Optical and Infrared Interferometry

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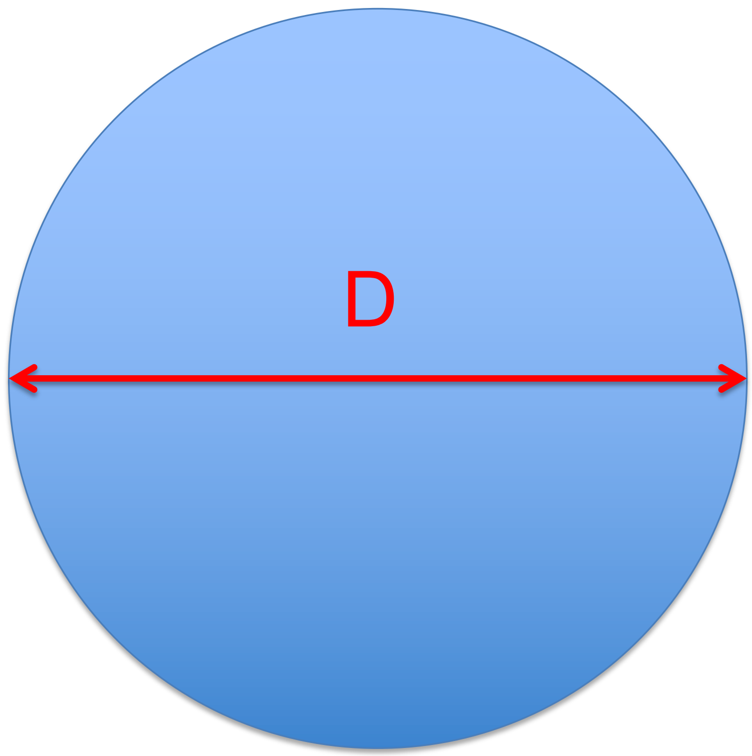
Outline

- General Principles of Interferometry
- Aperture Synthesis Imaging
- Spotlight on Optical Interferometry

- Readings
  - Volume 2 of Planets, Stars and Stellar System
  - Thompson/Moran/Swenson Chap 2-3, 10-11
  - Monnier Review 2003
  - Michelson Summer School 1999 Notes
    - https://core.ac.uk/download/pdf/79046071.pdf
  - Eisenhauer, Monnier, Pfuhl ARAA 2023

Angular Resolution



Diffraction-limit

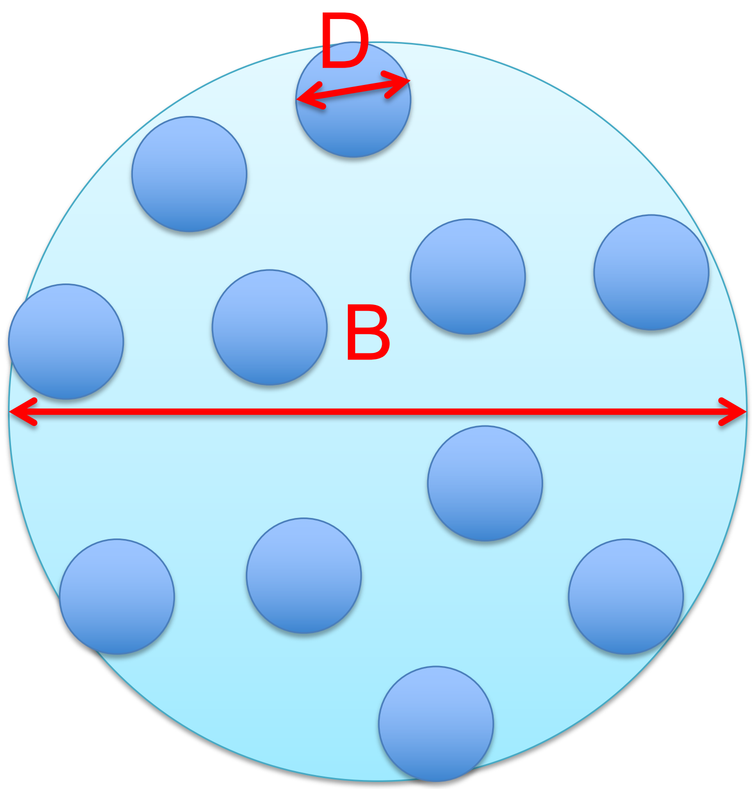
$$\Theta \sim \frac{\lambda}{D}$$

wavelength

Angular resolution in radians

diameter

Angular Resolution



Diffraction-limit

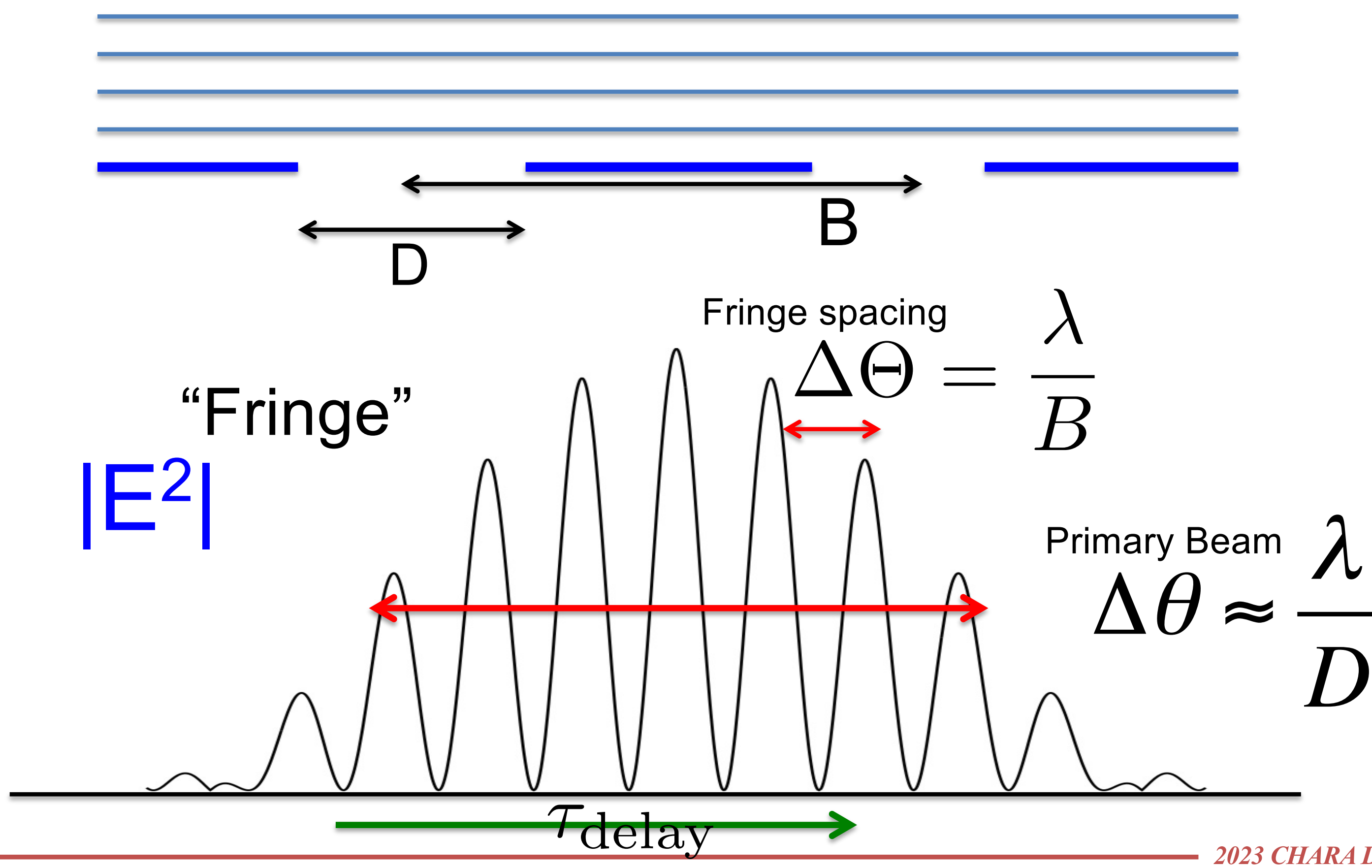
$$\Theta \sim \frac{\lambda}{B}$$

wavelength

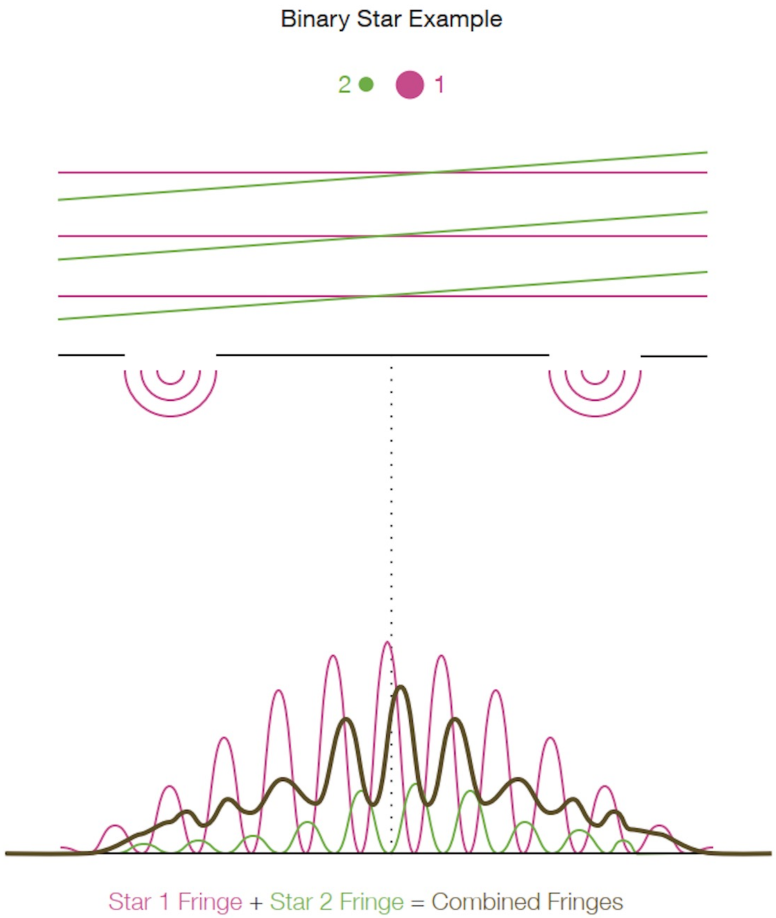
Angular resolution in radians

Baseoine

Optical View of Interferometry



van Cittert-Zernike Theorem



Eisenhauer, Monnier, Pfuhl 2023

Complex Visibility

$$\tilde{\mathcal{V}}\left(\frac{B_{\rm proj}}{\lambda}\right) = \int I_\lambda(\delta)\cos\left(2\pi\frac{B_{\rm proj}}{\lambda}\delta\right)d\delta$$

One Fourier Component of the Image

Basics

•The amplitude of fringe corresponds to Fourier amplitude of a single Fourier component of brightness distribution

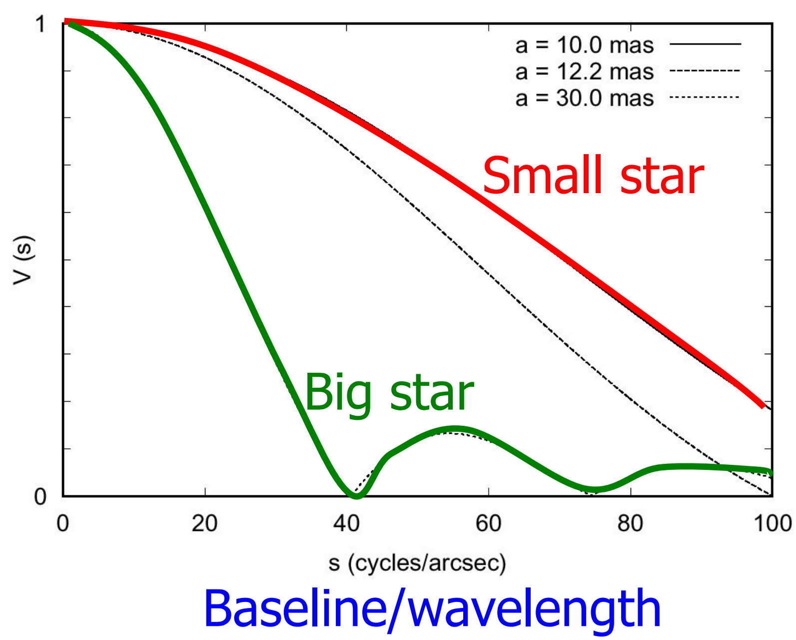
Normalized to flux, 0 → 1

For data analysis reasons, we often fit to **V^2**, not the amplitude

•The phase corresponds to the Fourier phase

•You need amplitudes & phases for imaging

Examples of Visibilities: Stars

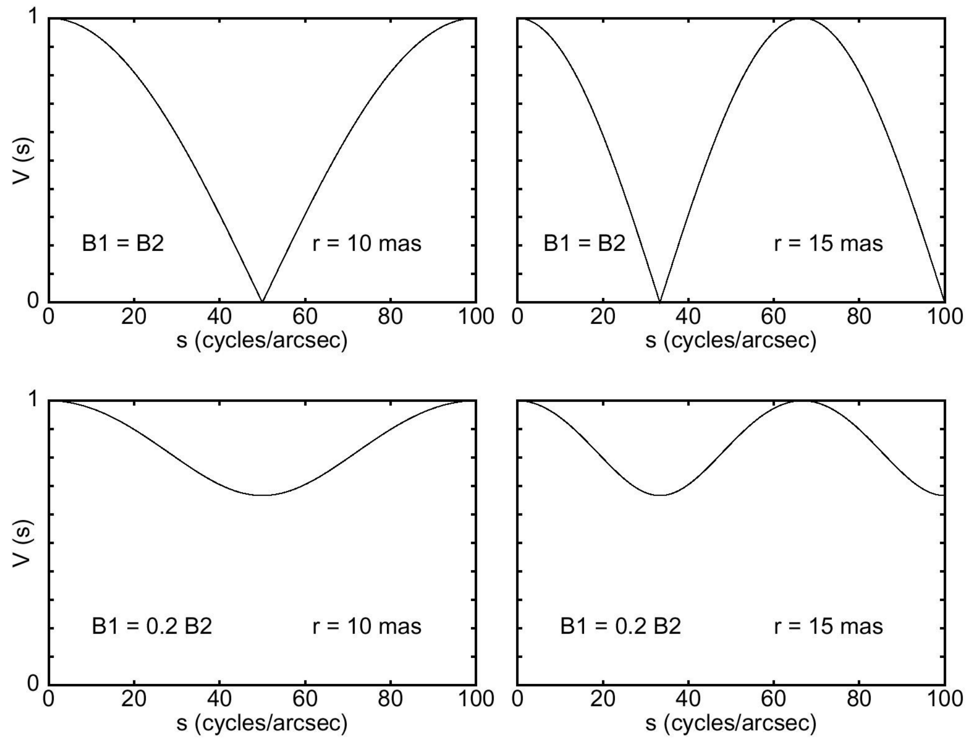


Uniform Disk:

