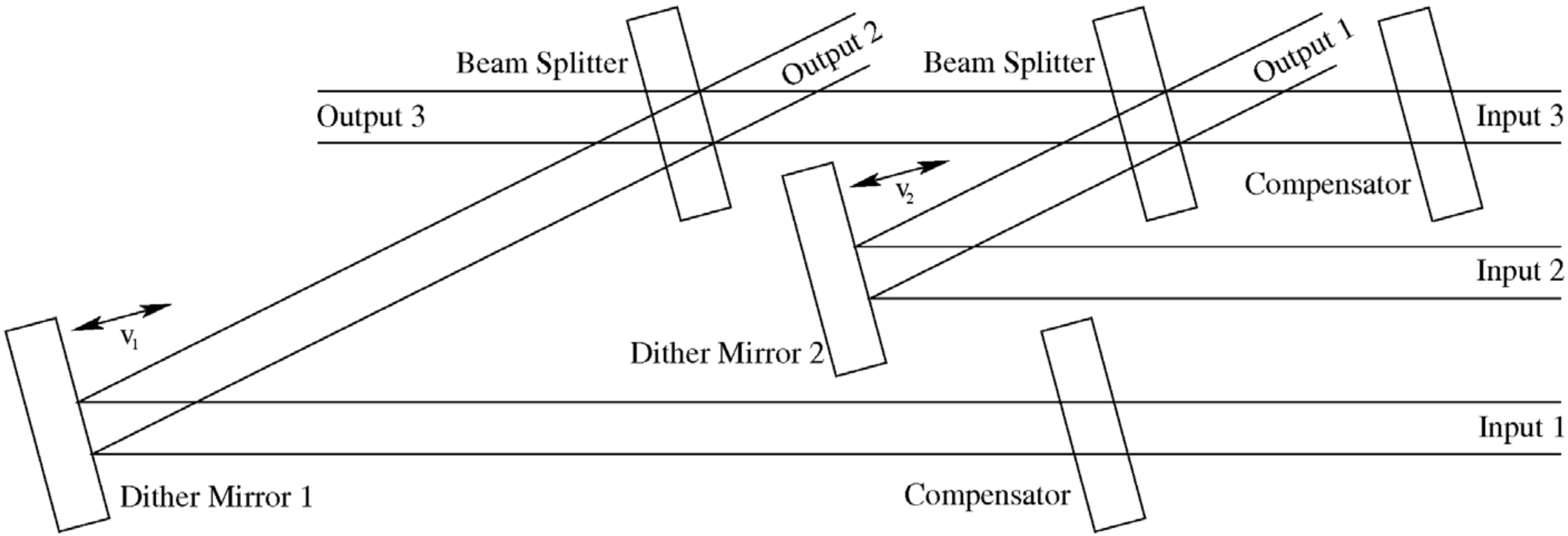
CHARA Climb Data Reduction



The math - sorry about this.

We can now write the detected signal as

$$S(k) = \frac{1}{2}\sum_{i=1}^{N} A_i^2 + \sum_{i=1}^{N-1}\sum_{j=i+1}^{N} A_i A_j \cos\left[k(x_i - x_j) + (\phi_i - \phi_j)\right]. \tag{7}$$

If we use the signal intensity $I_i = A_i^2$ this becomes

$$S(k) = \frac{1}{2}\sum_{i=1}^{N} I_i + \sum_{i=1}^{N-1}\sum_{j=i+1}^{N} \sqrt{I_i I_j} \cos\left[k(x_i - x_j) + (\phi_i - \phi_j)\right]. \tag{8}$$

Since visibility amplitudes are normalized, that is they have a value between zero and one, the next step is to divide this signal by the mean intensity $\frac{1}{2}\sum_{i=1}^{N} I_i$ resulting in

$$N(k) = 1 + \sum_{i=1}^{N-1}\sum_{j=i+1}^{N} T_{ij} \cos\left[k(x_i - x_j) + (\phi_i - \phi_j)\right] \tag{9}$$

where we have introduced the transfer function

$$T_{ij} = \frac{2\sqrt{I_i I_j}}{\sum_{i=1}^{N} I_i}. \tag{10}$$

$$x_i - x_j = (v_i - v_j)\ t = v_{ij}\ t. \tag{12}$$

It is also convenient to use the wave number

Visibility Amplitude and Phase

$$\sigma = \frac{1}{\lambda} = \frac{k}{2\pi}. \tag{13}$$

When we substitute Equations 12 and 13 into the fringe equation 11 we get

$$N(\sigma) = 1 + \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} T_{ij} V_{ij} \cos\left[2\pi\sigma v_{ij} t + (\phi_i - \phi_j) + \Phi ij\right]. \tag{14}$$

