Complex closure amplitudes

Useful?
Effects of the atmosphere and instruments

\[ \nu_{12}^{\text{true}} = |\nu_{12}^{\text{true}}| e^{i\Phi_{12}^{\text{true}}} \]

\[ \nu_{12}^{\text{obs}} = |G_1||G_2| |\nu_{12}^{\text{true}}| e^{i(\Phi_{12}^{\text{true}} + \phi_2 - \phi_1)} \]
Closure phase

\[
\Phi_{12}^{obs} = \Phi_{12}^{true} + (\phi_2 - \phi_1)
\]

\[
\Phi_{23}^{obs} = \Phi_{23}^{true} + (\phi_3 - \phi_2)
\]

\[
\Phi_{31}^{obs} = \Phi_{31}^{true} + (\phi_1 - \phi_3)
\]

\[
CP_{123}^{obs} = \Phi_{12}^{obs} + \Phi_{23}^{obs} + \Phi_{31}^{obs}
\]
Closure phase

\[ \Phi_{12}^{\text{obs}} = \Phi_{12}^{\text{true}} + (\Phi_2 - \Phi_1) \]

\[ \Phi_{23}^{\text{obs}} = \Phi_{23}^{\text{true}} + (\Phi_3 - \Phi_2) \]

\[ \Phi_{31}^{\text{obs}} = \Phi_{31}^{\text{true}} + (\Phi_1 - \Phi_3) \]

\[ \text{CP}_{123}^{\text{obs}} = \Phi_{12}^{\text{obs}} + \Phi_{23}^{\text{obs}} + \Phi_{31}^{\text{obs}} \]

\[ = \Phi_{12}^{\text{true}} + \Phi_{23}^{\text{true}} + \Phi_{31}^{\text{true}} \]
Closure phase

\[ CP_{123}^{\text{obs}} = \Phi_{12}^{\text{obs}} + \Phi_{23}^{\text{obs}} + \Phi_{31}^{\text{obs}} \]

\[ \Phi_{12}^{\text{true}} + \Phi_{23}^{\text{true}} + \Phi_{31}^{\text{true}} \]

Independent Closure Phases \( \frac{(N-1)(N-2)}{2} \)

Fraction of phase information recovered \( \frac{(N-2)}{N} \)

3 Telescopes (CLIMB, PAVO) 33%
4 Telescopes (VLTI) 50%
6 Telescopes (MIRC) 67%
21 Telescopes (PFI) 90%
**Closure amplitude**

\[ CA_{1234} = \frac{\left| \nu_{13}^{\text{obs}} \ || \nu_{12}^{\text{obs}} \right|}{\nu_{13}^{\text{obs}}} = \frac{\left| C_1 \ || C_2 \ || \nu_{12}^{\text{true}} \ || \nu_{34}^{\text{true}} \right|}{C_1 \ || C_3 \ || C_2 \ || C_4} \]

Independent Closure Amplitudes

\[ \frac{N(N-3)}{2} \]

Fraction of amplitude information recovered

\[ \frac{(N-3)}{(N-1)} \]

- 2-3 Telescopes: none
- 4 Telescopes: 33%
- 6 Telescopes: 60%
- 21 Telescopes (PFI): 90%
Sources of amplitude variations

- Amplitude variations come from:
  - **ATMOSPHERE**
    - Fast atmosphere changes scintillation, strong in radio
      negligible in visible/IR
    - Slow atmosphere changes: “transfer function”
      - We use calibrators for $|V|^2$
      - Closure amplitudes do not need this calibration
  - **TELESCOPES/BEAM TRAIN**
    - conventional throughput losses
    - adaptive optics
  - **INSTRUMENTS**
    - spatial filtering, fiber injection
  - **BASELINES**
    - polarization
    - *Closure amplitudes are still affected by baseline-related
decorrelation effects*
Quirks of closure amplitudes

• Loosing zero-flux value → OK: unlike radio, we did not have it anyway
• Basic bias emerging from error propagation:

\[
|T4|_{1234}^{\text{biased}} = \left| \frac{\mathcal{V}_{12} \mathcal{V}_{34}}{\mathcal{V}_{14} \mathcal{V}_{23}} \right| \approx |T4|_{1234}^{\text{debiased}} \left( 1 + \frac{\sigma_{41}^2}{|\mathcal{V}|_{14}^2} + \frac{\sigma_{23}^2}{|\mathcal{V}|_{23}^2} \right)
\]

• Error on closure amplitude

\[
\sigma_{|T4|}^2 \approx |T4|_{1234}^{-2} \left( \frac{\sigma_{12}^2}{|\mathcal{V}|_{12}^2} + \frac{\sigma_{34}^2}{|\mathcal{V}|_{34}^2} + \frac{\sigma_{14}^2}{|\mathcal{V}|_{14}^2} + \frac{\sigma_{23}^2}{|\mathcal{V}|_{23}^2} \right)
\]

• This is not even taking into account inherent bias from read-noise/Poisson
• Noise definitively not Gaussian-distributed
  • Division !
    • Though this is not unlike calibration with transfer function
  • Bad if denominator visibilities are low
    • Better compute inverse closure amplitude if higher SNR at numerator
Complex closure amplitude

Analogous to bispectrum $T_3$, one can form a hereby-called $T_4$

\[
T_{1234}^{obs} = \frac{\nu_{12}^{obs} \nu_{34}^{obs}}{\nu_{14}^{obs} \nu_{23}^{*obs}} = C A_{1234} e^{i Q P_{1234}}
\]

\[
T_{1234}^{obs} = \frac{|\nu_{true_{12}}||\nu_{true_{34}}|}{|\nu_{true_{14}}||\nu_{true_{23}}|} \frac{e^{i(\Phi_{12}^{true} + \phi_2 - \phi_1)} e^{i(\Phi_{34}^{true} + \phi_4 - \phi_3)}}{e^{i(\Phi_{14}^{true} + \phi_4 - \phi_1)} e^{-i(\Phi_{23}^{true} + \phi_3 - \phi_2)}}
\]

\[
= T_{1234}^{true}
\]

"Quad Closure Phase" \[QP_{1234} = \Phi_{12}^{true} + \Phi_{23}^{true} + \Phi_{34}^{true} + \Phi_{41}^{true}\]
Quad closure phases

• Quad phases are part of the kernel phase
  • free observables
  • they are only partially redundant with closure phase: they have different noise statistics
    • worse SNR, being made of 4 phases instead of 3
    • number of independent quad phases is the same as the number of independent closure phases
• Like closure phases, they measure assymetric flux
• Quad phases may be more independent of flux variations than closure phases
  • Provided closure amplitudes work
MIRC pipeline mod for T4: modulus (closure amp)

- Borrowing an idea from radio interferometry...
- Talk about kernel phase in masking

Calibrator: beta Lyrae

Calibrators: 
- calibrator
MIRC pipeline mod for T4: phase (quad phase)

calibrator

beta Lyrae

calibrator
Meanwhile in the closure phase world... higher SNR

Error bars ~ half that of quad phases
Work in progress

- Ground libraries: julia and C code for handling T4 (OIFITSlib)
- Image reconstruction using T4
  - In progress, simulations of T4 noise to improve debiasing from noise terms, similar to work by Gordon and Buscher (2012).
- Covariance matrix with closures phases
- First image from closure quantities only from good SNR data (but calibrated cphases)

Bet Lyr from closure amps + closure phases, no V2, no T3amp