$$V(s) = \left| \frac{2J_1(\pi a s)}{\pi a s} \right|$$

Dyck (Michelson Summer School Notes 2000)

Examples of Visibilities: Binaries



Equal Brightness

Brightness Ratio 5:1

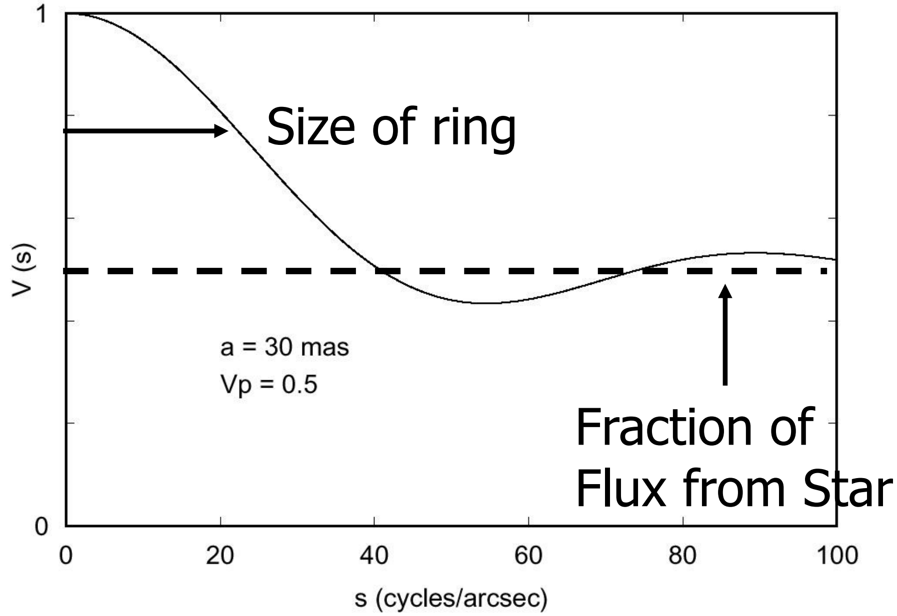
Increasing Binary Separation →

Dyck (Michelson Summer School Notes 2000)

Star + Dust Shell

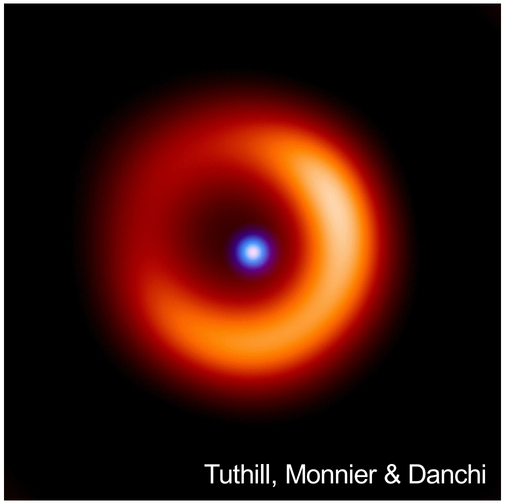
Fourier Transforms are linear, so you can add up components in complex visibility

Visibility function for Star + Ring



Baseline/wavelength

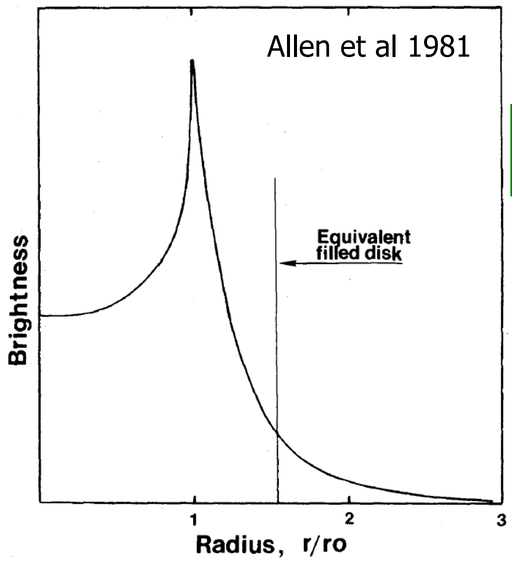
LkHα 101 (young star with disk)

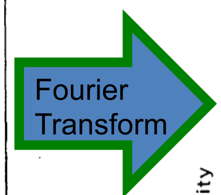


Dyck (Michelson Summer School Notes 2000)

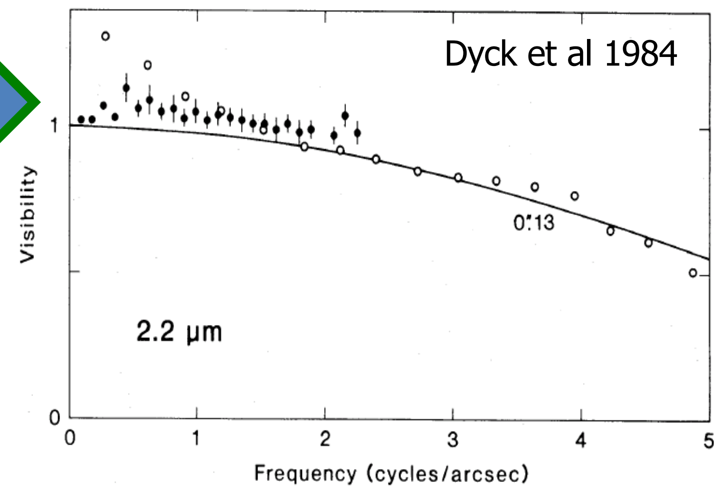
Radiative Transfer Modeling

Radial Profile of an Optically-Thin Dust Shell Model



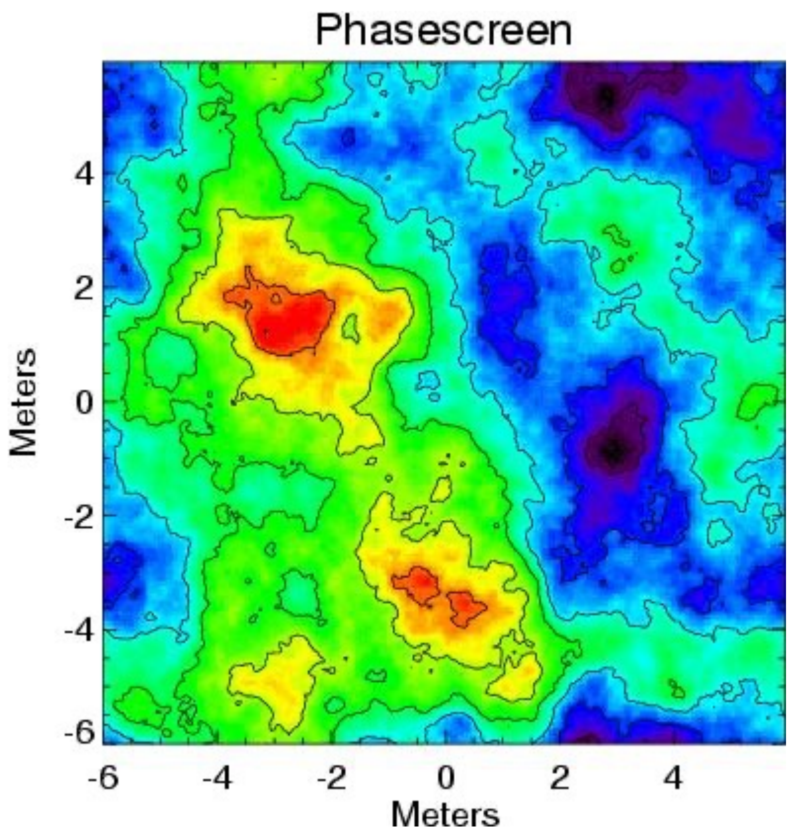


Calculated and Observed Visibilities



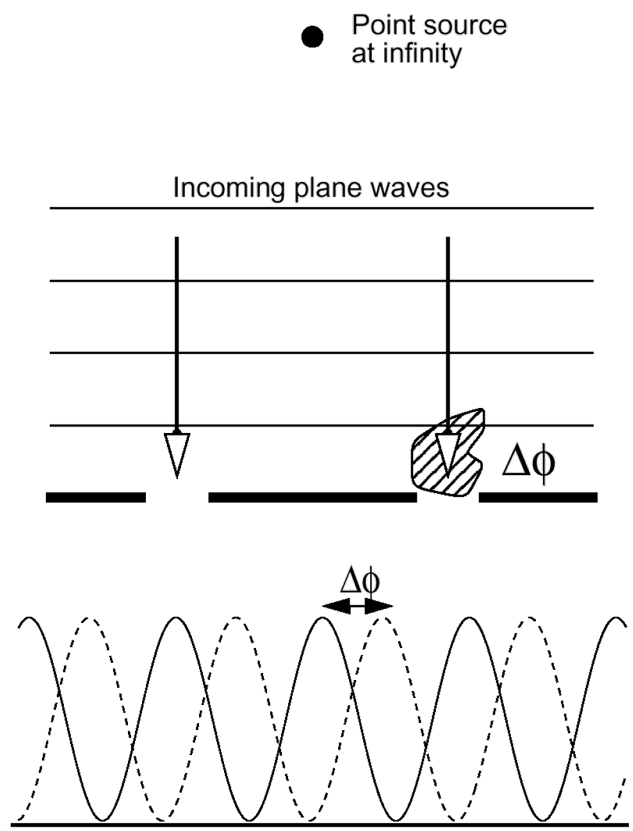
here the dust shell is 'barely' resolved and no details show up in the visibility curve

What about Phases? Atmosphere is bad.