Phase Imposed by Optics and Atmosphere

The final step is to introduce a finite bandwidth and integrate the fringe equation across this bandwidth. Let us for the sake of simplicity assume that the bandwidth is square, centered on σ_0 and with a width of $\Delta\sigma$. We then have

$$\begin{aligned}
N(\sigma_0, \Delta\sigma) &= \frac{1}{\Delta\sigma} \int_{\sigma_0 - \frac{\Delta\sigma}{2}}^{\sigma_0 + \frac{\Delta\sigma}{2}} N(\sigma) d\sigma \\
&= \frac{1}{\Delta\sigma} \int_{\sigma_0 - \frac{\Delta\sigma}{2}}^{\sigma_0 + \frac{\Delta\sigma}{2}} \left(1 + \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} T_{ij} V_{ij} \cos\left[2\pi\sigma v_{ij} t + (\phi_i - \phi_j) + \Phi ij\right]\right) d\sigma \\
&= 1 + \frac{1}{\Delta\sigma} \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} T_{ij} V_{ij} \int_{\sigma_0 - \frac{\Delta\sigma}{2}}^{\sigma_0 + \frac{\Delta\sigma}{2}} \cos\left[2\pi\sigma v_{ij} t + (\phi_i - \phi_j) + \Phi ij\right] d\sigma. \qquad (15)
\end{aligned}$$

$$N(\sigma_0, \Delta\sigma) = 1 + \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} T_{ij} V_{ij} \, \mathrm{sinc}\left[\pi \Delta\sigma v_{ij} t\right] \cos\left[2\pi\sigma_0 v_{ij} t + (\phi_i - \phi_j) + \Phi ij\right]. \qquad (19)$$

This is the result we're all used to seeing: oscillating terms for the fringes from each baseline, whose fringe period is a function of the central wavelength of the optical filter, modulated by an envelope function, in this case a sinc function, whose size is inversely proportional to the width of the optical filter. In this analysis we have assumed that the optical filter is square, or a 'top hat' function, whose Fourier Transform is a sinc function, and as we shall see, this works in general. That is, the fringe envelope function is the Fourier transform of the filter function. This is the basis for Fourier Transform Spectroscopy, but that is beyond the realm of this technical report.

Everything that now follows is now based on Equation 19, the generalized fringe equation for an N-way beam combiner.

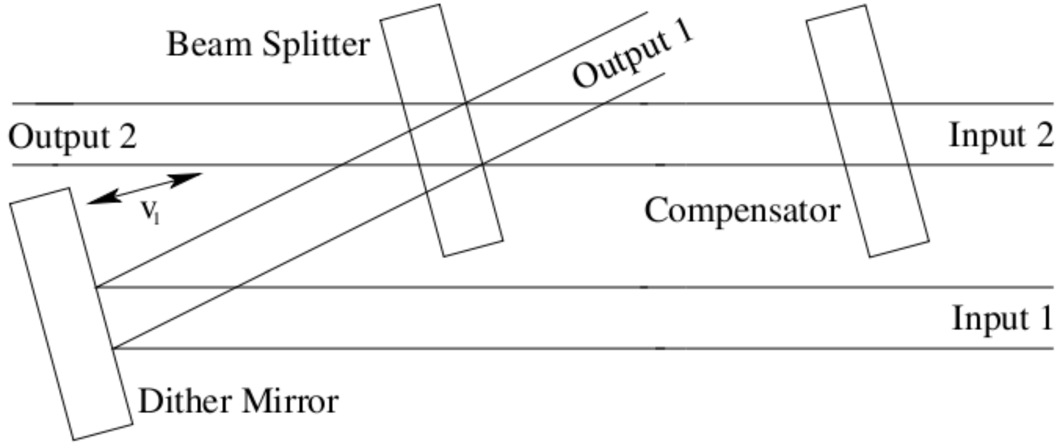


FIGURE 1. Schematic of the optical layout of the CLASSIC beam combiner.

