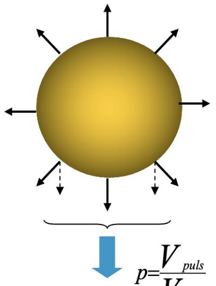
The Surface Brightness Color Relation of Cepheids : calibration of the distance scale in the universe

M.C. Bailleul

(Paper accepted for publication + preliminary result from CHARA survey)

● Context of my work

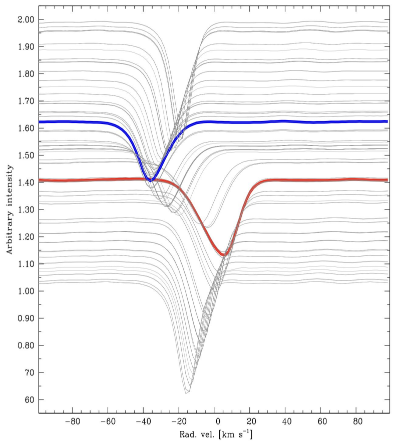
- ❏ Calibration of a **new Surface Brightness Color Relation** for Cepheids
- ❏ **Increase the precision** and the physical understanding of the Baade-Wesselink (BW) method of distance determination



$p = \frac{V_{puls}}{V_{rad}}$

$$\Delta R = \int_{t=0}^{t=P} V_{\mathrm{puls}} dt$$

linear diameter



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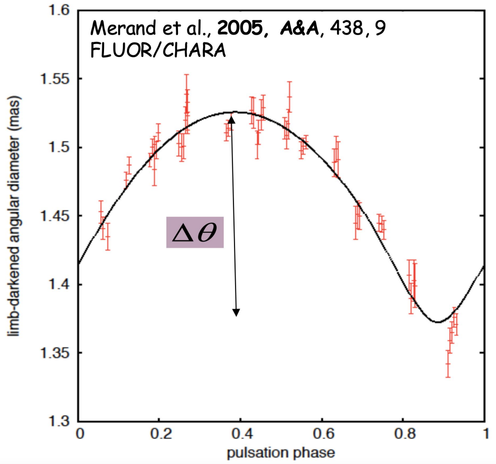


angular diameter

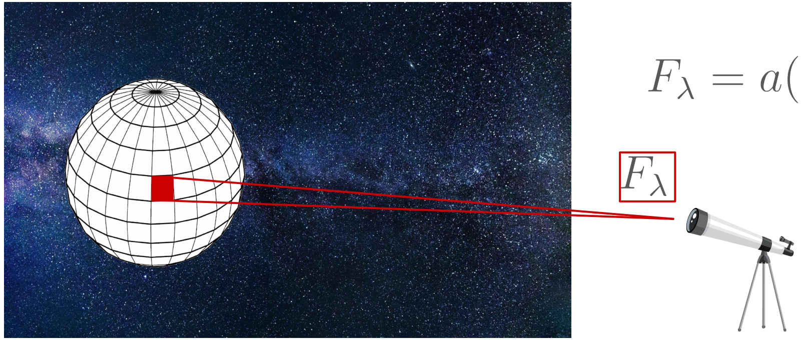
=

$$d \propto \frac{\Delta R}{\Delta \theta}$$

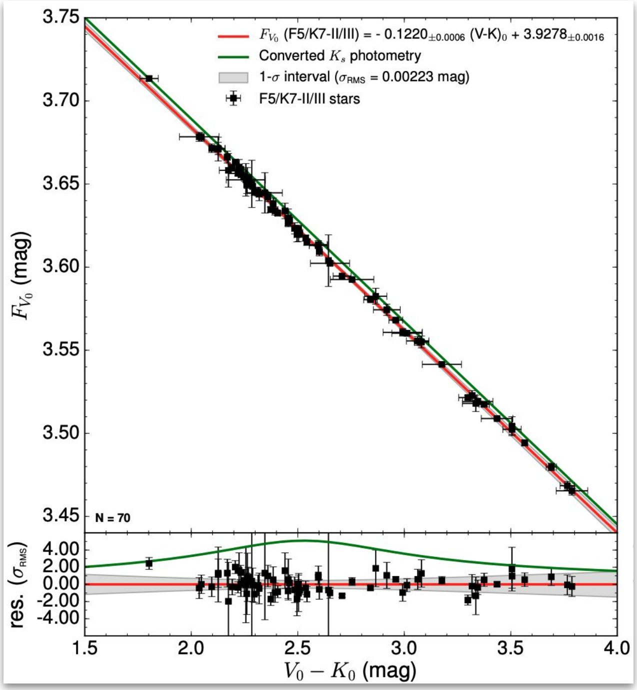
- ❏ The objective of the **ANR Unlock p-factor project** (2023-2028) is to open the road to the BW distance of cepheids in the MW and in the local group in the context of Gaia, LSST and ELT

● The Surface brightness color relation (SBCR)

Surface brightness = flux density received per unit of solid angle expressed in magnitude.



$$F_\lambda = a(m_{\lambda_1} - m_{\lambda_2}) + b$$



- ❑ 2 magnitudes → F_λ → angular diameter

$$\theta_{LD} = 10^{8.4392 - 0.2 m_{\lambda_1} - 2F_\lambda}$$

Salsi et al. (2021)