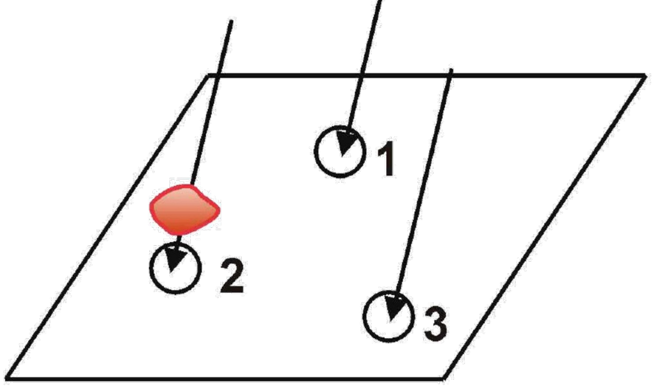
A Keck-sized patch of atmosphere during typical good seeing

Each contour is one radian of phase delay of 2-micron light



Monnier (Michelson Summer School Notes 2000)

The "Closure Phase" Is Not Corrupted



Observed — Intrinsic — Atmosphere

$$\Phi(1\text{-}2) = \Phi_o(1\text{-}2) + [\phi(2)-\phi(1)]$$
$$\Phi(2\text{-}3) = \Phi_o(2\text{-}3) + [\phi(3)-\phi(2)]$$
$$\Phi(3\text{-}1) = \Phi_o(3\text{-}1) + [\phi(1)-\phi(3)]$$

Closure Phase (1-2-3) $= \Phi_o(1\text{-}2)+\Phi_o(2\text{-}3)+\Phi_o(3\text{-}1)$

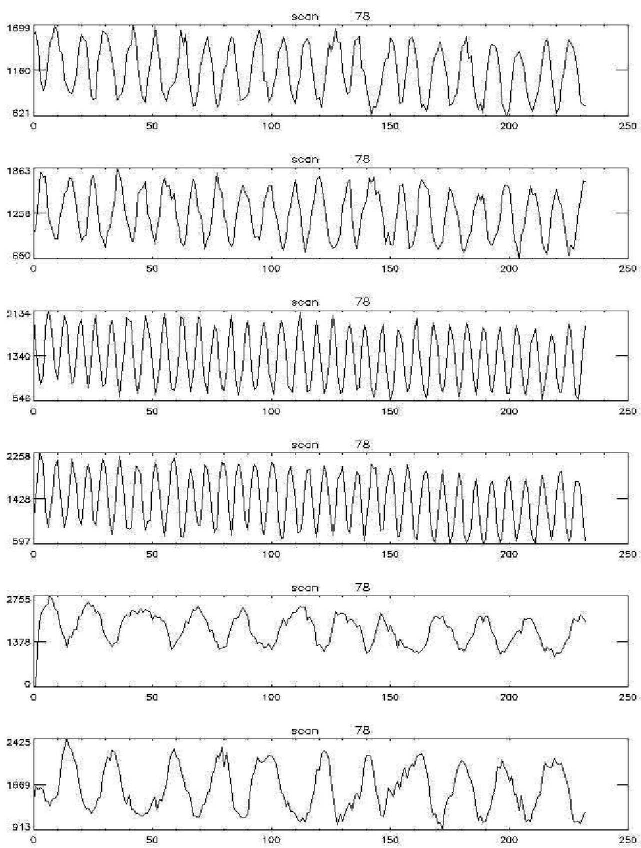
Monnier (Michelson Summer School Notes 2000)

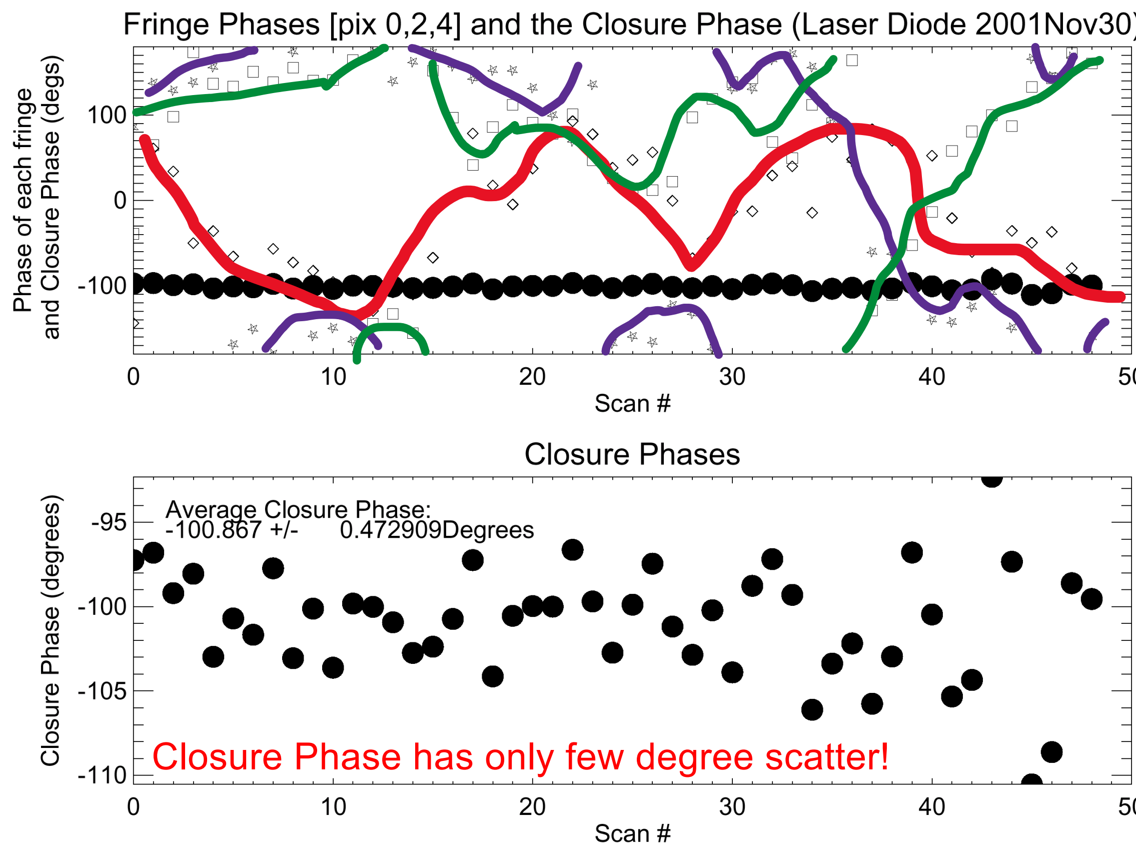
Properties of Closure Phases

- 0° /180° for "centro"-symmetric objects
  - E.g., uniform disks, rings, elliptical disks, equal binaries
  - But otherwise not easy to interpret
- Not dependent on position on sky, so therefore no astrometry
- Related to 'skewness' of a distribution
- Immune to most calibration errors, so very robust!

Closure Phase is a Good Observable

Pair-wise Combination at IOTA



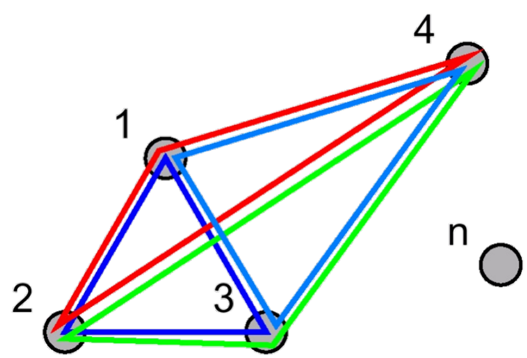


How Much Phase Information?

Closure Phases are not all independent from each other.

Number of Closure Phases

$$\binom{N}{3} = \frac{(N)(N-1)(N-2)}{(3)(2)},$$



Number of Fourier Phases

$$\binom{N}{2} = \frac{(N)(N-1)}{2}$$

Number of Independent Closure Phases

$$\binom{N-1}{2} = \frac{(N-1)(N-2)}{2}$$

Number of Telescopes	Number of Fourier Phases	Number of Closing Triangles	Number of Independent Closure Phases	Percentage of Phase Information
3	3	1	1	33%
7	21	35	15	71%
21	210	1330	190	90%
27	351	2925	325	93%
50	1225	19600	1176	96%

Monnier (Michelson Summer School Notes 2000)

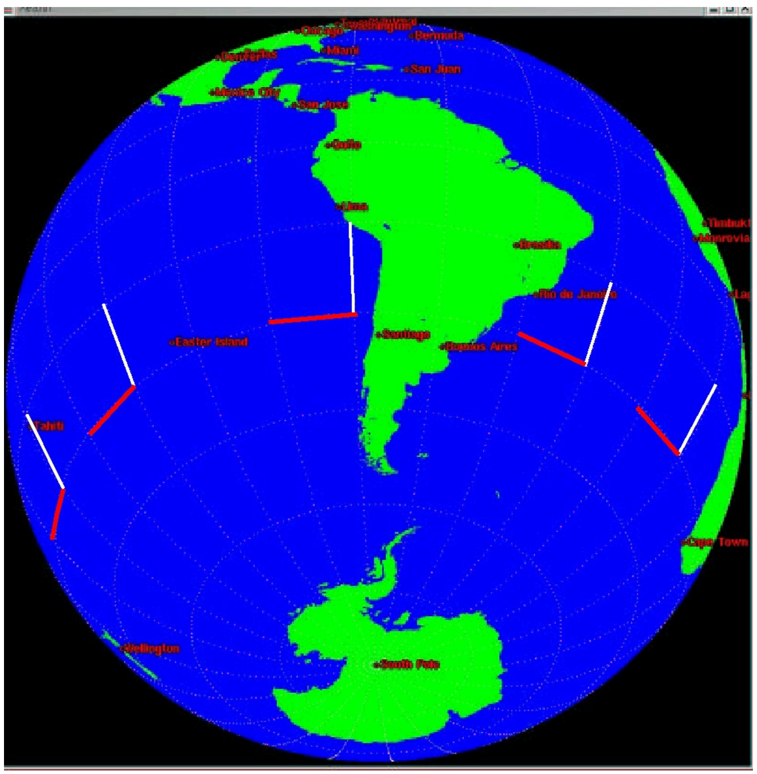
Basics of Interferometric Imaging

- How much data do you need?
  - The number of filled pixels ~> number of independent visibility measurements (degrees of freedom argument)
- What range of baselines?
  - Longest baselines sets your highest resolution
  - Diffraction-limit of individual telescope usually sets the maximum field-of-view of the interferometer
- Dynamic Range expected to be 1000:1 to 100:1
- How can you get enough data with only a few telescopes?

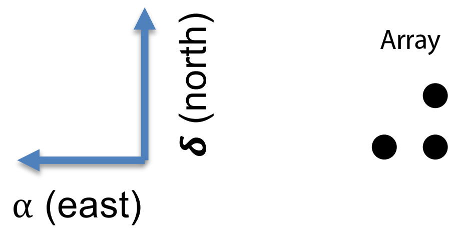
$$\mathrm{FT}(I(\alpha,\delta)) = \tilde{\mathcal{V}}(u,v) = \int I_\lambda(\alpha,\delta) e^{-2\pi i (\vec{u,v})\cdot(\vec{\alpha,\delta})} d\alpha d\delta$$

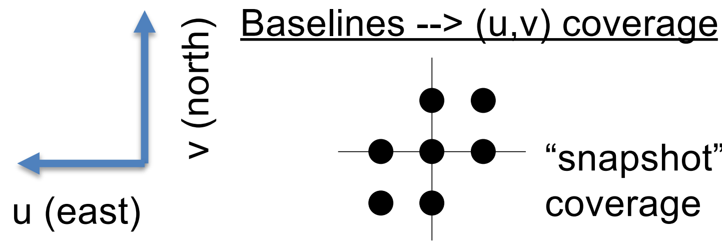
(u,v) coverage

Earth Rotation Aperture Synthesis

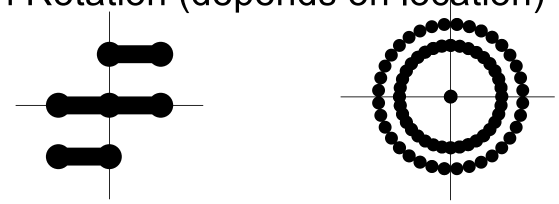


Telescope positions





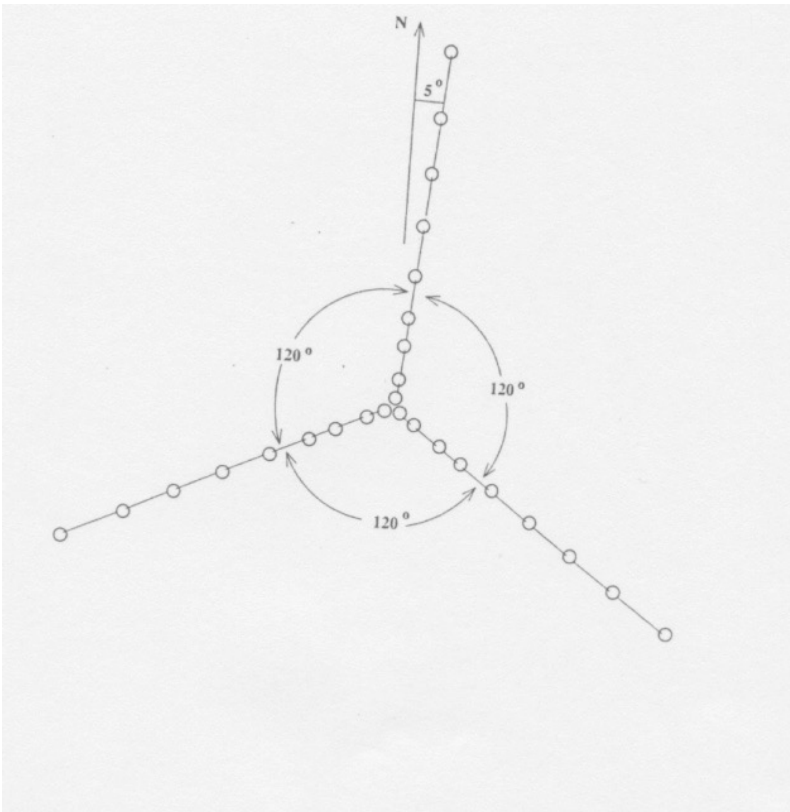
w/ Earth Rotation (depends on location)