In this simple case of a two beam combiner the transfer function in Equation 10 reduces to

$$T_{12} = \frac{2\sqrt{I_1 I_2}}{I_1 + I_2}. \tag{20}$$

$$\phi_{11} = 0,\ \phi_{12} = \pi,\ \phi_{21} = 0,\ \text{and}\ \phi_{22} = 0. \tag{21}$$

We now have a π phase difference between the two outputs giving us the bright fringe on one side and a dark fringe on the other, as we know it should be, for only then is energy conserved.

We can therefore write the fringe equation for output i of CLASSIC as

$$N_i(\sigma_0, \Delta\sigma) = 1 + (-1)^i\ T_{i12}\ V_{12}\ \mathrm{sinc}\left[\pi\Delta\sigma v t\right] \cos\left[2\pi\sigma_0 v t + \Phi_{12}\right]. \tag{22}$$

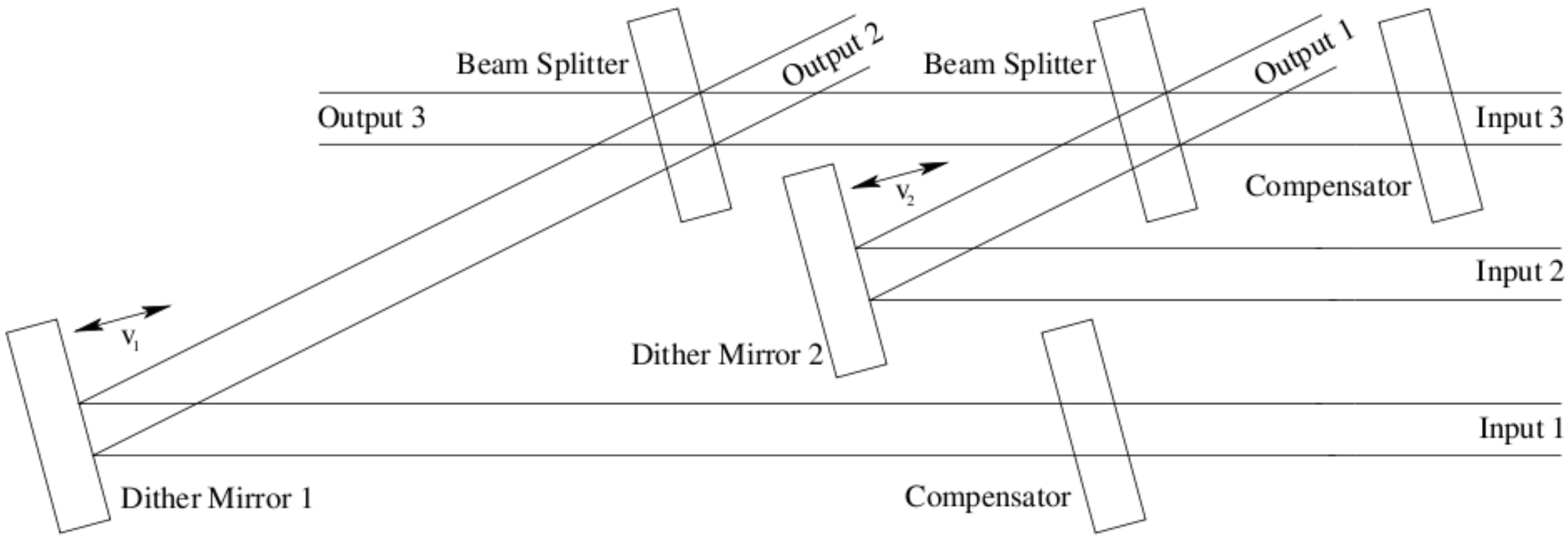


FIGURE 3. Schematic of the optical layout of the CLIMB beam combiner.

so for output i and input beams j and k we write

$$T_{ijk} = \frac{2\sqrt{I_{ij}I_{ik}}}{I_{i1} + I_{i2} + I_{i3}} \qquad (23)$$

TABLE 2. CLIMB transfer functions for perfect optics. Note that beam 1 does not reach output 1 and so no phase is given.