The Surface brightness color relation (SBCR)

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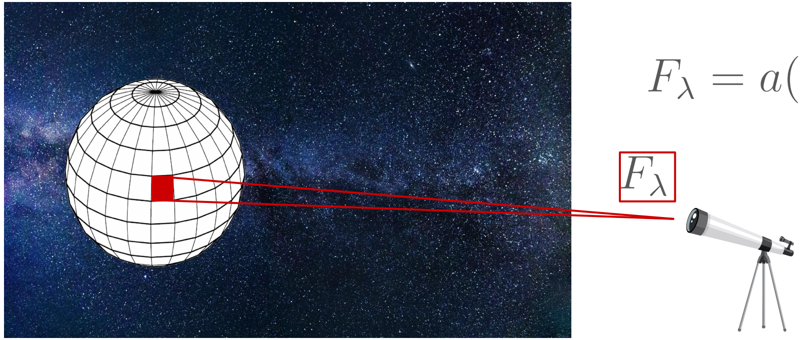

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- ❑ 2 magnitudes → F_λ → angular diameter

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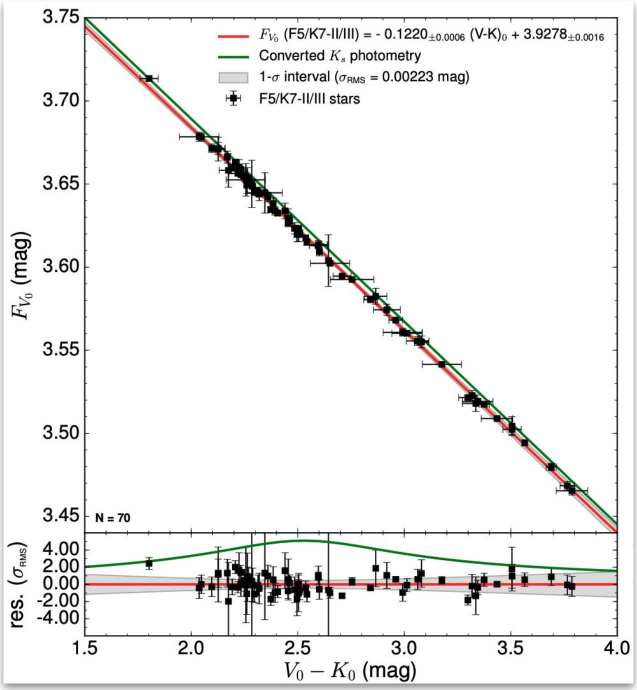
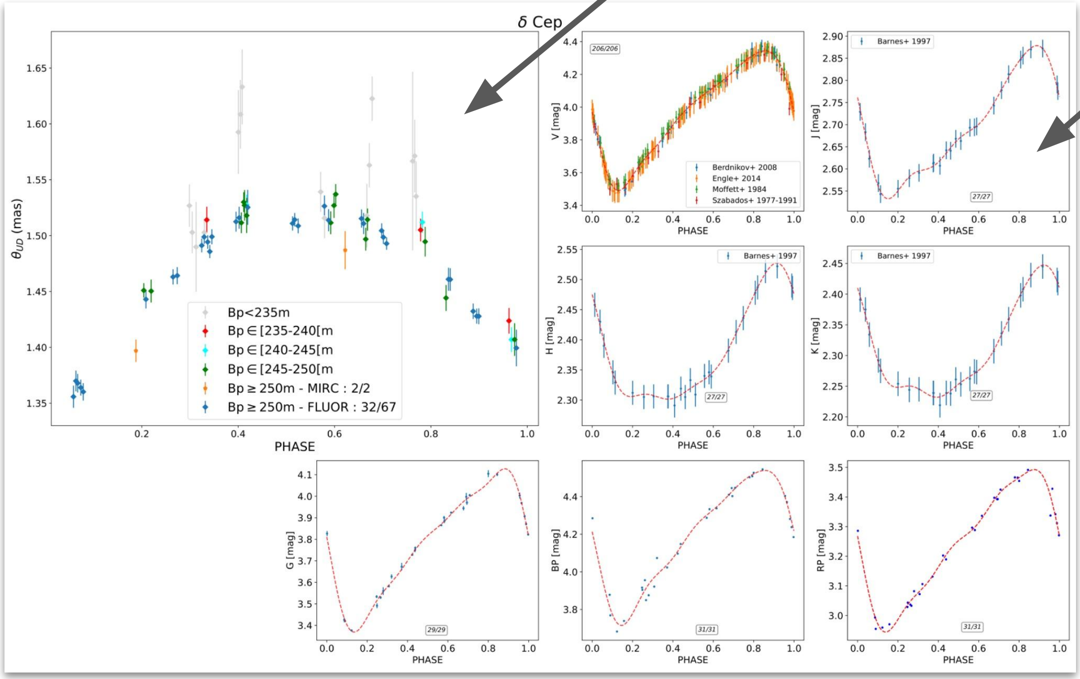
- ❑ This **empirical relationship** needs to be calibrated by **interferometry**