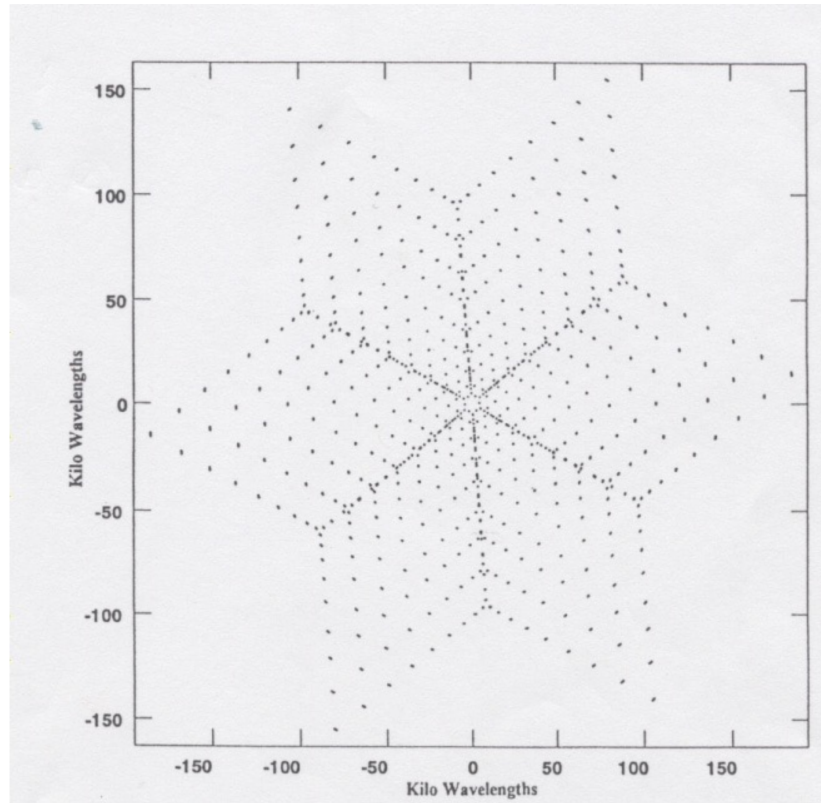


a) dec=0deg, lat=0deg
(12hr integration)

b) dec=90deg, lat=90deg
(24hr integration)
symbols are smaller to see better

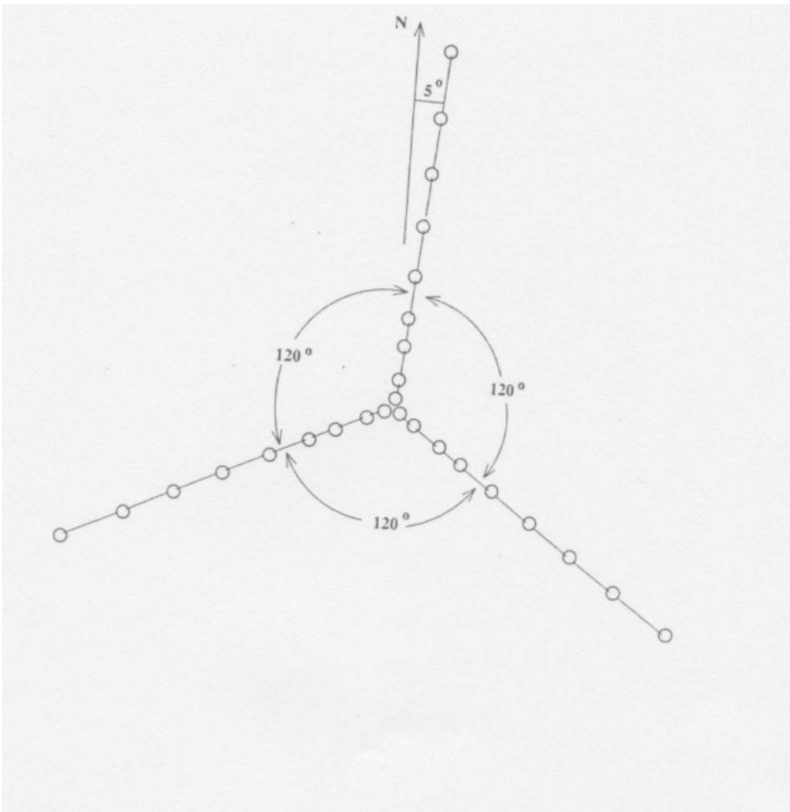
Very Large Array (VLA)

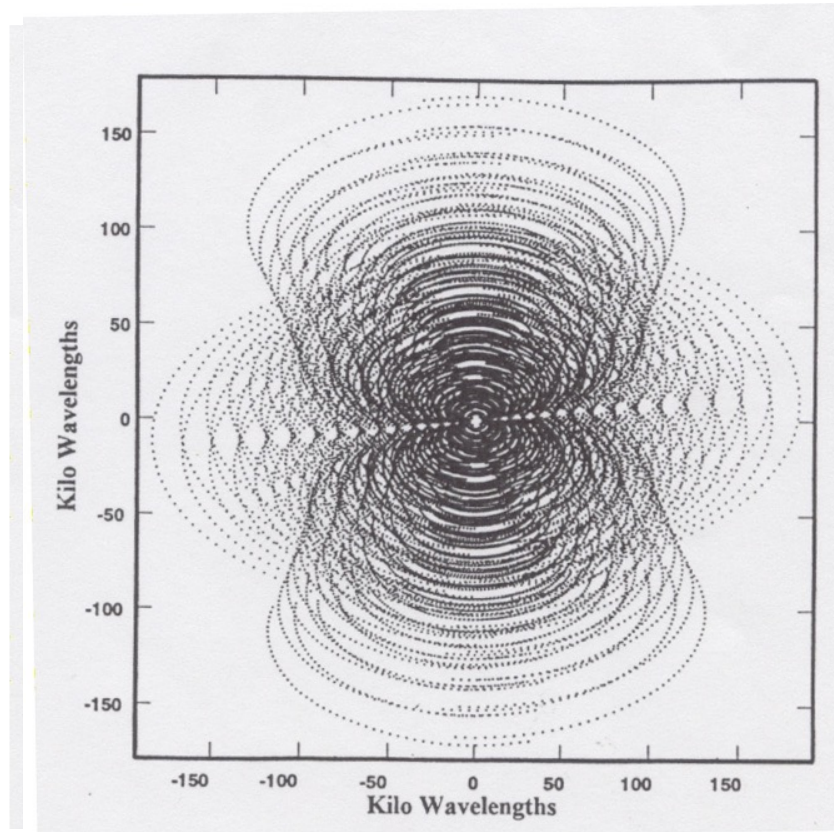




Thompson, Moran and Swenseon

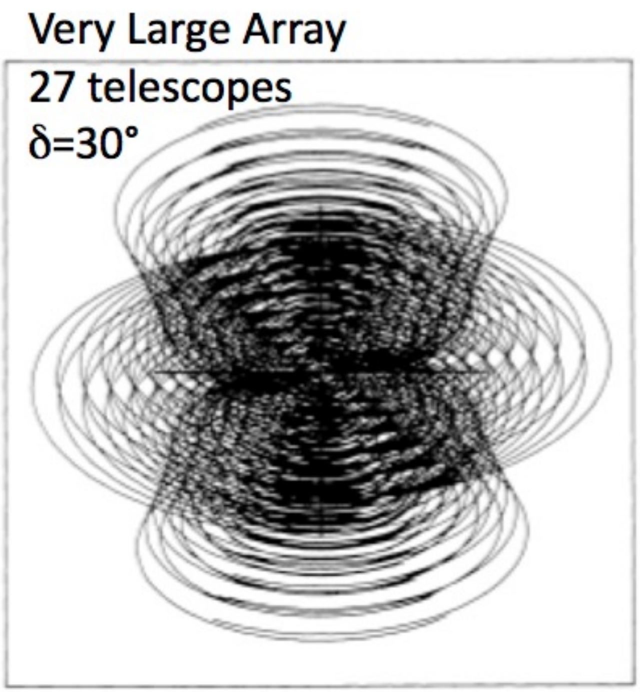
Very Large Array (VLA)

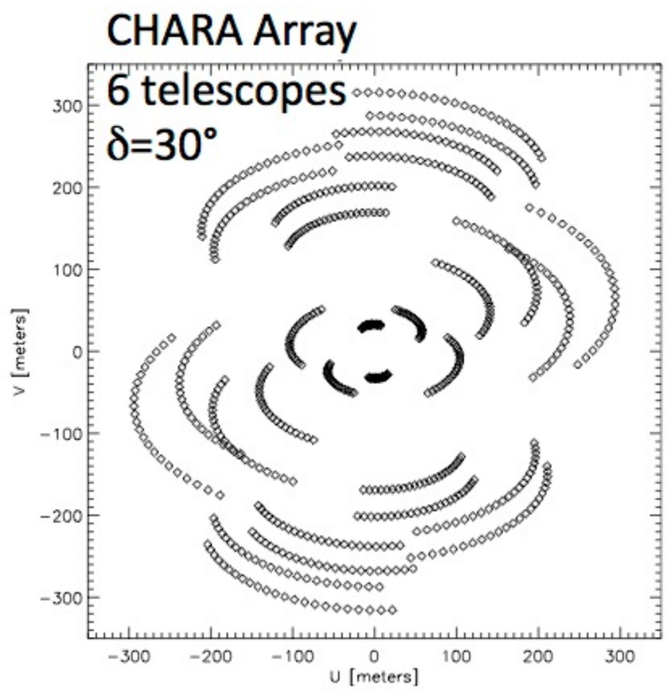




Thompson, Moran and Swenseon

UV coverage: VLA (27) vs CHARA (6)



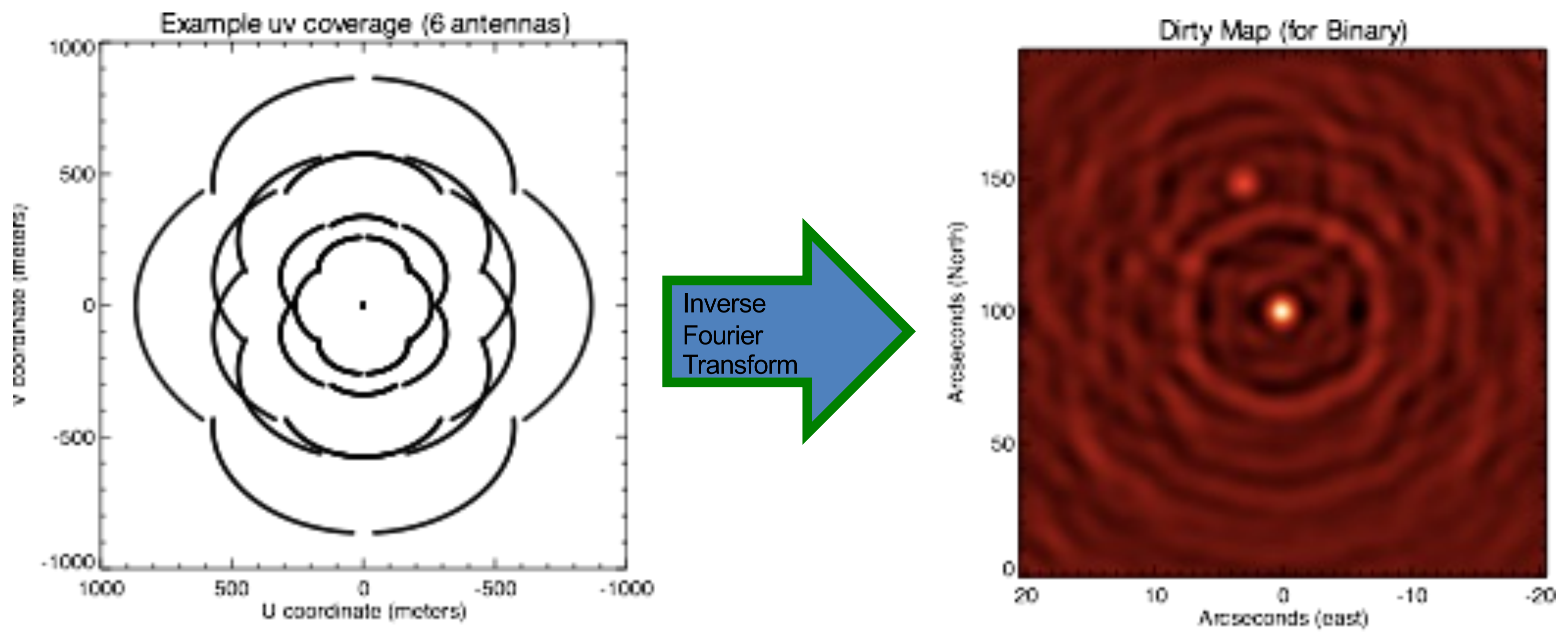


Deconvolution & Aperture Synthesis

- To reconstruct an image from sparsely sampled (u,v) data, one must interpolate into regions where data does not exist.
  - This is Identical to multiplying the true Complex Visibility by an Aperture Function.
- Since Multiplication in the (u,v) space is the same as Convolution in image space (see Convolution Theorem), the problem can be re-cast as a Deconvolution problem.
- Popular methods of Deconvolution include CLEAN and the Maximum Entropy Method.

