Improving Visibility Estimation in Two-Beam Interferometers

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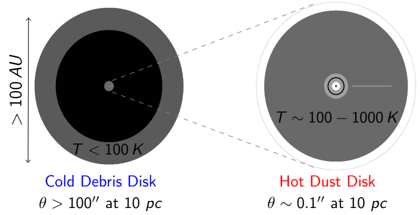
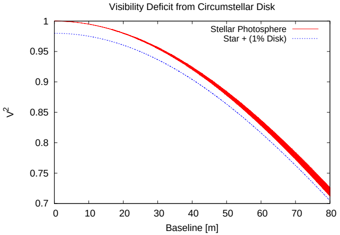
Motivation: Hot Circumstellar Dust Detection

- Must be understood to ensure success of future direct imaging missions
- Can be used to learn about planet formation & dynamical state





- Hot dust is detected in a $\sim 25\%$ fraction of main sequence stars. (Absil et al. 2013)
- Generally a $\sim 1\%$ deficit in visibility. Can benefit from data analysis improvements

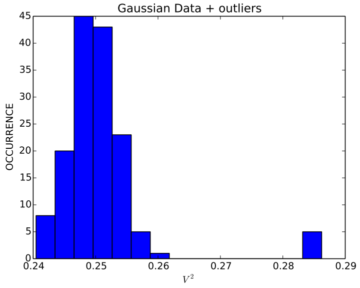
Visibility Estimators

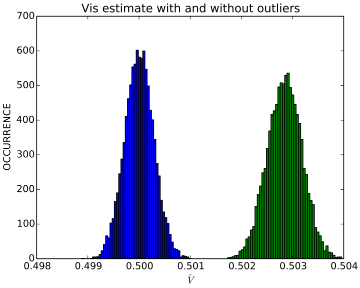
Problem: estimate uncalibrated V from a sample of measured V^2 scans.

- The sample mean is a biased estimator

$$\langle V^2 \rangle = \langle V \rangle^2 + \sigma^2$$

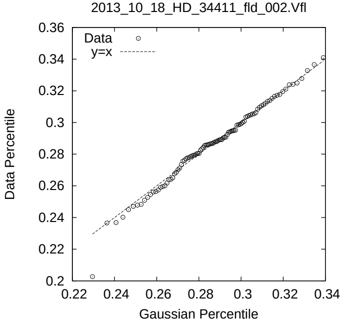
  Also very sensitive to outliers
  Example: 5 outliers at $10\sigma \Rightarrow \sim \%0.5$ bias in visibility

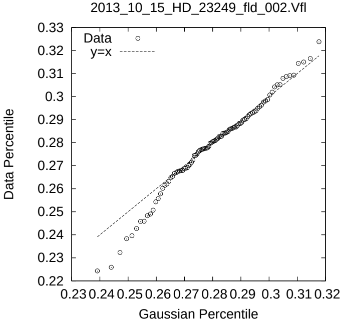




Unbiased Visibility Estimators

- The FLUOR/CLASSIC pipeline offers several unbiased estimators, e.g. V_NORM, V_SQRT, V_LOGNORM
- All of the current estimators assume that the measured Visibility follows Gaussian statistics (?)





In general, data are not Gauss distributed

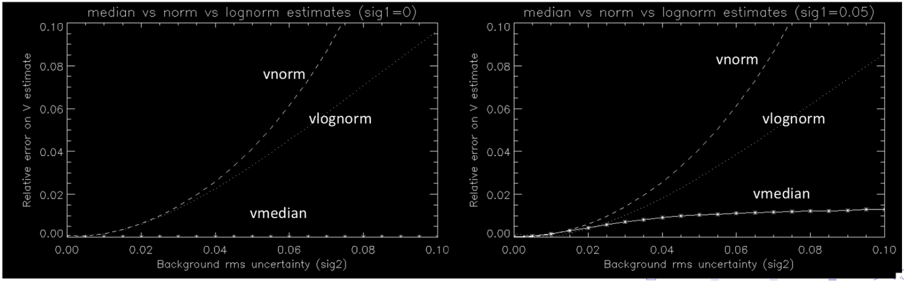
The Median as an unbiased estimator

For each fringe scan we measure

$$V^2 = (V_{true} + N_1)^2 + N_2,$$

where N_1 and N_2 are noise sources (e.g. piston and photon noise).

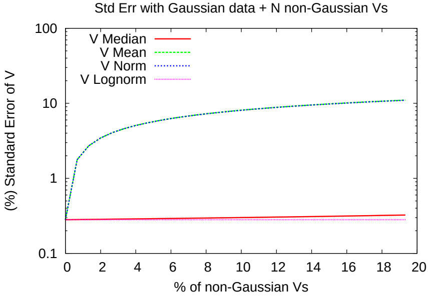
- If $Med(N_1) = N_2 = 0 \Rightarrow \boxed{\sqrt{Med(V^2)} = Med(V_{true})}$ $(Med(V_{true}) = V_{true})$
- In general, $V_{median} = \sqrt{Med(V^2)}$ has a smaller bias than other estimators



Standard Error of the Median

Bootstrapping:

- From the sample of N V^2s, draw N random V^2s to create a bootstrap sample
- Calculate $\sqrt{Med(V^2)}$ for the bootstrap sample and repeat many times
- Find the 84^{th} and 16^{th} percentiles of the ensemble of V_{median}
- Note: No assumption is made on the statistics of V