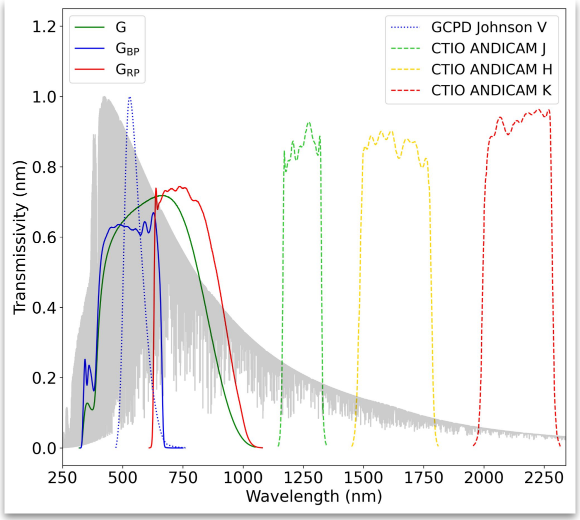
Salsi et al. (2021)

● My work

❑ Interferometric data



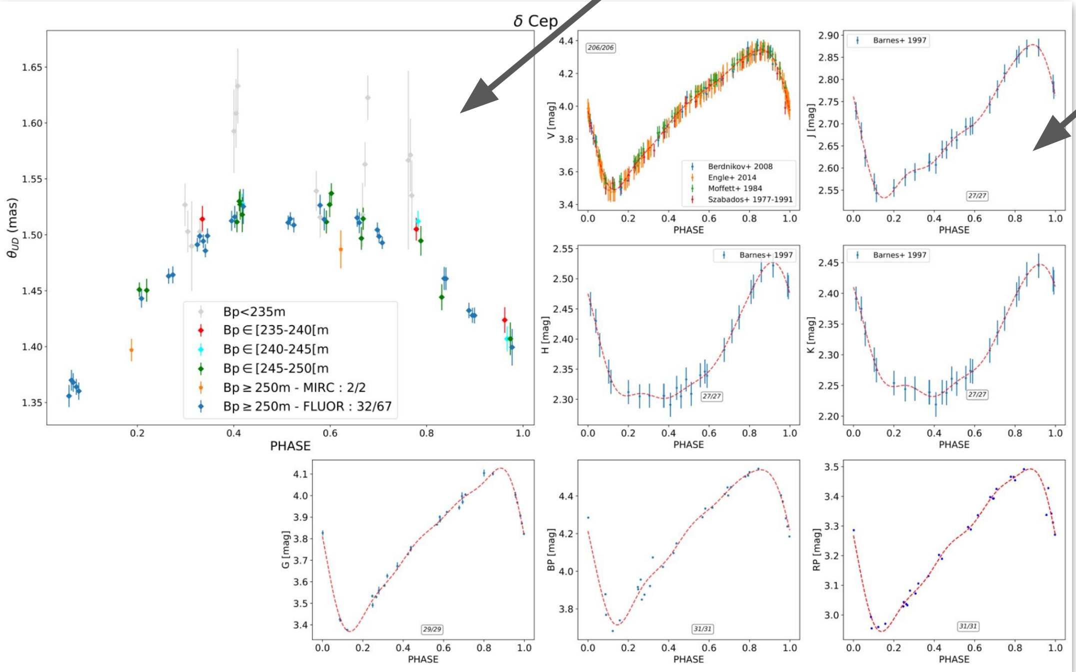
❑ Different photometric bands



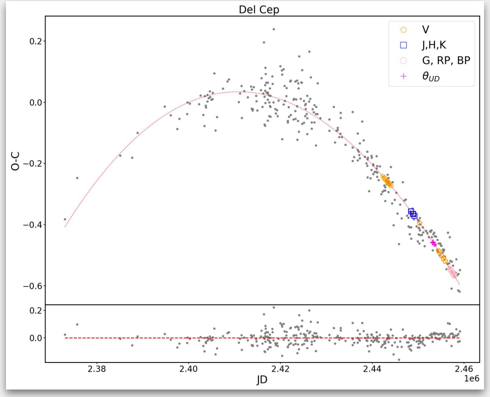
- # My work

❑ Interferometric data

❑ Different photometric bands

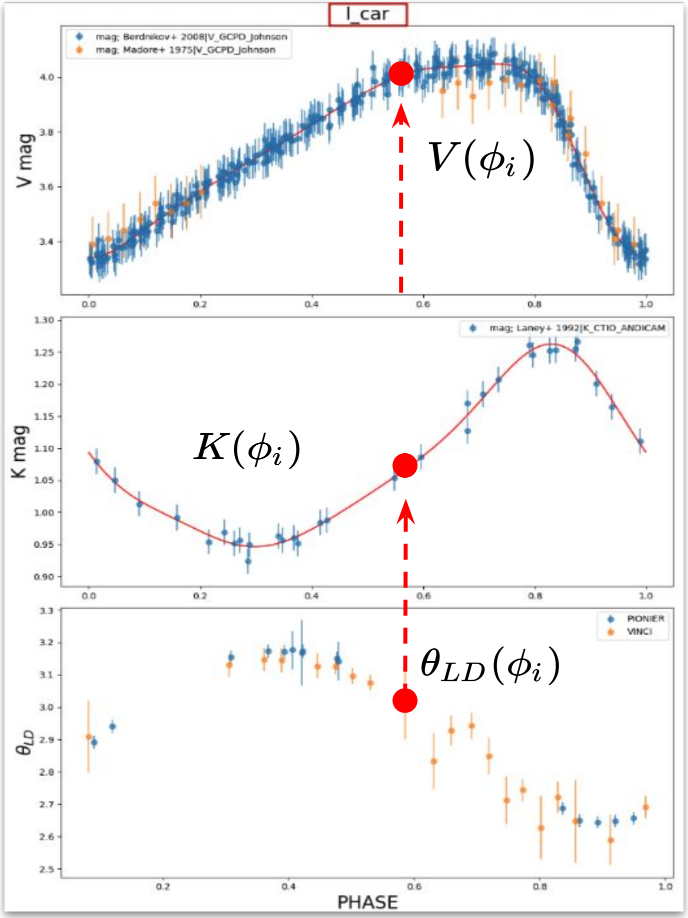