Imaging Methods: Direct Fourier Transform

- Poor UV coverage leads to artifacts in your image

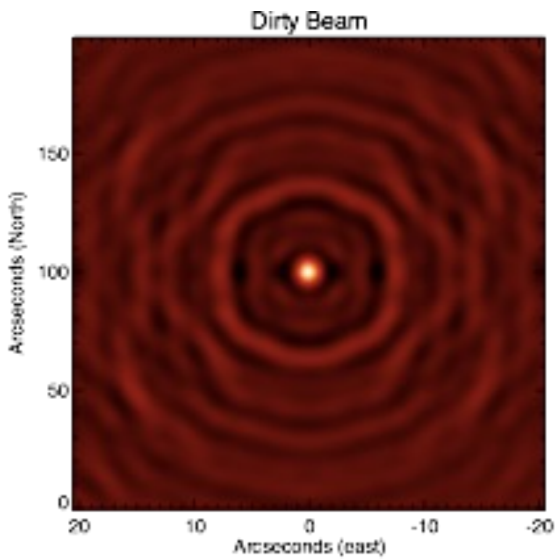


Monnier & Allen

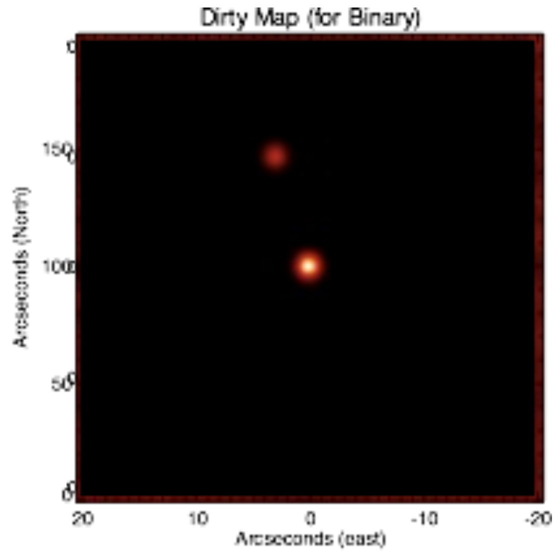
Deconvolution with CLEAN

*CLEAN requires phases, so we incorporate closure phases iteratively "self-calibration"

"Point Spread Function"
From known UV coverage



Deconvolution



Monnier & Allen

Forward Model Example: Maximum Entropy Method (MEM)

With finite (u,v) coverage and with noisy data, there are an infinite number of images which will fit the data.

So how do we choose?

- Find "smoothest" image consistent with data ($\chi^2 \sim 1$)
- MEM uses the "entropy" S to parameterize the "smoothness."
- MEM is one of many possible "regularizers," other include total variation, sparsity, 'UD' regularizer, etc.

Fraction of flux in pixel i

Skilling & Bryan (1984)

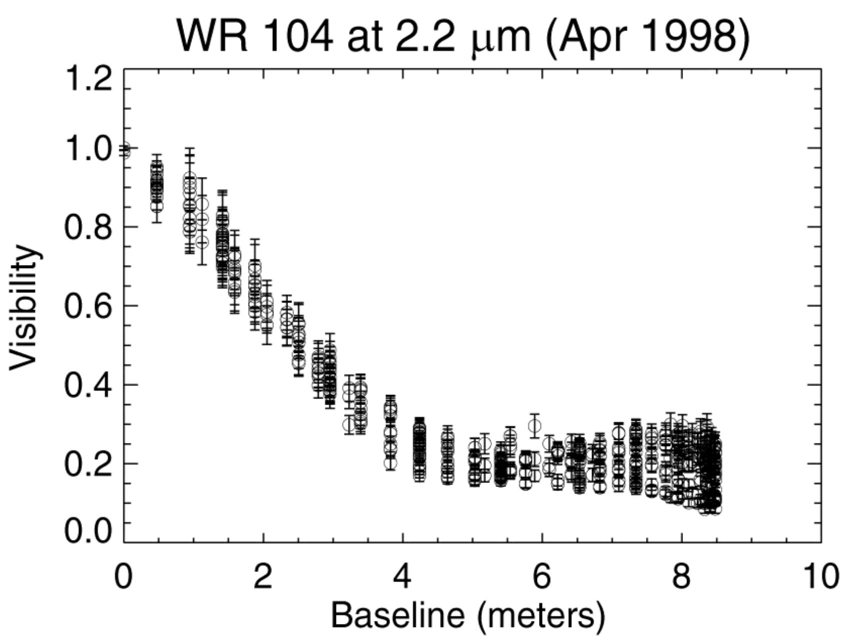
Entropy

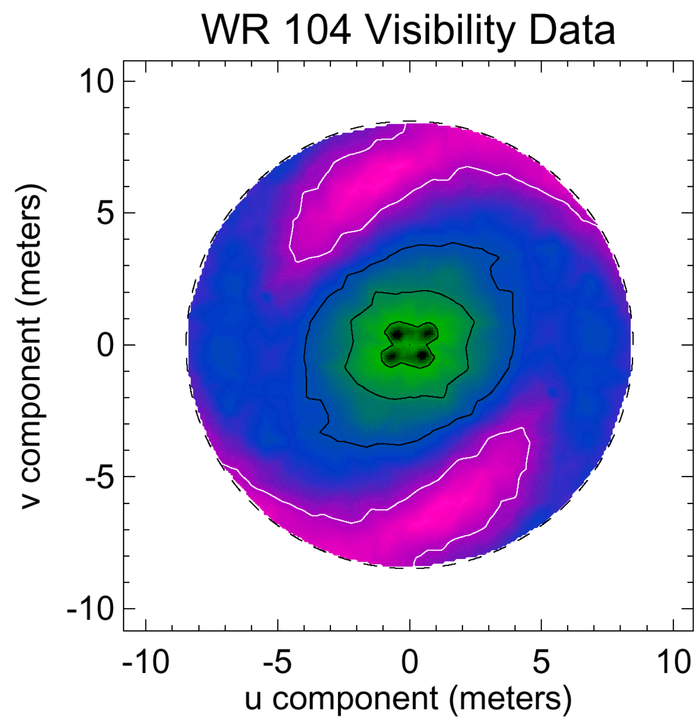
$$S = -\sum_i f_i \ln \frac{f_i}{I_i}$$

Image prior

Sum over all pixels

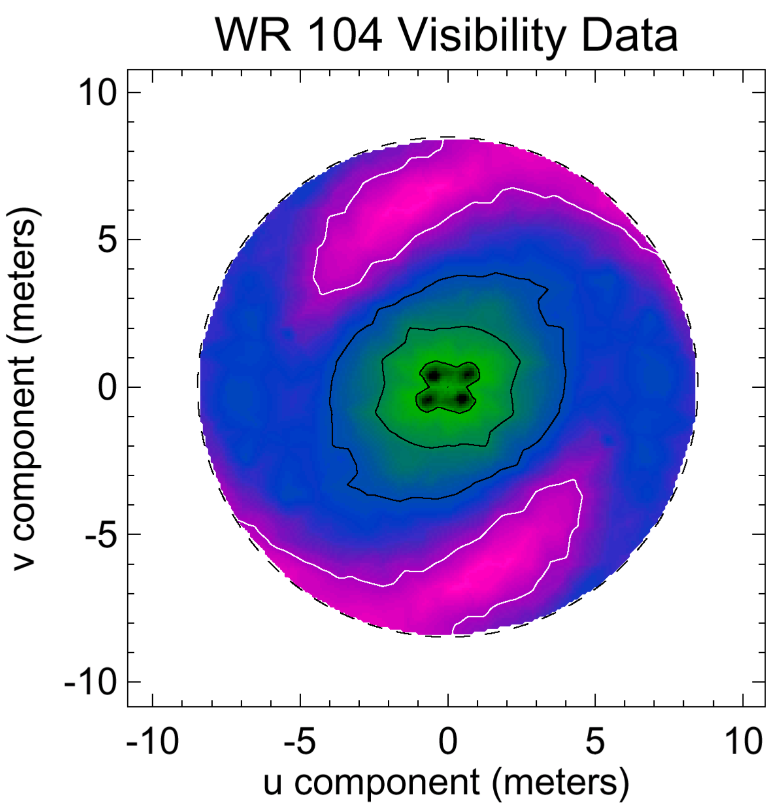
WR 104 Data


WR 104 at 2.2 μm (Apr 1998)
Visibility
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
10
Baseline (meters)

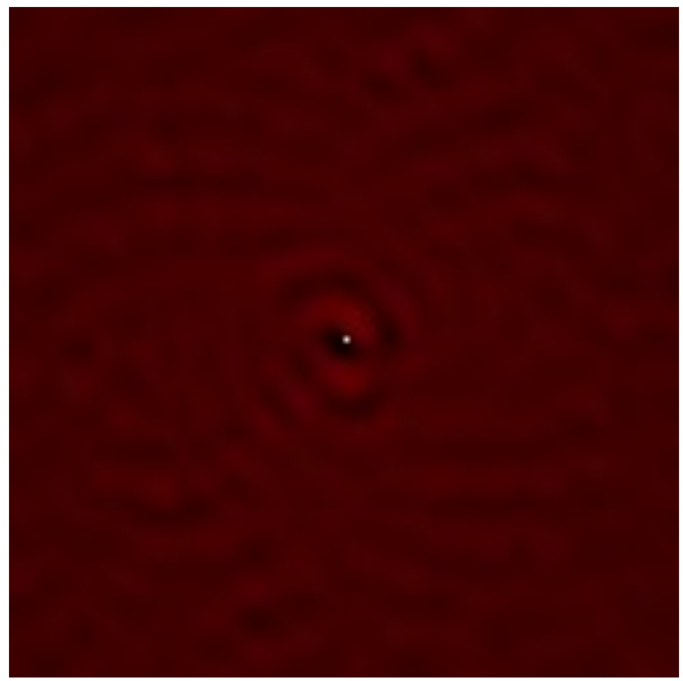

WR 104 Visibility Data
v component (meters)
10
5
0
-5
-10
-10
-5
0
5
10
u component (meters)