Output	ϕ_1	ϕ_2	ϕ_3
1	-	0	π
2	0	π	π
3	0	0	0

We write the three outputs of the CLIMB beam combiner as

$$N_1 = 1 - T_{123}\, V_{23}\, \text{sinc}\,[2\pi\Delta\sigma vt]\cos\,[4\pi\sigma_0 vt + \Phi_{23}] \tag{26}$$

$$\begin{aligned} N_2 = 1 &- T_{212}\, V_{12}\, \text{sinc}\,[3\pi\Delta\sigma vt]\cos\,[6\pi\sigma_0 vt + \Phi_{12}] \\ &+ T_{223}\, V_{23}\, \text{sinc}\,[2\pi\Delta\sigma vt]\cos\,[4\pi\sigma_0 vt + \Phi_{23}] \\ &- T_{231}\, V_{31}\, \text{sinc}\,[\pi\Delta\sigma vt]\cos\,[2\pi\sigma_0 vt + \Phi_{31}] \end{aligned} \tag{27}$$

$$\begin{aligned} N_3 = 1 &+ T_{312}\, V_{12}\, \text{sinc}\,[3\pi\Delta\sigma vt]\cos\,[6\pi\sigma_0 vt + \Phi_{12}] \\ &+ T_{323}\, V_{23}\, \text{sinc}\,[2\pi\Delta\sigma vt]\cos\,[4\pi\sigma_0 vt + \Phi_{23}] \\ &+ T_{331}\, V_{31}\, \text{sinc}\,[\pi\Delta\sigma vt]\cos\,[2\pi\sigma_0 vt + \Phi_{31}] \end{aligned} \tag{28}$$

and the power spectrum will therefore be

$$\mathrm{PS}(f(t)) = \frac{V^2}{4\Delta\sigma^2 v^2}\left[\Pi\left(\frac{f-\sigma_0 v}{\Delta\sigma v}\right)+\Pi\left(\frac{f+\sigma_0 v}{\Delta\sigma v}\right)\right]^2. \tag{42}$$

The last step is to integrate this power spectrum, which since it is symmetric, we will only do over the positive frequencies

$$\begin{aligned} S &= \int_0^\infty \mathrm{PS}(f(t))df \\ &= \frac{V^2}{4\Delta\sigma^2 v^2}\int_0^\infty \Pi^2\left(\frac{f-\sigma_0 v}{\Delta\sigma v}\right)df. \end{aligned} \tag{43}$$

So, the total power S in the fringe power spectrum will be

$$S = \frac{V^2}{4\Delta\sigma v}, \tag{46}$$

which gives us our estimator for V^2

$$V^2 = \frac{4\Delta\sigma v}{S}. \tag{47}$$

This is the same as the result in Benson et al. (1995) except for a factor of 2. I do not know why this difference exists and can only assume it derives from a different normalization of the Fourier transform equations.

There is one more correction that can be made to the visibility estimate. Because of atmospheric turbulence, the visibility is changing constantly and so can be considered to be a random variable with some mean $\overline{V}$ and a variance σ_V^2. Since we are measuring the mean of the square of the visibility, we are actually measuring

$$\overline{V^2} = \overline{V}^2 + \sigma_V^2, \tag{48}$$

and thus all estimates of the square of the visibility are biased by the variance of the visibility. Unfortunately, it is not possible to take the square root of S as, due to the statistical nature of the measure, it is sometimes negative. It is, however, possible to square S, resulting in an estimate of $\overline{V^4}$. If one assumes the statistical distribution of the correlation is normal, one can then form the unbiased estimator for the correlation

$$\overline{V} = \left(\frac{3\overline{V^2}^2 - \overline{V^4}}{2}\right)^{\frac{1}{4}} \tag{49}$$

with the corresponding variance estimate

$$\sigma_V^2 = \sqrt{\overline{V}^4 - \frac{1}{2}(\overline{V^2}^2 - \overline{V^4})} - \overline{V}^2. \tag{50}$$

In the data reduction pipeline this is known as the V_NORM estimator.