All the measurements were phased using O-C diagram from Csornyei+2022.



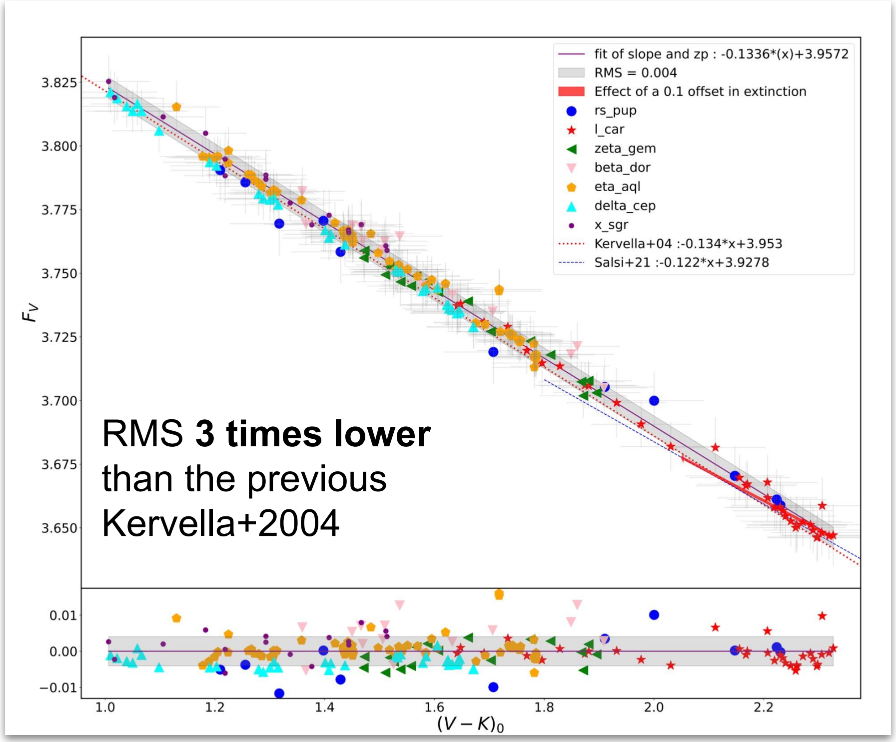
Methodology

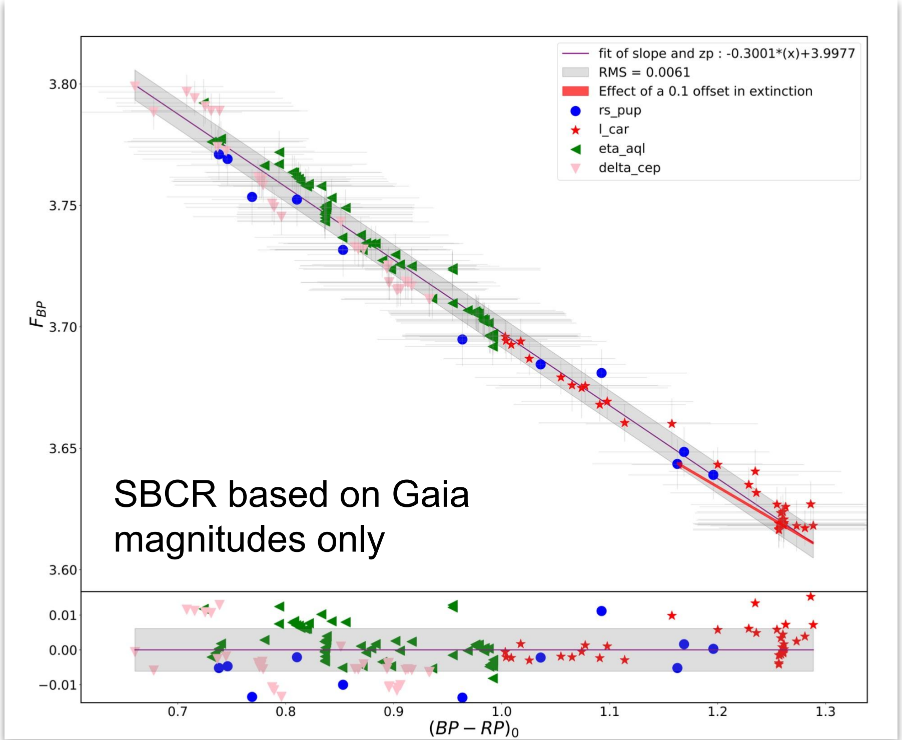
- ❑ $\theta_{UD} \rightarrow \theta_{LD}$ (Claret+2011)
- ❑ Interpolation of the light curves
- ❑ Correction from extinction



Main results

SBCRs with data from 7 Cepheids.