BSMEM developed first by Dave Buscher

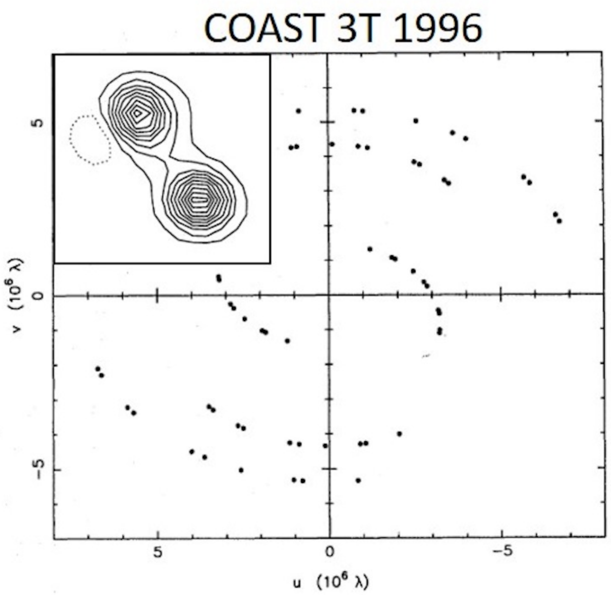
WR 104 MEM Reconstruction

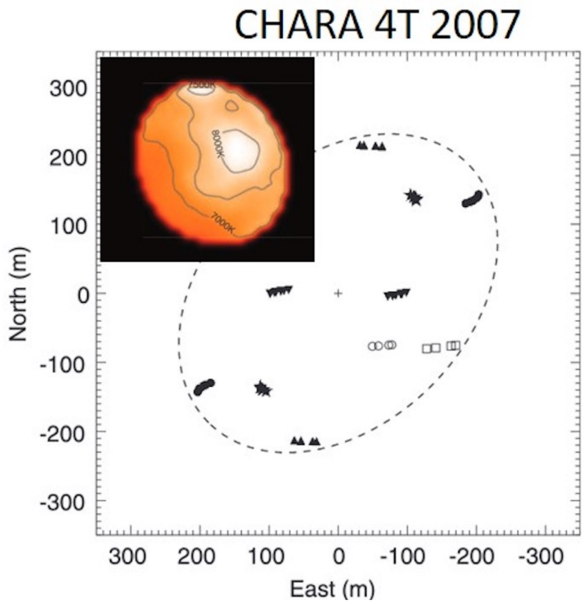


Iterations 1 to 30

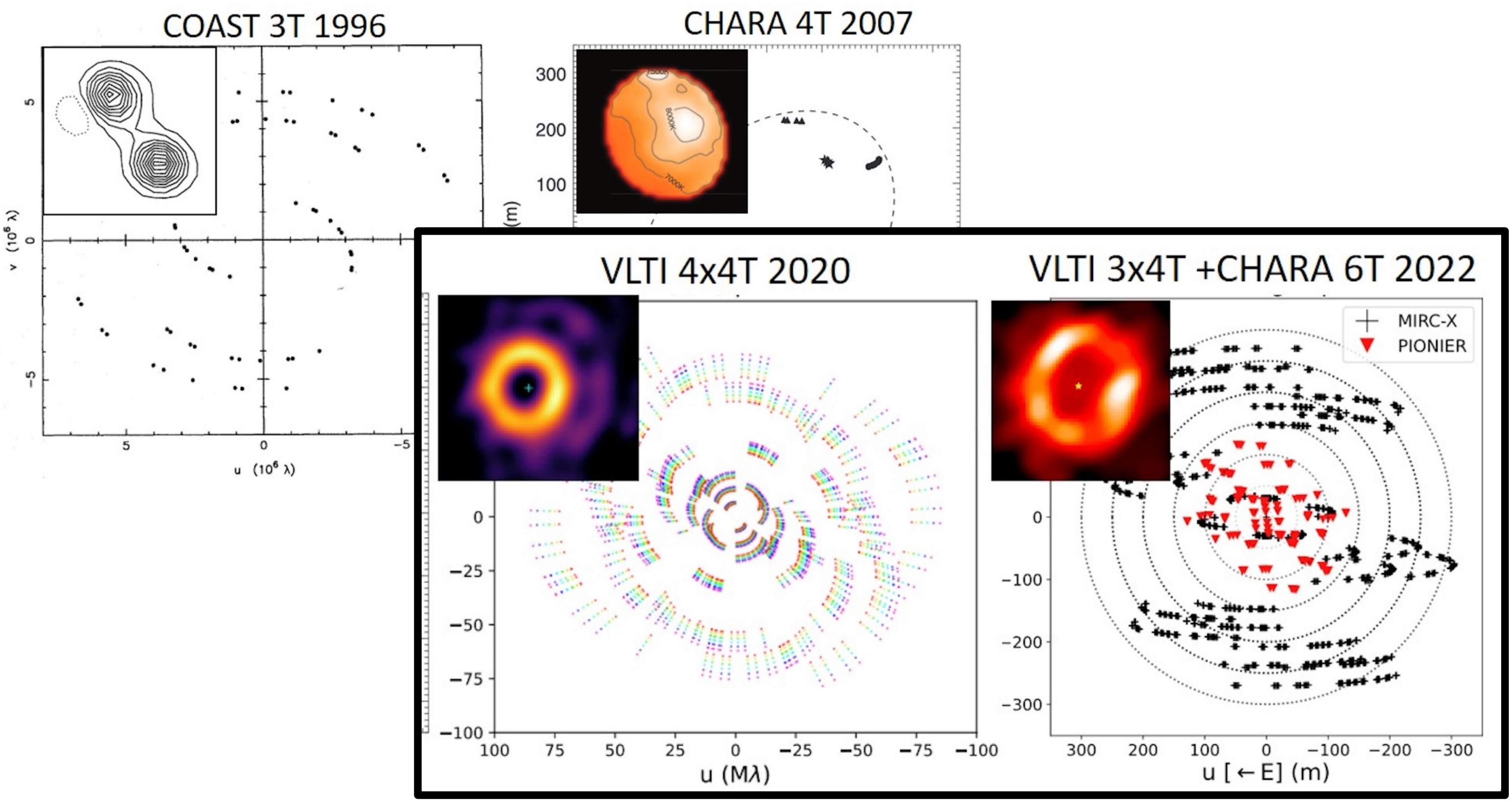


WR 104 (2.2 microns)





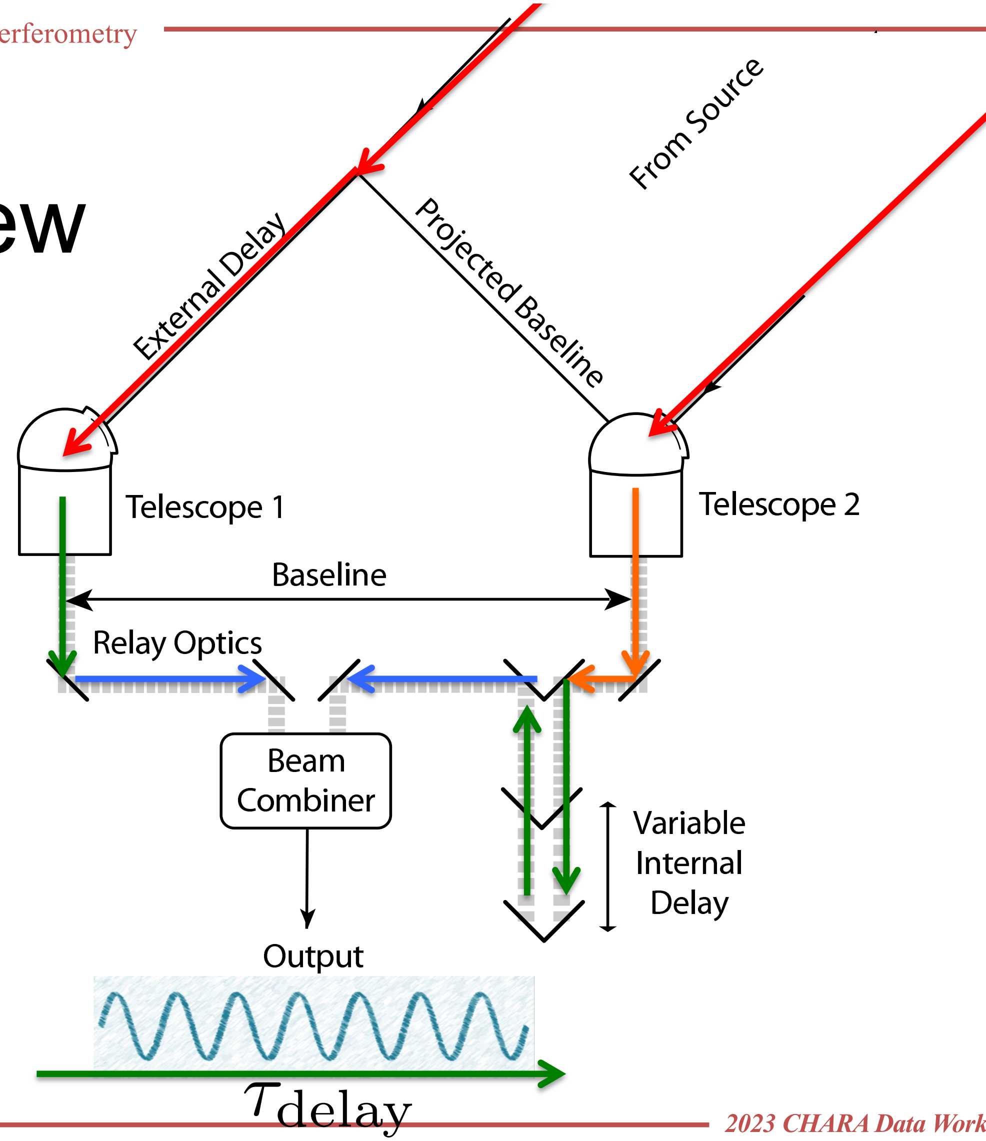
Eisenhauer, Monnier, Pfuhl 2023



Eisenhauer, Monnier, Pfuhl 2023

Spotlight:
Optical Interferometry

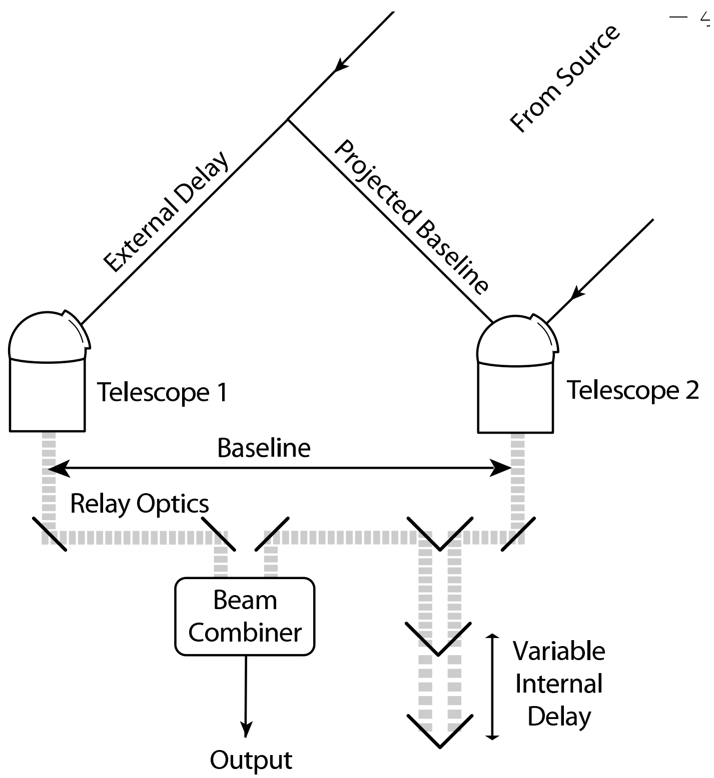
More Realistic View



Monnier & Allen 2012

Realistic Interferometers

Light Collectors



Monnier & Allen 2012

(a)

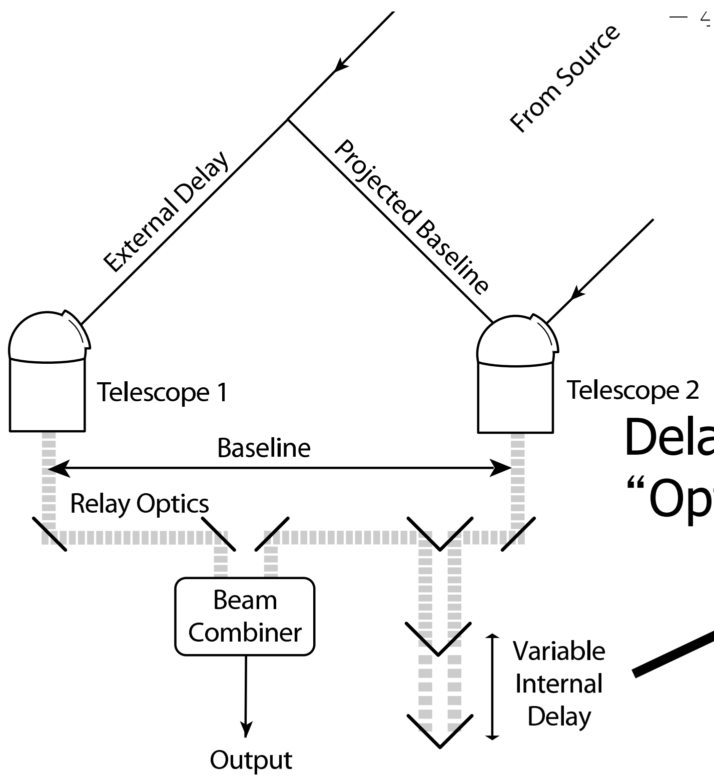
(b)

(c)

(d)

Some images from ten Brummelaar (Michelson Summer School 2000)

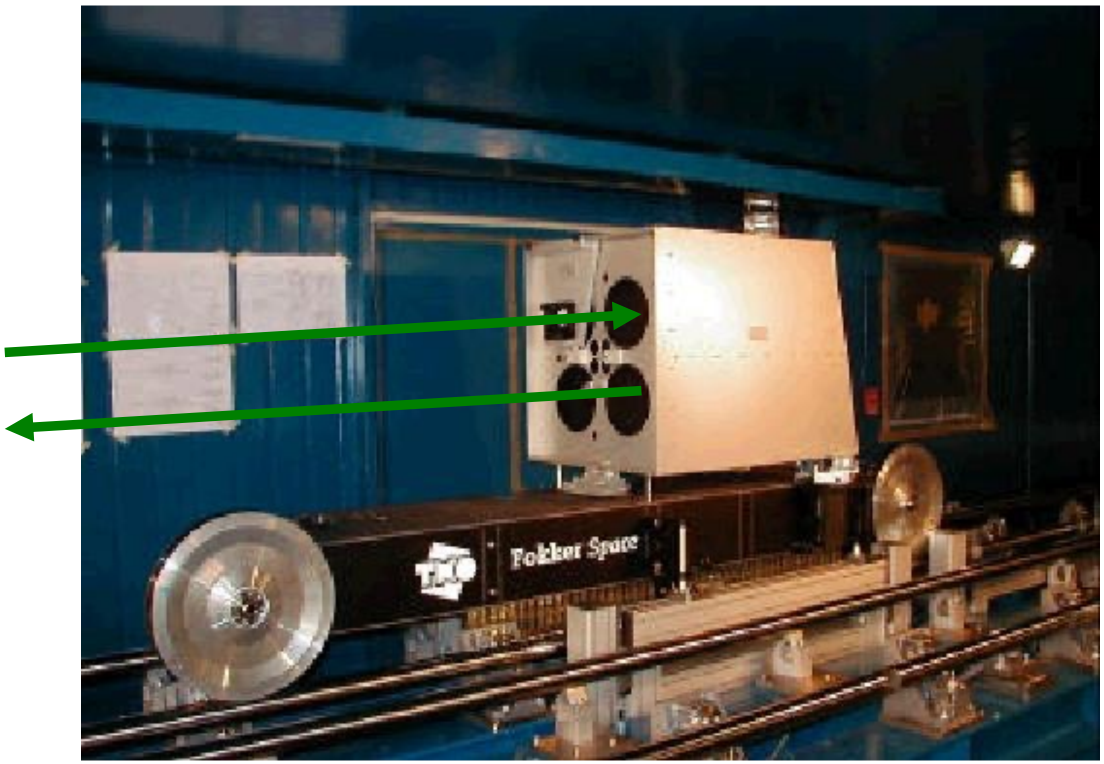
Realistic Interferometers



Monnier & Allen 2012

Delay Lines
"Optical Trombones"