Since the real visibility can never be negative, it is sometimes better to use a log-normal distribution, normally parametrized using the variables μ and σ^2. These can be determined using

$$\mu = \frac{1}{4} \ln \frac{\overline{V^2}^4}{\overline{V^4}} \tag{51}$$

and

$$\sigma^2 = \frac{1}{4} \ln \frac{\overline{V^4}}{\overline{V^2}^2} \tag{52}$$

from which we get the unbiased correlation estimate

$$\overline{V} = \exp(\mu + \frac{1}{2}\sigma^2) \tag{53}$$

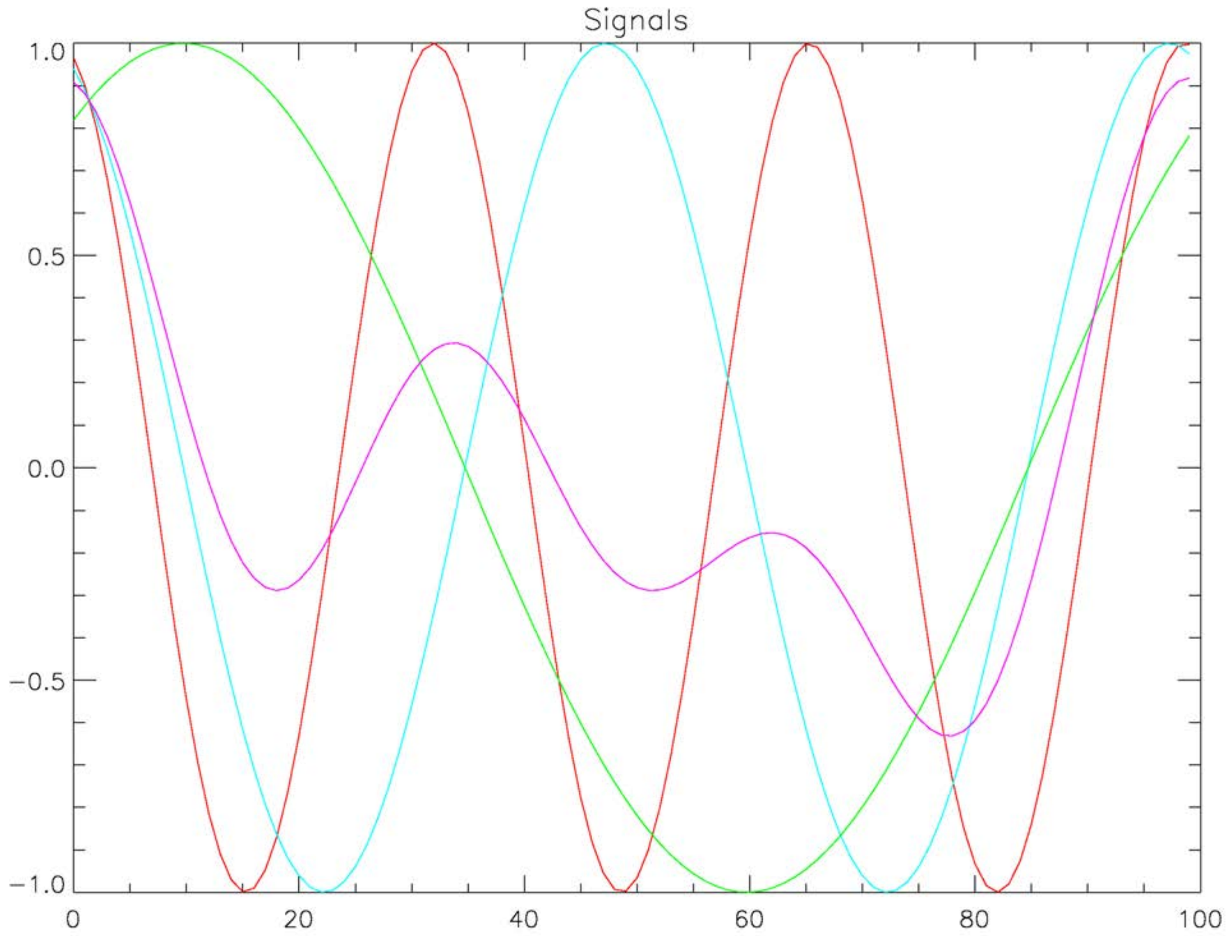
with the variance

$$\sigma_V^2 = \exp(2\mu + 2\sigma^2) - \exp(2\mu + \sigma^2). \tag{54}$$

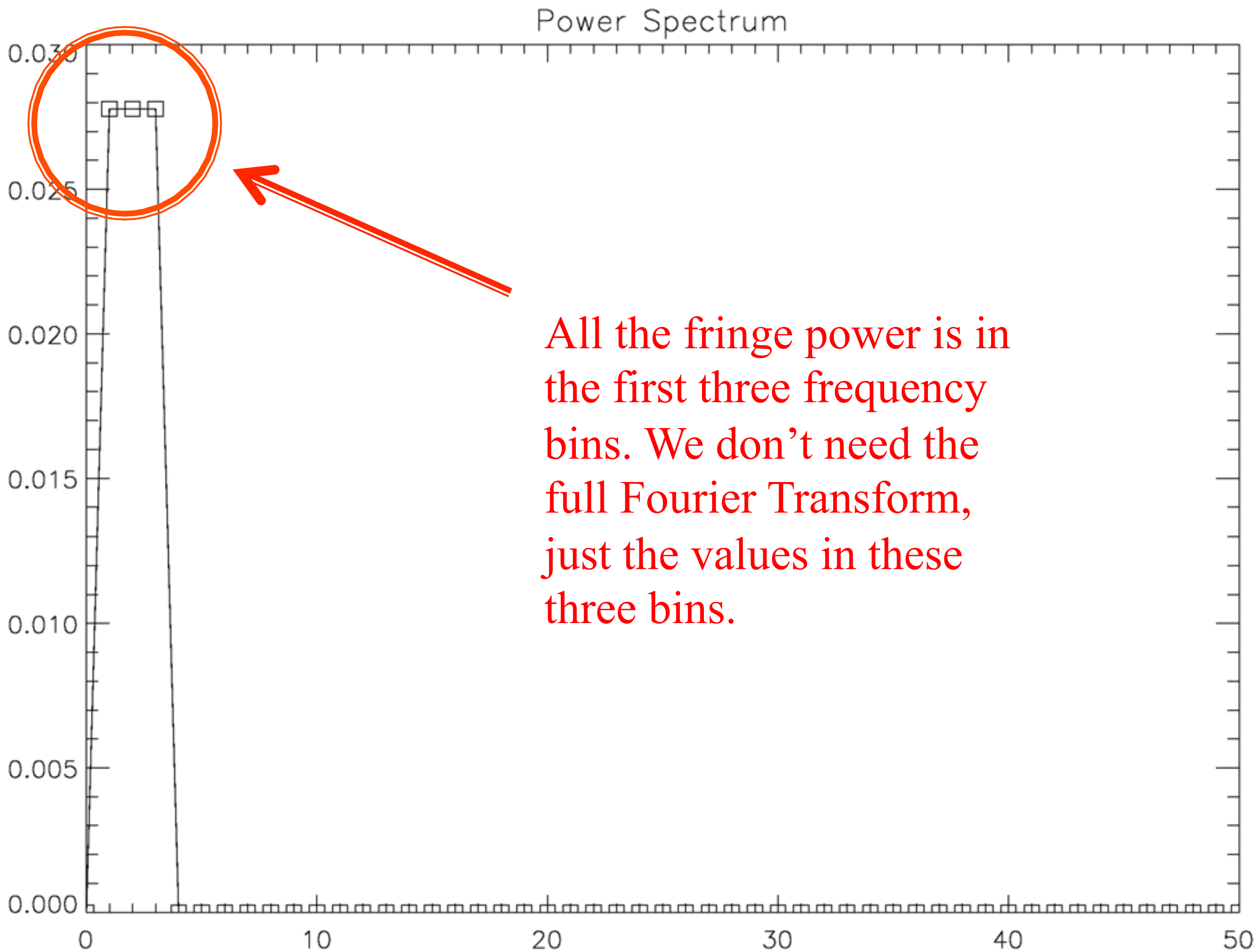
In the data reduction pipeline this is known as the V_LOGNORM estimator.