RMS = 0.004 mag

RMS = 0.0061 mag

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Surface brightness–colour relations of Cepheids calibrated by optical interferometry

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ABSTRACT

Context. Surface brightness–colour relations (SBCRs) are widely used to determine the angular diameters of stars. They are in particular used in the Baade-Wesselink (BW) method of distance determination of Cepheids. However, the impact of the SBCR on the BW distance of Cepheids is about 8%, depending on the choice of SBCR considered in the literature.
Aims. We aim to calibrate a precise SBCR dedicated to Cepheids using the best quality interferometric measurements available as well as different photometric bands, including the *Gaia* bands.
Methods. We selected interferometric and photometric data in the literature for seven Cepheids covering different pulsation periods. From the phased photometry in the different bands ($VJHKGG_{BP}G_{RP}$) corrected from extinction and the interferometric limb-darkened angular diameters, we calculated the SBCR associated with each combination of colours.
Results. We first find that the seven Cepheids have consistent SBCRs as long as the two magnitudes considered are not too close in wavelengths. For the SBCR $(F_V, V-K)$: $F_V = -0.1336_{\pm 0.0009}(V-K)_0 + 3.9572_{\pm 0.0015}$, we obtain a root mean square (RMS) of 0.0040 mag, which is three times lower than the latest estimate from 2004. Also, for the first time, we present an SBCR dedicated to Cepheids based on *Gaia* bands only: $F_{G_{BP}} = -0.3001_{\pm 0.0030}(G_{BP}-G_{RP})_0 + 3.9977_{\pm 0.0029}$, with an excellent RMS of 0.0061 mag. However, using theoretical models, we show that this SBCR is highly sensitive to metallicity. From this empirical multi-wavelength approach, we also show that the impact of the CircumStellar Environment (CSE) of Cepheids emission is not negligible and should be taken into account in the future.
Conclusions. With this study, we improve the calibration and our understanding of the SBCR of Cepheids. The overall goal of this project is to provide a purely empirical SBCR version of the BW method that takes into account the metallicity and the CSE emission of Cepheids and that could be applied to individual Cepheids in the local group in the context of JWST and ELT.

Key words. techniques: interferometric – stars: atmospheres – stars: distances – stars: fundamental parameters – stars: variables: Cepheids

- Survey of Cepheids with CHARA SPICA/MIRCX/MYSTIC

Observed stars : Del Cep & Zeta Gem + 2 expected binaries (S Sge & V1334 Cyg)

✅ 7 nights + 1 : August 26-27, September 29-30, ~~October 29~~, November 11 & 14 (by Jeremy), December 17.

❌ No observation of the binary stars : too faint.

We secured :

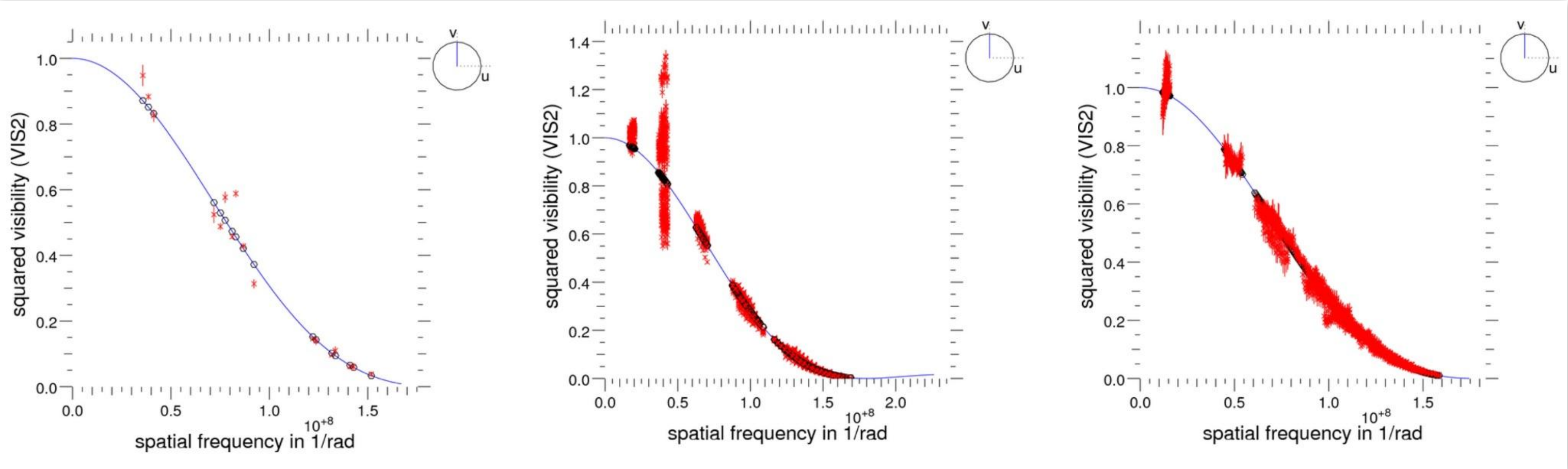
- Del Cep : **2 Spica, 7 MIRCX, 7 MYSTIC**
- Zeta Gem : **0 Spica, 4 MIRCX, 4 MYSTIC**

Long term goal
We want to **measure** for the first time the **limb-darkening** of Cepheid in the **visible** + LD variation + caracterise the **Circumstellar environment** + improve the **SBCR** of Cepheids.