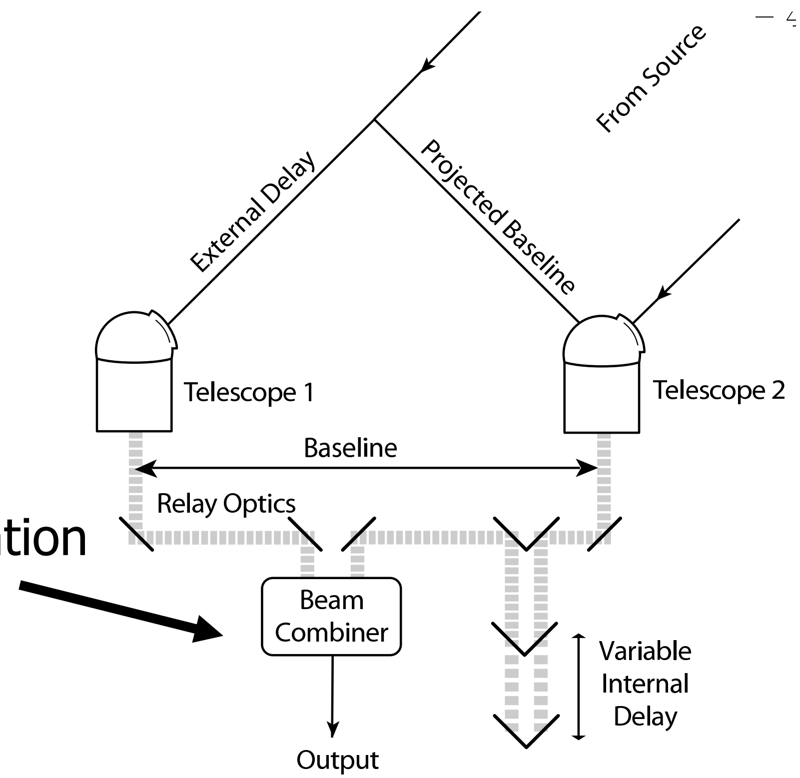
Delay Lines



Some images from ten Brummelaar (Michelson Summer School 2000)

Realistic Interferometers

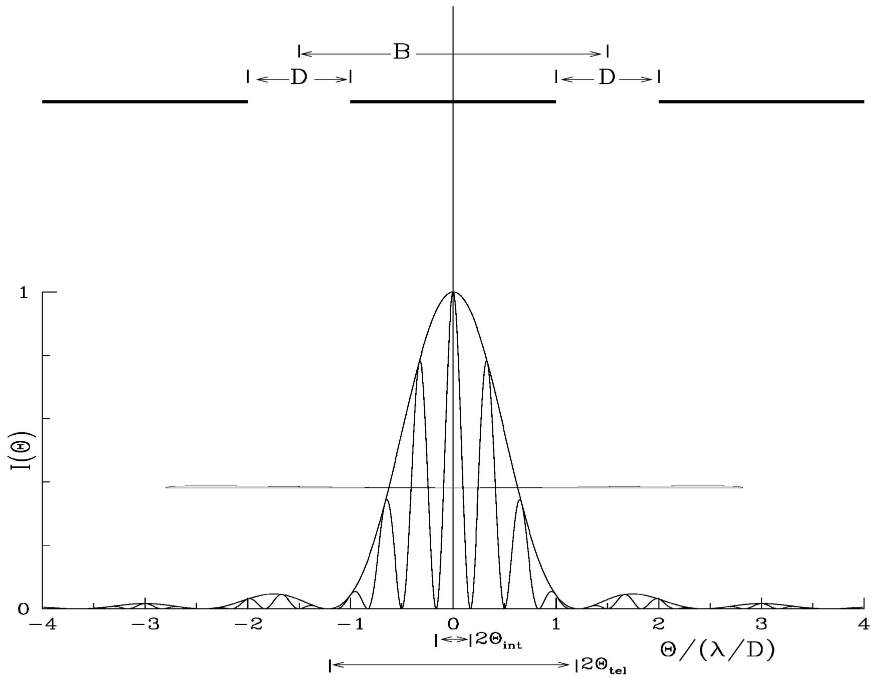
Beam Combination



Monnier & Allen 2012

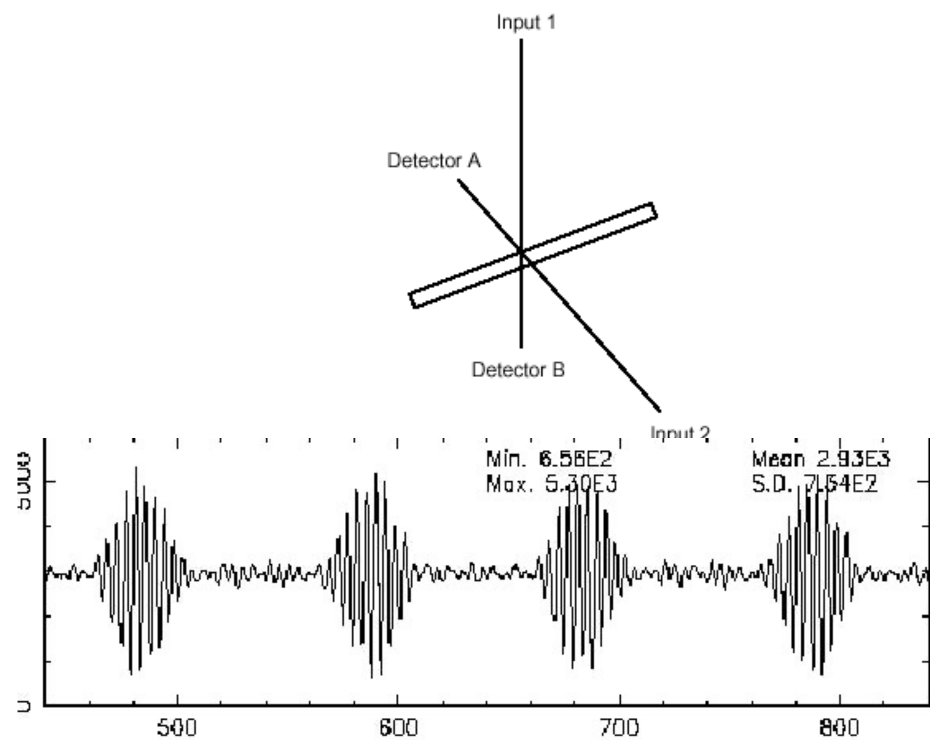
Beam Combination

Image-Plane Combination



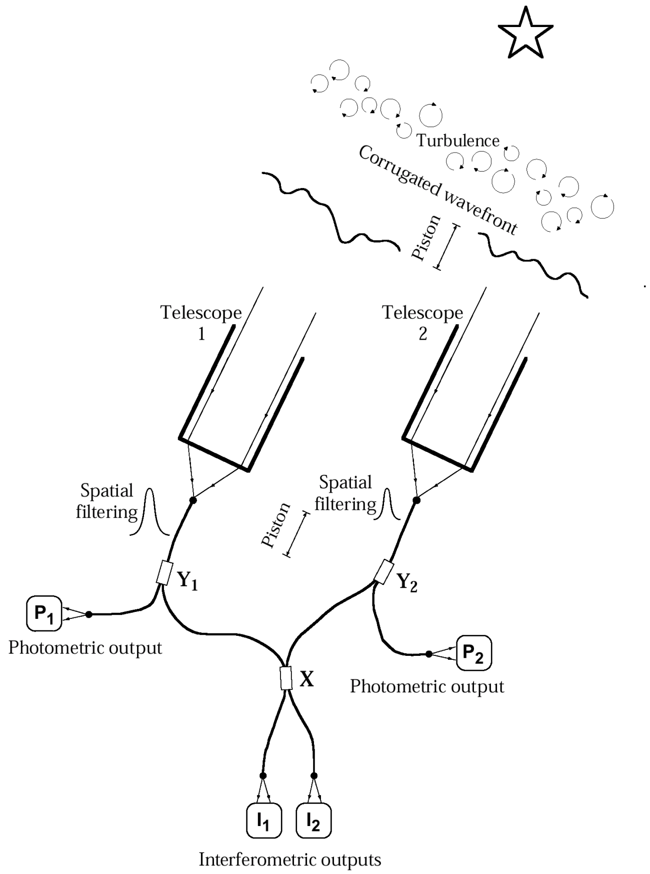
Traub (Michelson Summer School Notes 2000)

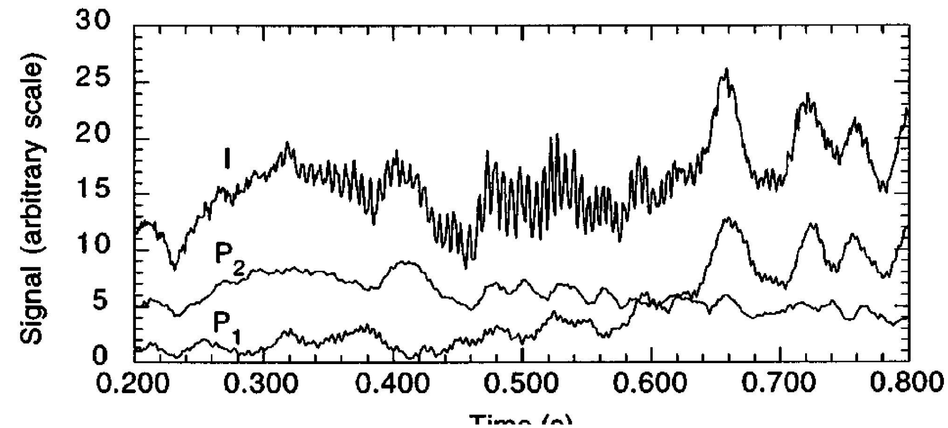
Pair-wise "pupil plane" With temporal scanning

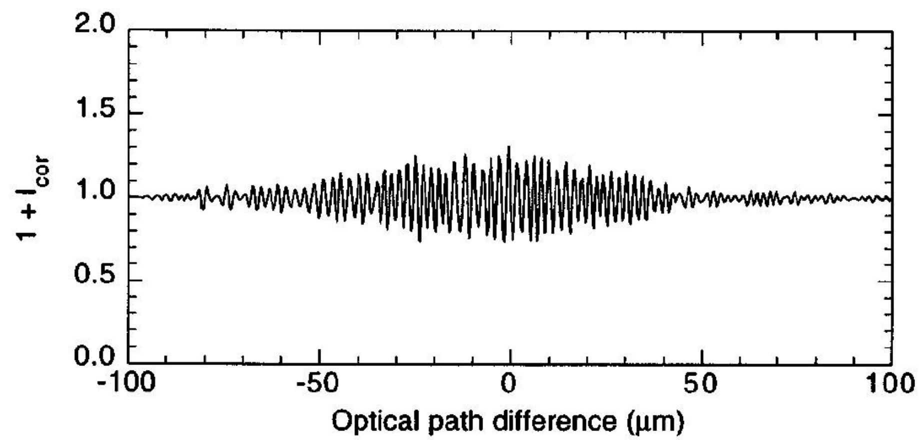


Mozurkwich (Michelson Summer School Notes 2000)

