CHARA Technical Report

No. 100

Dynamical Modeling of CHARA AO

Version 0.1

2020-05-27

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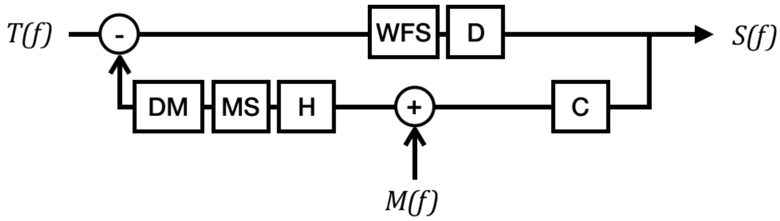
1 Scope of this document

This document presents a basic dynamical model for the CHARA AO loop.

2 Model of blocks

2.1 Block diagram

The system is modelled with the following diagram. By convention, I consider that the output of the system is the measured slopes already processed by the reconstructor. It simplifies the comparison for internal/turbulence disturbances. The block diagram of the system is:



where the variables:

- $T(f)$: input phase perturbation, that is turbulence;
- $S(f)$: measured slopes processed through reconstructor (actuator space)
- $M(f)$: internal modulations of the command (e.g sine)

are propagated through the systems:

- WFS : wavefront sensor
- D : get data out of camera, compute the slopes, and go through reconstructor
- C : controller
- H : holder (the computed command is freeze until the next one is available)
- MS : optional multi-stepping applied by ALPAO driver
- DM : mechanical transfer function of the DM

The transfer functions of the various elements of the system are expressed using the Laplace coordinate $s = 2j\pi\, f$ where j is the complex number and f the frequency.

2.2 Transfer Function of the WFS and Delay

The WFS is modelled as an integration $[t, t + T_i]$ over the integration time $T_i = 2$ms whose transfer function is:

$$W(s) = \frac{1 - e^{-T_i\, s}}{T_i\, s}$$

This model already includes a time shift $T_i/2$ due to the sliding window. The additional delays to transfer the frame $T_{ft} \approx 0.27$ms, to read the frame $T_{ro} \approx 1.75$ms[1], and to compute the slopes and apply reconstructor $T_{rtc} \approx 0.7$ms[2] are modelled as a pure delay:

$$D(s) = e^{-(T_{ft}+T_{ro}+T_{rtc})\,s}$$

2.3 Transfer Function of the Controller

The controller is a discrete, leaky integrator. This is equivalent as a first-order low-pass filter. Its transfer function depends on the integration gain $K_i \approx 0.4$, the memory gain $K_m \approx 0.99$, the cycle period $T_c \approx 1/440$Hz:

$$C(s) = \frac{K_i\, K_m}{1 - K_m\, e^{-T_c\, s}}$$

Let's define the memory factor $\alpha_m = K_m/(1 - K_m) \approx 99$. At frequencies lower than $1/(2\pi\alpha_m T_c) \approx 0.8Hz$, the integrator becomes a proportional with gain $\alpha_m\, K_i \approx 40$.

2.4 Transfer Function of the Holder and DM

The command is hold during the cycle period of the loop $[t, t + T_c]$. This hold is modelled by:

$$H(s) = \frac{1 - e^{-T_c\, s}}{T_c\, s}$$

This model already includes a time shift $T_c/2$ due to the sliding window of the hold.

The transfer function of the BAX187 from ALPAO can be crudely modelled as a sum of 4 second-order resonances of frequencies 470, 480, 500, 525 Hz, damping 0.07, 0.1, 0.1, 0.1 and amplitudes 0.0625, 0.3125, 0.3125, 0.3125:

$$DM(s) = \sum_i \frac{a_i}{\left(\frac{s}{s_i}\right)^2 + 2\xi_i \frac{s}{s_i} + 1}$$

This transfer function is extremely fast at the frequencies of the loop. At 100Hz, the phase lag of the DM is equivalent to a delay of 0.065ms so we can ignore it when doing crude estimation of total delay.