● Example of calibrated visibility : □ Cep

Preliminary results

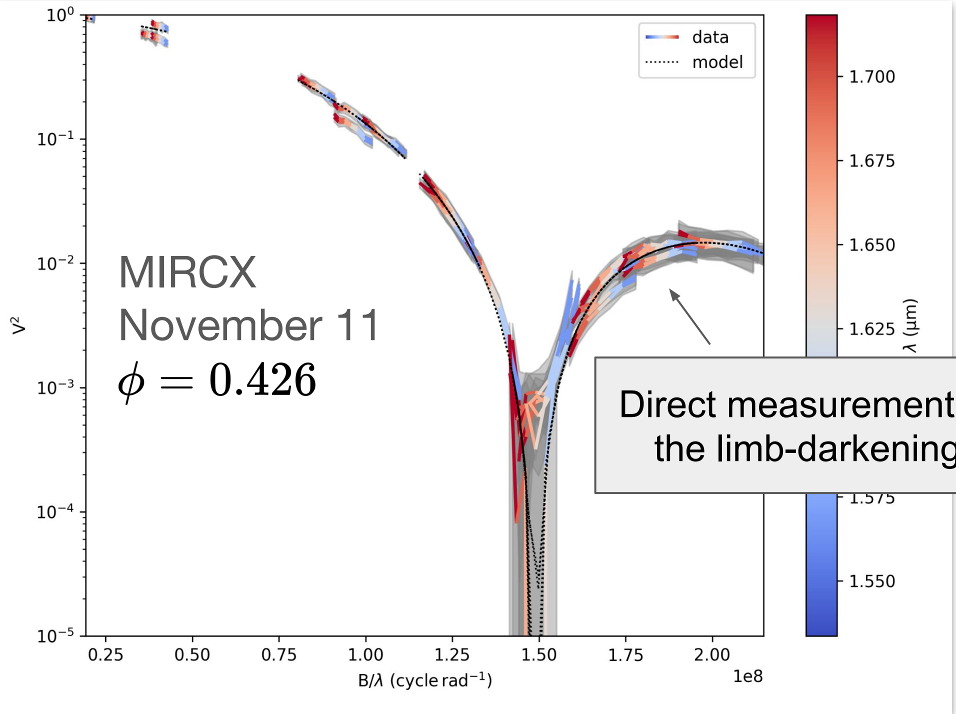
SPICA MIRCX MYSTIC



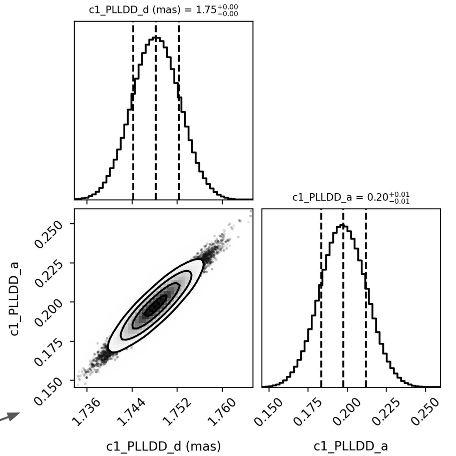
➔ August 27

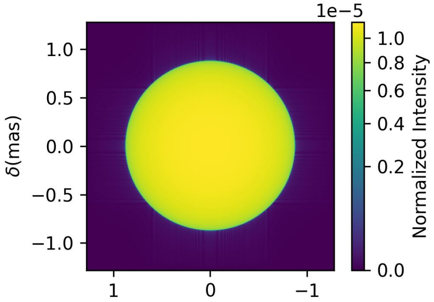
• Example of calibrated visibility : ζ Gem - oimodeler

Preliminary results



Direct measurement of the limb-darkening



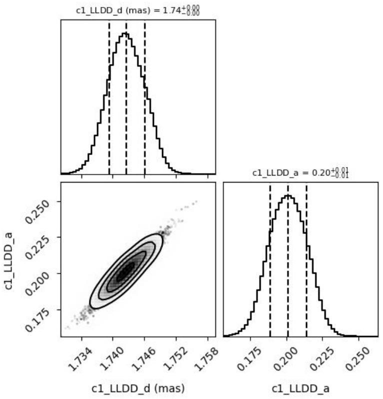


- Example of calibrated visibility : ζ Gem - **linear law**

$$1 - u_\lambda(1-\mu)$$

Preliminary results

OIMODELER



$$\theta_{LD} = 1.742 \pm 0.004$$
$$u_H = 0.200 \pm 0.014$$

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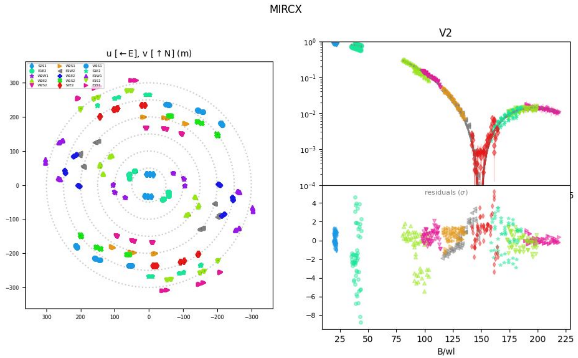
PMOIRED



