$

At this point I used the IDL function IGAMMA,

whereas some folks used a more elegant expansion

$$L=8\pi I_e R_e^2\ e^{7.67}(7.67)^{-8}(2520)$$

Thus $L_e=0.5L_{tot}$

c) Integrate to get Gamma function once again!

$$L=\int_0^\infty I(R)\ 2\pi R\ dR = I(R) = 2\pi I_d\int_0^\infty R\exp(-R/R_d)\ dR$$

Let $t=R/R_d$, $dt=dR/R_d$

$$L=2\pi I_d R_d^2\int_0^\infty t\ e^{-t}dt = 2\pi I_d R_d^2(1!)$$

$$L=2\pi I_d R_d^2$$