The multi-stepping stepping is currently deactivated in the software because analysis demonstrate that it does not improve performance. Moreover, it makes impossible to send multiple commands to the DM within 1ms. If necessary, the transfer function of multi-stepping is a sum of several impulses time-shifted by multiples of delta:

$$MS(s) = \sum_n a_n\, e^{-s\, n\, \Delta}$$

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[1] We have determined these quantities for the standard camera setup. The cycle time is the maximum between the sum of the integration and frame-transfer times or readout time, that is $T_c = \max(T_i + T_{ft}, T_{ro})$

[2] The time in the software is known accurately by mean of tagging and telemetry. The processing time is completely dominated by the time to measure the slopes out of the raw image, not the reconstructor.

3 Open and closed loop Transfer function

3.1 Open Loop measurement with modulations

Our standard Open Loop tests consist into measuring the output slopes $S_{ol}(f)$ created by an internal modulation $M_{ol}(f)$. Therefore, for these tests, the transfer function is given by

$$\frac{S_{ol}(f)}{M_{ol}(f)} = D.WFS.DM.MS.H$$

In the following, we call this function the Open Loop Transfer Function of the system $OL(f)$. Note that the controller is not part of this function. According to this model, the total delay in our Open Loop measurements should be:

$$T = \frac{T_c}{2} + T_{ft} + T_{ro} + T_{rtc} + \frac{T_i}{2}$$

This equation gives $T \approx 4.9ms$ for $T_i = 2ms$.

3.2 Closed Loop measurements

Our standard Close Loop tests consist into measuring the ratio between the slopes measured in closed-loop $S_{cl}(f)$ and open-loop $S_{ol}(f)$ against turbulence or against internal modulation. In both cases, the function linking these measurements is given by

$$\frac{S_{cl}(f)}{S_{ol}(f)} = \frac{1}{1 + DM.MS.H.C.D.WFS} = \frac{1}{1 + C.OL}$$

In the following, we call this function the Close Loop Rejection Function $CL(f)$. As expected, the controller is now part of this function.

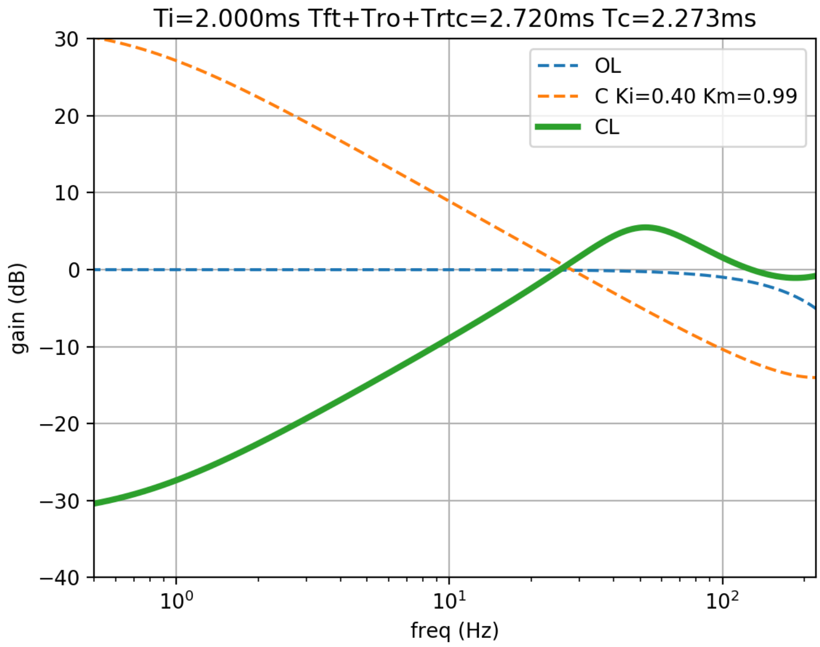
3.3 Stability margins and bandwidth

Increasing the integrator gain K_i increases the bandwidth and thus reduces the residual, but is also reduces the stability margin… until the system eventually destabilises by itself from the input noise.

It is standard to request the following margins when designing a closed loop system. These margins correspond to the amount of actual jitter in the gain or in the phase that the system will accept before going unstable.