$$\theta_{LD} = 1.743 \pm 0.006$$
$$u_H = 0.270 \pm 0.025$$

>

≠

CLARET

- Teff(φ), logg(φ) for the star (Trahin+2021)
- Find the closest (Teff,logg) on Claret Table → u_λ
- Hambury Brown et al. 1974 :

$$\frac{\theta_{LD}}{\theta_{UD}} = \left[\frac{1-\frac{u_\lambda}{3}}{1-\frac{7u_\lambda}{15}}\right]^{1/2}$$

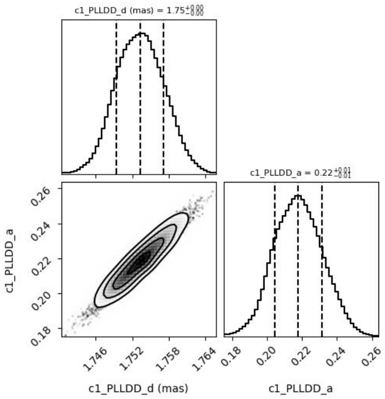
$$\theta_{LD} = 1.723 \pm 0.007$$
$$u_H = 0.249$$

Preliminary results

Example of calibrated visibility : ζ Gem - **power law**

μ^{α}

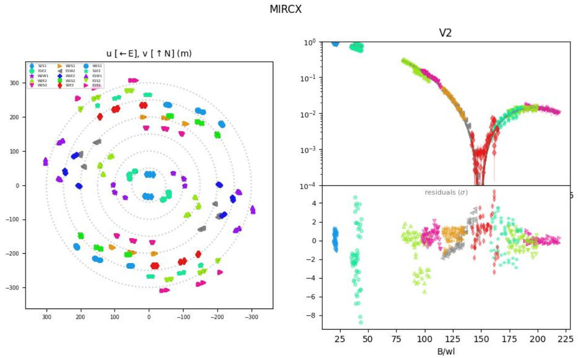
OIMODELER



$$\theta_{LD} = 1.753 \pm 0.004$$
$$\alpha_H = 0.218 \pm 0.013$$

≈

PMOIRED



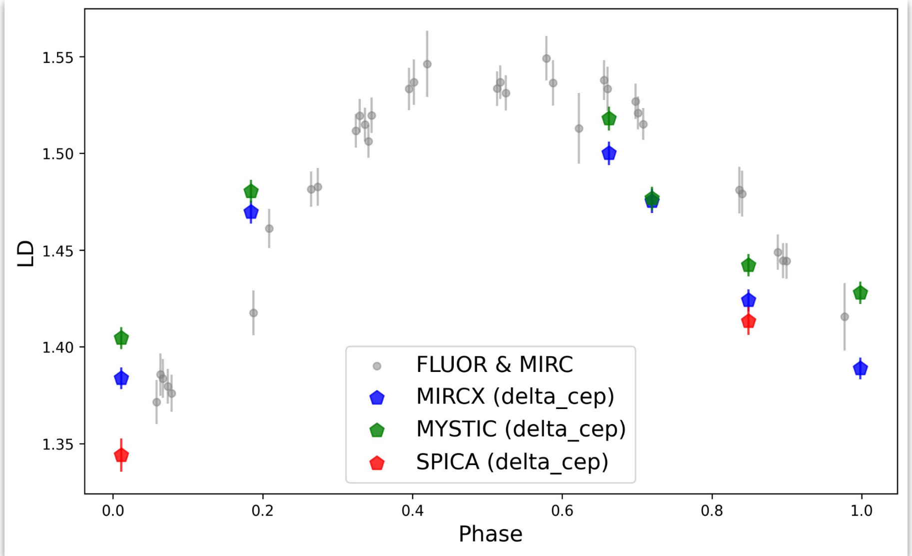
$$\theta_{LD} = 1.761 \pm 0.006$$
$$\alpha_H = 0.221 \pm 0.023$$