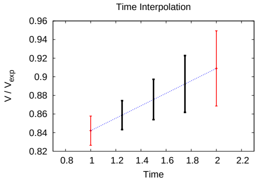
Visibility Calibration

Problem: to estimate V from V_{obs}, and calibrators V_1, and V_2.

$$V = V_{obs} \left\{ \alpha_1 \frac{V_1}{V_{1_{exp}}} + \alpha_2 \frac{V_2}{V_{2_{exp}}} \right\} \frac{1}{\alpha_1 + \alpha_2}$$

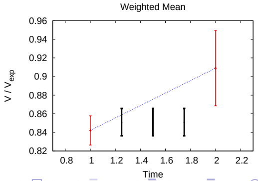
Time Interpolation

$$\alpha_1(t) = \left(\frac{t_2 - t}{t_1 - t_2} \right); \qquad \alpha_2(t) = \left(\frac{t - t_1}{t_1 - t_2} \right)$$



Weighted Mean

$$\alpha_1 = \frac{1}{\sigma_1^2}; \qquad \alpha_2 = \frac{1}{\sigma_2^2}$$

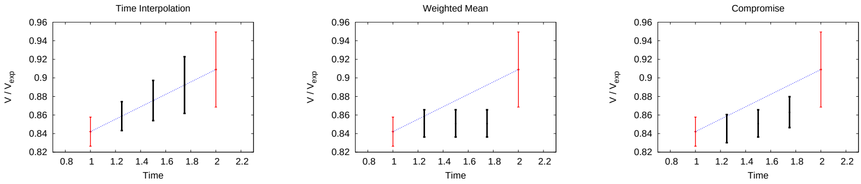


Visibility Calibration: a compromise

$$\mathcal{P}(T, t_1) \sim \exp \underbrace{\left(\frac{-(T - T_1)^2}{2\sigma_1^2} \right)}_{\chi_1^2}; \qquad \mathcal{P}(T, t_2) \sim \exp \underbrace{\left(\frac{-(T - T_2)^2}{2\sigma_2^2} \right)}_{\chi_2^2}$$

$$\mathcal{P}(T, t) \sim \exp \left\{ \left(\frac{t_2 - t}{t_1 - t_2} \right) \chi_1^2 + \left(\frac{t - t_1}{t_1 - t_2} \right) \chi_2^2 \right\}$$

$$\frac{\partial \chi^2}{\partial T_1} = \frac{\partial \chi^2}{\partial T_2} = 0 \Rightarrow \boxed{\alpha_1(t) = \frac{1}{\sigma_1^2} \left(\frac{t_2 - t}{t_1 - t_2} \right)} \quad \boxed{\alpha_2(t) = \frac{1}{\sigma_2^2} \left(\frac{t - t_1}{t_1 - t_2} \right)}$$



Final Visibility Error

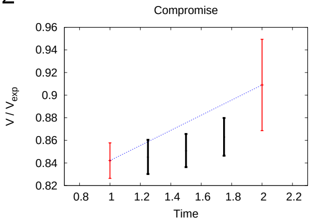
$$\sigma_V = \sqrt{\sigma_{stat}^2 + \sigma_{syst}^2}$$

Accounts for statistical variability of target and calibrators

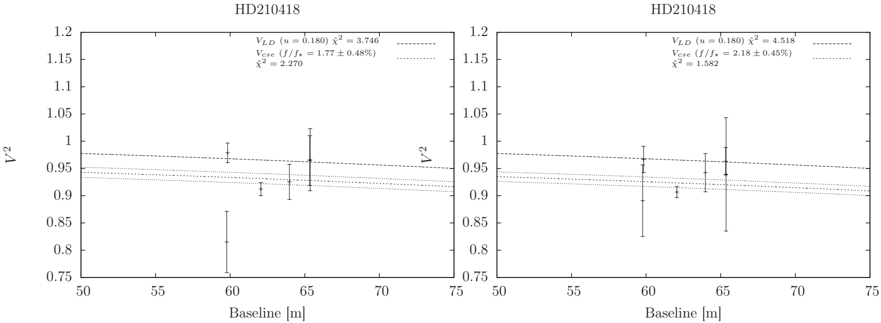
$$\sigma_{stat} = \sqrt{\left(\frac{\partial V}{\partial V_{obs}}\sigma_{obs}\right)^2 + \left(\frac{\partial V}{\partial T_1}\sigma_1\right)^2 + \left(\frac{\partial V}{\partial T_2}\sigma_2\right)^2}$$

Accounts for time variability of transfer function

$$\sigma_{syst} = \sqrt{\frac{1}{2}((V - V_{obs}T_1)^2 + (V - V_{obs}T_2)^2)}$$



Old Analysis vs. New Analysis



Final Remarks

Raw Visibilities:

- The Mean may not be the best visibility estimator
- The Median is less biased, more precise, and does not assume any statistical distribution of the data
- The computation time cost in bootstrapping is negligible

Calibrated Visibilities:

- A (time) linear interpolation of the transfer function does not properly account for their uncertainties
- A weighted mean does not account for temporal variability of the transfer function
- A compromise between the time interpolation and weighted mean is a promising alternative

More to come!