- Gain margin at arg(C.OL)=180deg should be >6db
- Phase margin at gain(C.OL)=0db should be >45deg

Considering the previous model, we can increase the gain up to $K_i = 0.4$. The -3dB bandwidth in the CL for such gain is 19Hz. The corresponding Transfer Functions are shown in figure hereafter.



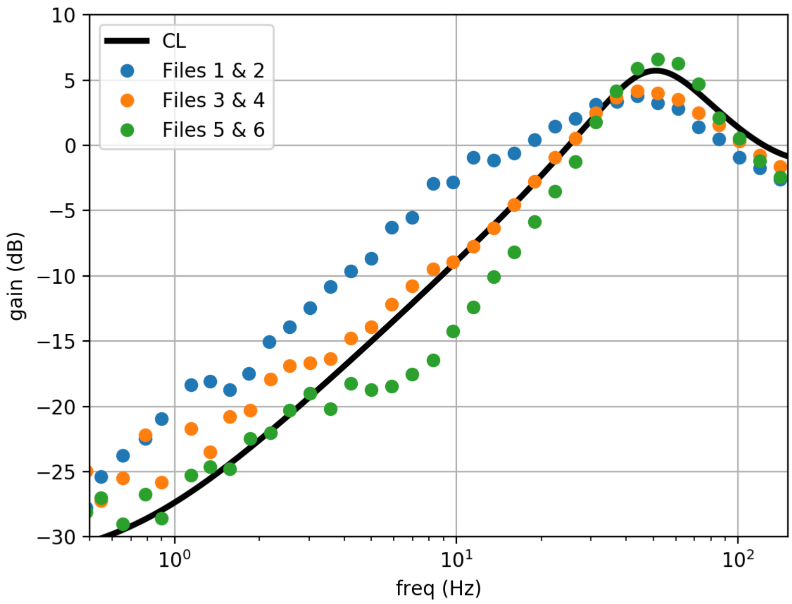
4 Further thoughts

4.1 Comparison with data

The data from 2020-05-22, with $T_i = 2ms$, $1/T_c = 440Hz$, $K_m = 0.99$. The loop status, K_i and damping coefficient are listed below:

2020_05_22_wfs_S2_telemetry_001.fits	CLOSED	0.4	0.
2020_05_22_wfs_S2_telemetry_002.fits	OPEN	0.4	0.
2020_05_22_wfs_S2_telemetry_003.fits	CLOSED	0.4	0.
2020_05_22_wfs_S2_telemetry_004.fits	OPEN	0.4	0.
2020_05_22_wfs_S2_telemetry_005.fits	CLOSED	0.6	0.1
2020_05_22_wfs_S2_telemetry_006.fits	OPEN	0.6	0.1

The final PSD density is the mean of the PSD of the 60 reconstructed signals coming out of the reconstructor. I think using the reconstructed actuator values introduces the necessary filtering to weight properly the poorly seen modes (indeed, there is no difference if we keep of filter out the outer actuators). Results are show below.



The significant difference between files 1&2 and 3&4 is striking because the configuration is the same. Files 5&6 use a larger gain and a damping parameter, which is not modelled here.

4.2 Comparison with other systems

CHARA-AO runs at 440 Hz and has expected -3dB CL bandwidth of 19Hz.

NAOMI runs at 500Hz and has -3dB CL bandwidth of 18Hz.

SPHERE runs at 1200Hz and has -3dB CL bandwidth of 50Hz.

4.3 Model is missing the damping

The model of controller is missing the “damping” coefficient which I don’t know how to model.

4.4 Open Questions about memory

The memory leak is necessary to remove all possible perturbations in the command that are not in the controlled space, for instance because of saturation or because of leftover of commands or in the actuator or the Zernike spaces.

When using a memory of 0.99, the closed-loop rejection saturates at -32dB for frequencies smaller than 0.8Hz. As a consequence, about 2.4% of the slow perturbations will go through the system. This should be OK against turbulence. This may contribute significantly to the residuals if we have very large static aberrations in the telescope, such as focus of coma.