>> Linear law results

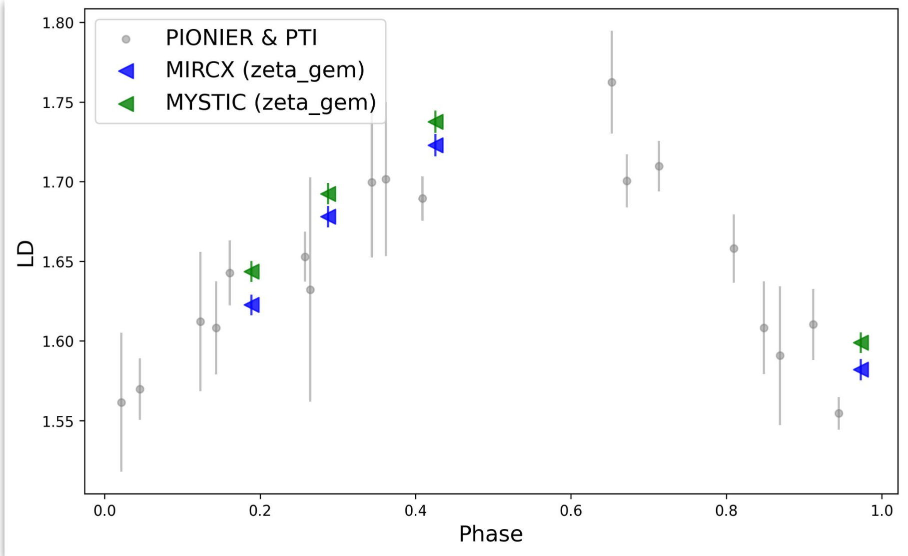
● **LD** angular diameter (oimodeler)

Preliminary results

❑ δ Cep (~5 days)

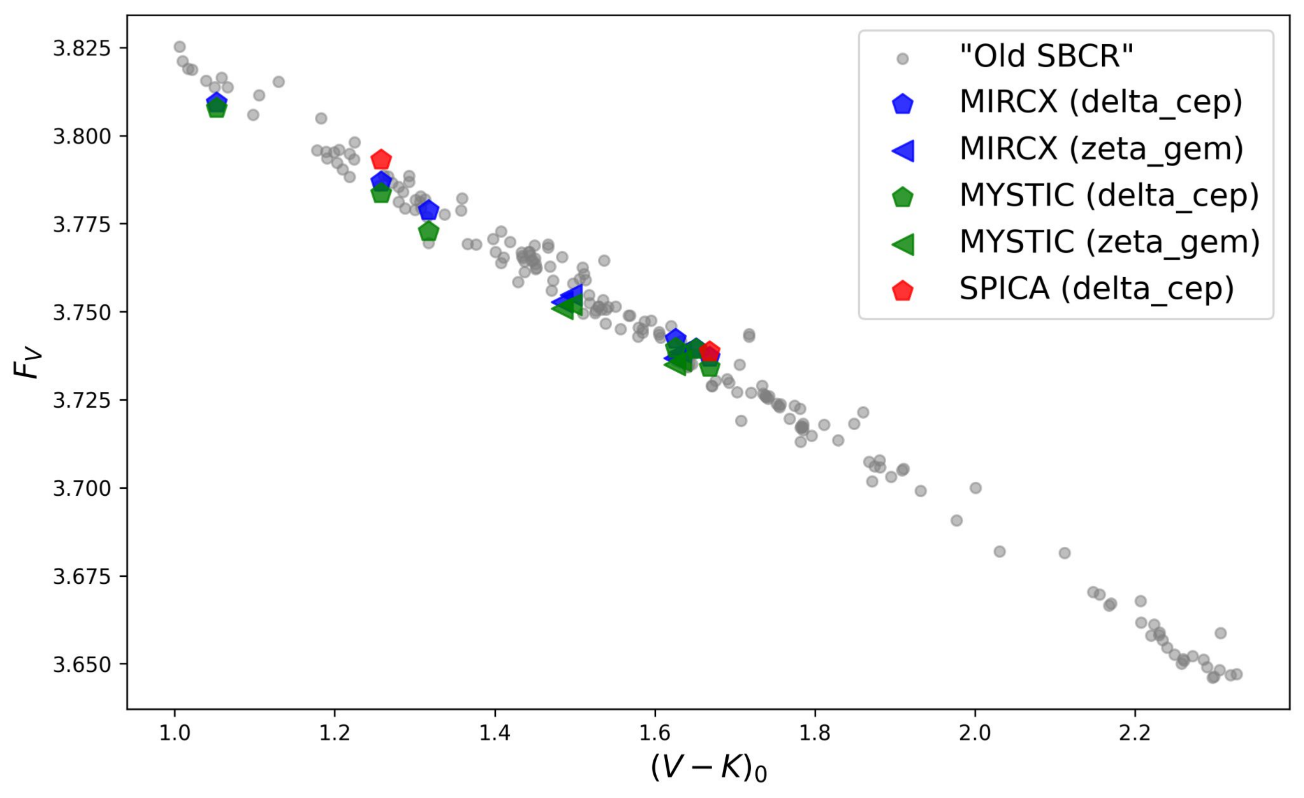


❑ ζ Gem (~10 days)



• "New" SBCR (oimodeler)

Preliminary results



SBCR with data from 7 Cepheids.

Conclusion

- We derived the SBCR(V, V-K) with an RMS 3 times lower than the latest estimate from 2004.
- For the first time we calibrated an SBCR based on Gaia magnitudes only for Cepheids

- We obtained :
  - 2 SPICA, 7 MIRCX, 7 MYSTIC observations for Delta Cep
  - 0 SPICA, 4 MIRCX, 4 MYSTIC observations for Zeta Gem

  Preliminary results are encouraging!

- For 2025A : We will observe Eta Aql of period 7.1 days (eight different half-nights)
- Proposal for 2025B : continuation of Delta Cep and Zeta Gem observation

- The analysis will be done in a consistent multi-chromatic way taking into account the circumstellar environment if present and the limb-darkening of each stars.