In practice the normal and log-normal equations give virtually the same results during times of good seeing, or high visibility, and typically diverge at very low signal to noise.

Closure Phase – Consider One Segment

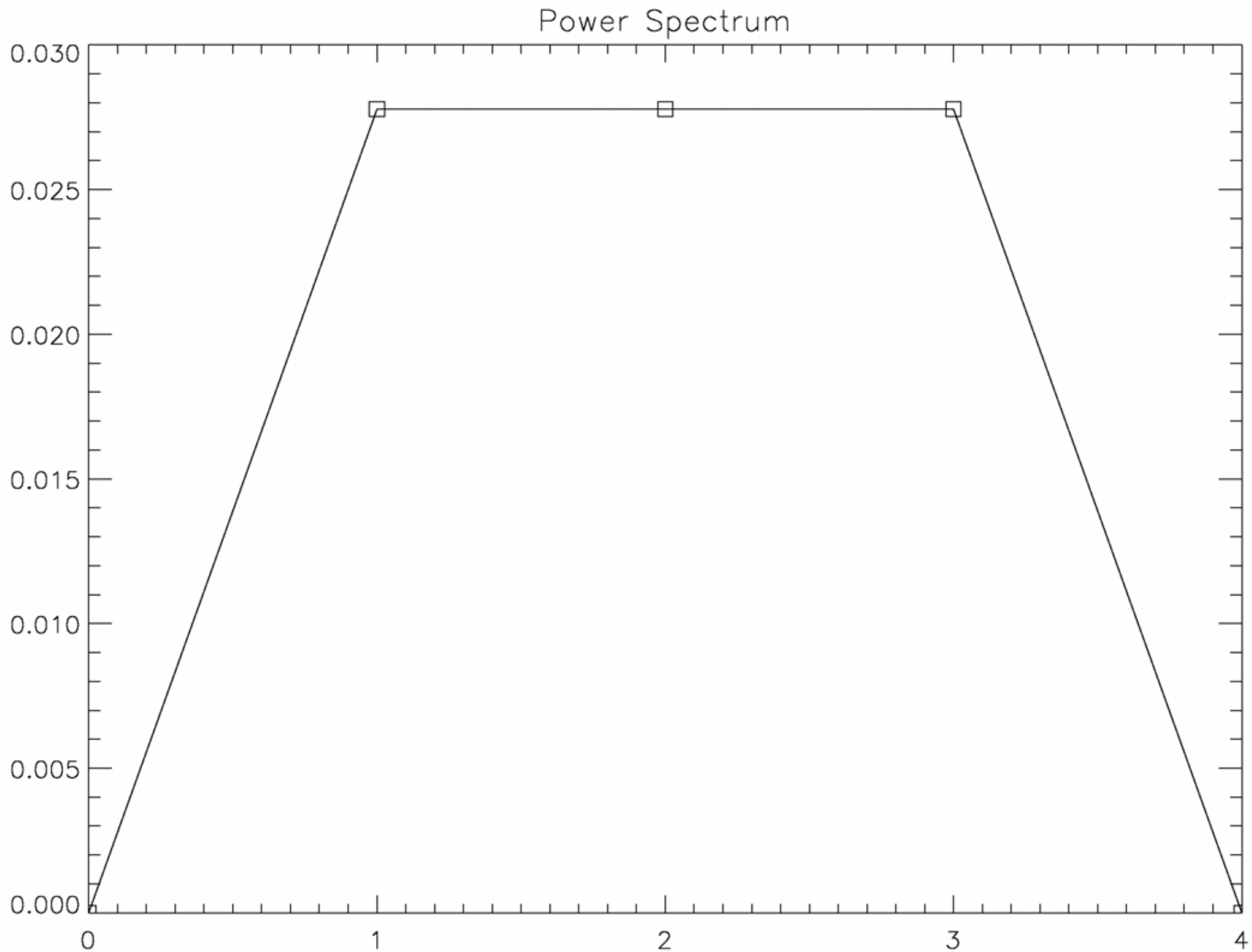


Take the Fourier (DFT) of that Segment



All the fringe power is in the first three frequency bins. We don't need the full Fourier Transform, just the values in these three bins.