SM Fibers: How They Work

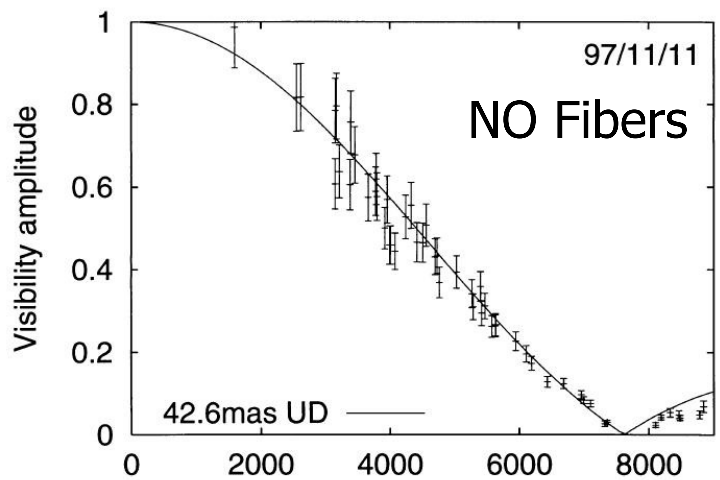


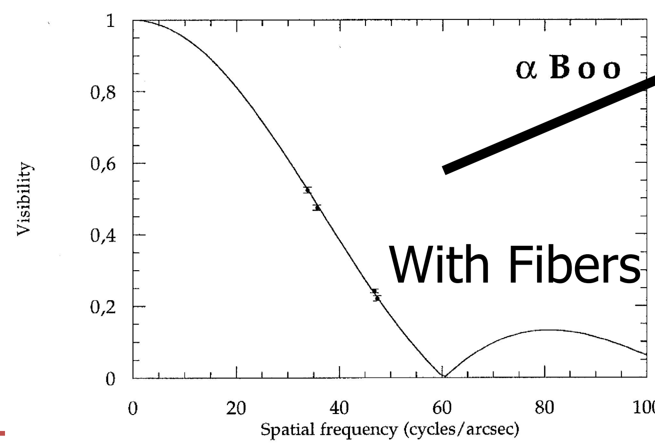




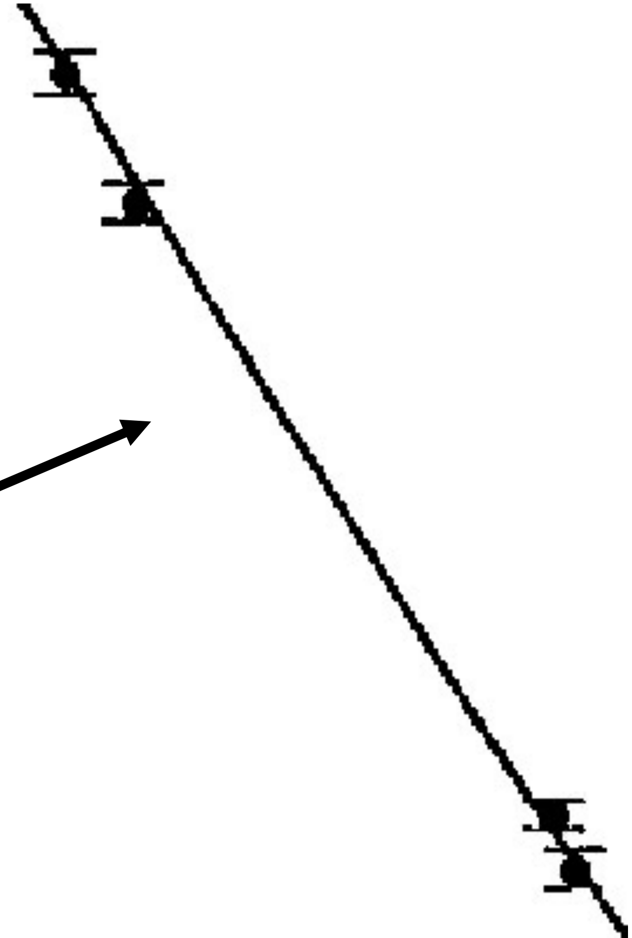
From Foresto et al 1997

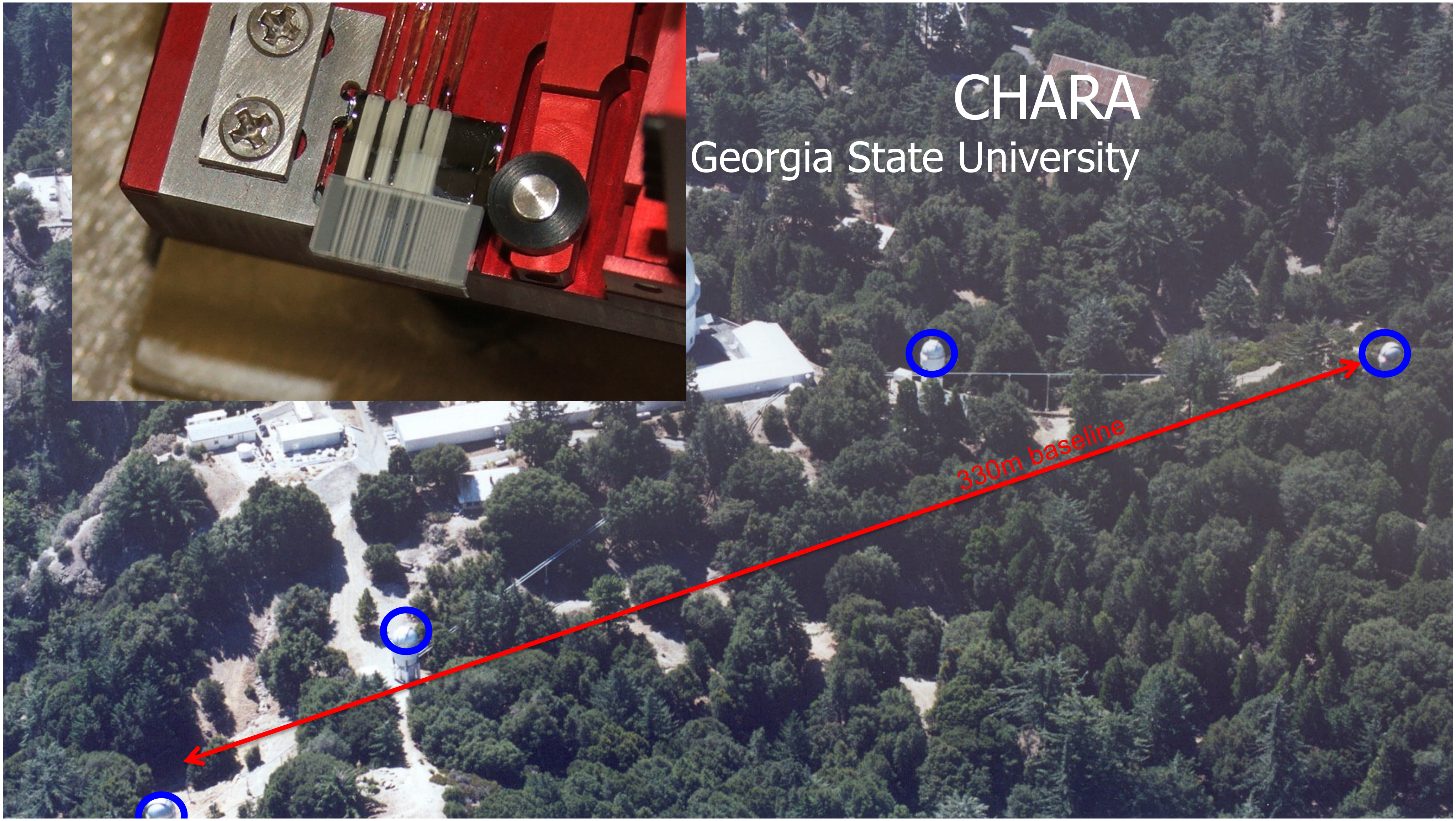
Calibration Improvement



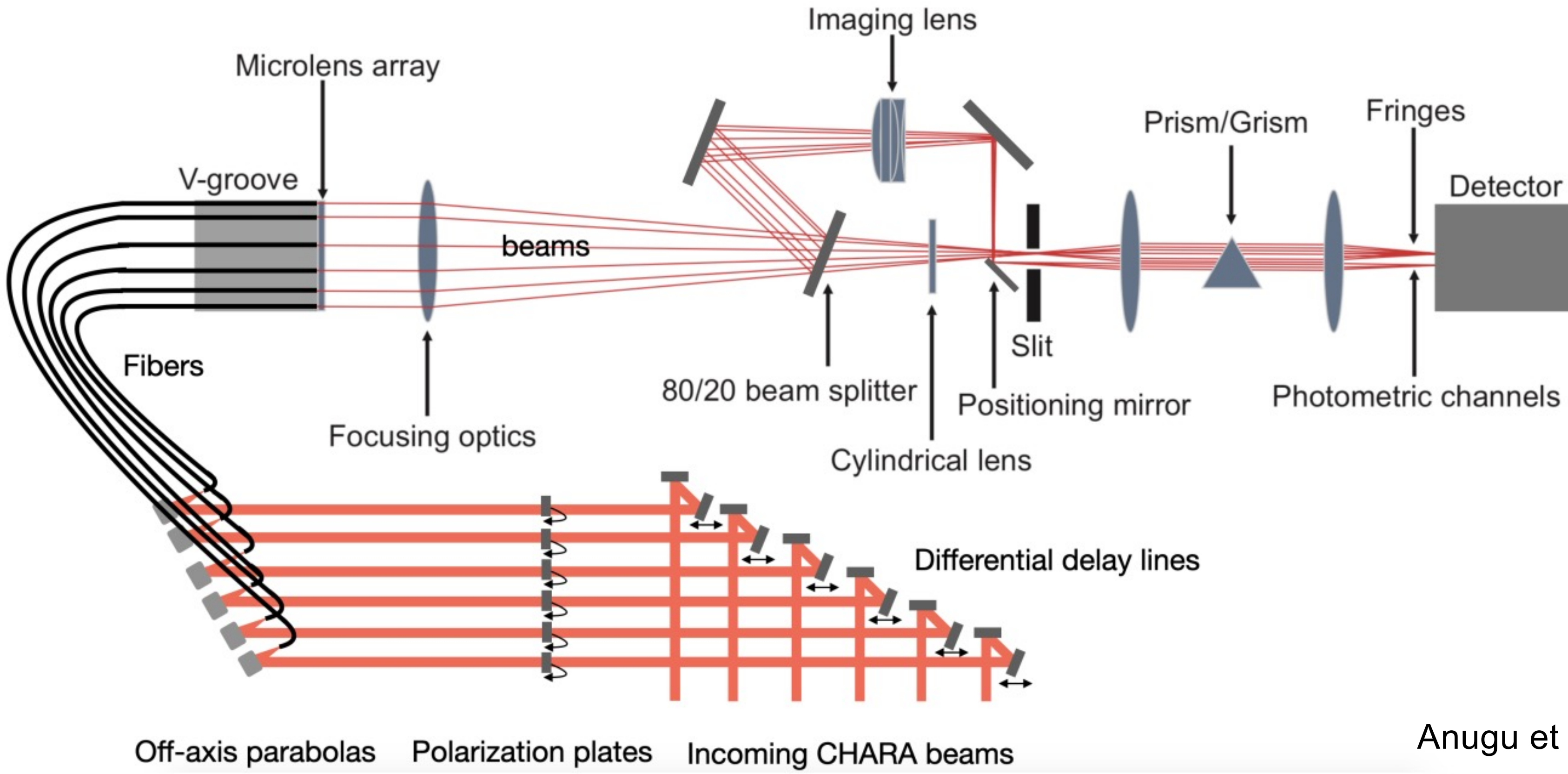


Zoom




CHARA
Georgia State University
330m baseline

"MIRC-style" All-in-One Combiners: MIRC, VISION, MIRC-X, MYSTIC, SPICA

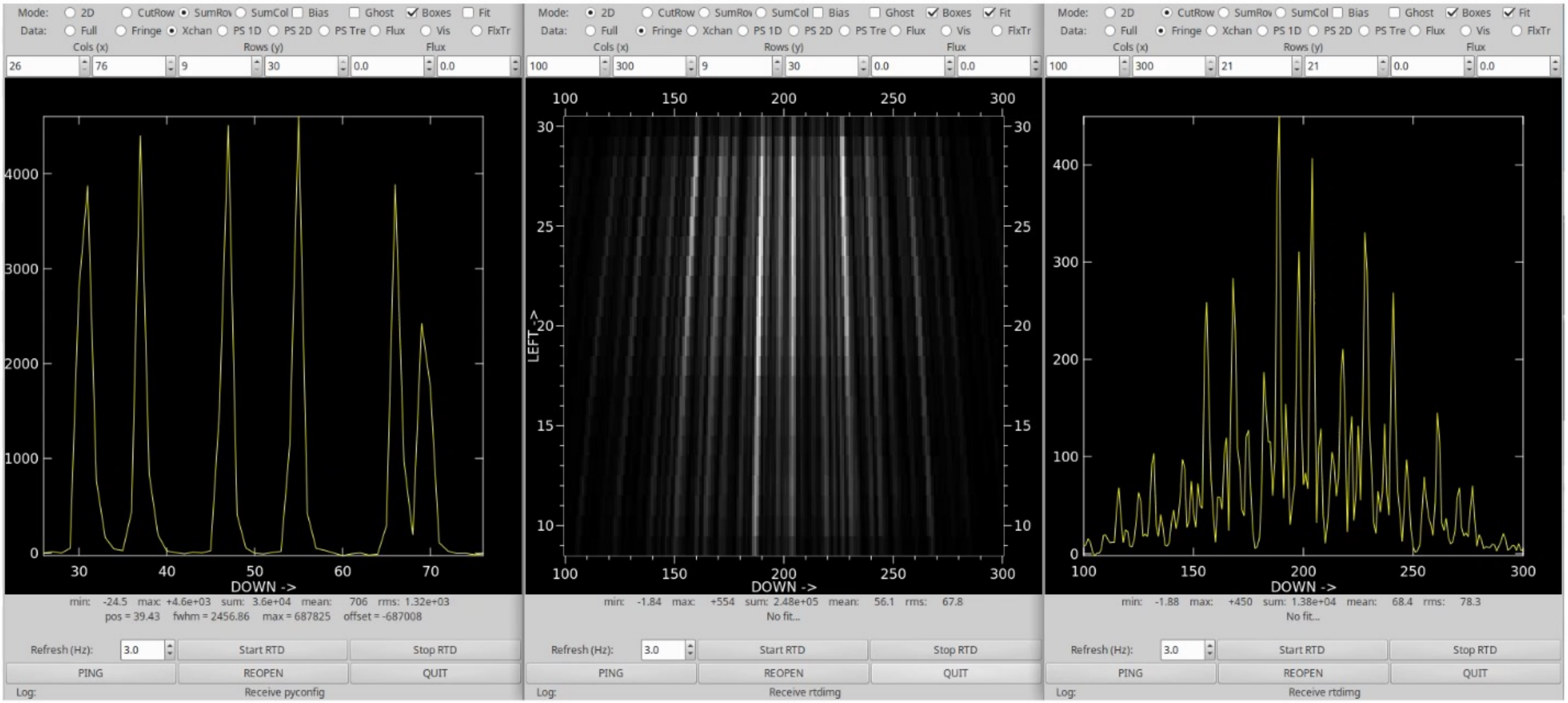


Anugu et al. 2022

Photometric Channels

Fringes vs Wavelength

Fringes:
One Spectral Channel



Essential Points

- Interferometers measure Fourier components of images
  - With enough data, imaging is possible
  - Most commonly, we fit models to the interferometric observable: V^2 and Closure Phases
- Optical/Infrared Interferometry is technically demanding, but now routine
  - Wavelengths are small; atmosphere is unkind
- ~1 milliarcsecond resolution is interesting
  - Stellar diameters and stellar evolution, Planet-forming disks, stellar surfaces, interacting binaries, mass loss, exoplanets, AGN, and more!