We really have fewer pixels in each segment.



Most recent version of software:

http://dl.dropbox.com/u/13523232/2012_02_29_reduceir_32bit.tgz

http://dl.dropbox.com/u/13523232/2012_02_29_reduceir_64bit.tgz

http://dl.dropbox.com/u/13523232/2012_02_29_reduceir_MacOSX.tgz

```
usage: redclimb [-flags] ir_datafile
Flags:
-a                 Toggle apodize for FFT (OFF)
-A                 Use only shutter sequence A (OFF)
-b                 Toggle redo background and beams (OFF)
-B                 Use only shutter sequence B (OFF)
-c                 Toggle pixels must agree (ON)
-C                 Toggle confirm (ON)
-d[0,1,2]          Set display level(1)
-D[Dir]            Directory for results (Basename)
-e                 Toggle edit scans (ON)
-E[weight]         Use fringe weight to edit for AMP (OFF)
-f[width_frac]     Envelope width fraction (0.35)
-F                 Toggle filtering of signal (ON)
-g[0-1]            Fraction of Gaussian to include in CP (0.1)
-G                 Toggle using AMP editing for CP (ON)
-h                 Print this message
-i                 Toggle manual integration range (OFF)
-I[12-12,23-23,31-31]   Set integration range
-m                 Toggle save means as text (ON)
-M                 Toggle manual data selection in scan (OFF)
-n                 Toggle use noise instead of signal (OFF)
-Nndata            Force data segment size (AUTOMATIC)
-o                 Toggle skip overlap test (OFF)
-p                 Toggle plot closure phases (ON)
-P[smooth_noise_size]   Change noise PS +-smooth size (4)
-r                 Toggle save raw data as text (OFF)
-R                 Toggle remote mode (OFF)
-s[n]              Scans to skip after shutter change (0)
-S[smooth_size]    Change +-smooth size (1)
-t[start,stop]     Truncate scans (OFF)
-u                 Toggle use dither freqs for fringes (OFF)
-U[freq]           Set DC suppression frequency (10.0 Hz)
-v                 Toggle verbose mode (OFF)
-w[weight]         Set minimum fringe weight for CP (1.0)
-W[width]          Set PS peak width for fit (10.0 Hz)
-z[pixmult]        Set pixel multiplier (2)
-#[start,stop]     Set range of scan to include (ALL)
```

Brackets show show alternatives for argument and should not be included.

No space between flag and argument.

Defaults shown here.

Verbose mode is helpful to learn what is going on, though is *very* verbose.

```
{di-centos:1078} calibir -h
usage: calibir [-flags] {OBJ CAL1,CAL2,CAL3...}
Flags:
-b[beta]                Set intensity ratio (Cal/Obj) (1.0)
-B[Vis Type]            Set Vis estimator for Object and Calibrator (V_LOGNORM)
-c                      Use CHARA number for identifier (OFF)
-C[Cal Vis Type]        Set Vis estimator for Calibrator (V_LOGNORM)
-d                      Use change in calibrator for error (OFF)
-f[oif]                 Set OIFITS filename (From object)
-F                      Toggle saving OIFITS file (OFF)
-h                      Print this message
-H                      Use HD number for identifier (OFF)
-i                      Use ID/Name for identifier (OFF)
-I[12|23|31]            Select CLIMB baseline (None)
-n                      Use standard error of mean for error (OFF)
-O[Obj Vis Type]        Set Vis estimator for Object (V_LOGNORM)
-r                      Print raw data (ON)
-s[diam1,diam2,...]     Size of calibrators in mas (0.0)
-S                      Self Calibrate mode (OFF)
-v                      Verbose mode (OFF)

Visibility estimators available are:
V_CMB V_FIT V_ENV V_MEAN_ENV_PEAK
V_MEAN_ENV_FIT BINARY_V_A BINARY_V_B BINARY_ENV_V_A
BINARY_ENV_V_B V2_SCANS V_SCANS V_NORM
V_LOGNORM
```