Telescope Primary Mirror Optical Requirements

S.T. Ridgway and L. Barr

1. INTRODUCTION TO THE CHARA ARRAY PROJECT

The Center for High Angular Resolution Astronomy (CHARA) of Georgia State University will build a facility for optical/infrared multi-telescope interferometry, called the CHARA Array. This array will consist of initially five (with a goal of seven) telescopes distributed over an area approximately 350 m across. The light beams from the individual telescopes will be transported through evacuated pipes to a central laboratory, which will contain optical delay lines, beam combination optics, and detection systems. The facility will consist of these components plus the associated buildings and support equipment, and will be located at the Mount Wilson Observatory in southern California. The CHARA Array is funded by Georgia State University and the National Science Foundation.

2. THE TELESCOPES

Each telescope will be an altitude-azimuth reflecting telescope of 1 m clear aperture. The primary-secondary mirrors of each telescope will constitute a beam compressor, providing an afocal output beam with a demagnification factor of 8:1.

The telescope primary, secondary and tertiary are planned to be passive. That is, there will be no active mechanical systems or active feedback mechanisms to maintain the optical figure quality or mirror alignment. (There will be fast tilt correction on flat mirrors #4 and/or #5, but this does not influence the instantaneous wavefront quality.)

The CHARA group has initiated subcontracts for the engineering design of the CHARA telescopes to be carried out by Mr. Larry Barr of Tucson, with engineering support from the National Optical Astronomy Observatories of Tucson.

3. A WAVEFRONT QUALITY REQUIREMENT BASED ON STREHL RATIO

The interferometric performance of the Array is conveniently described in terms of the visibility loss, \( \eta \), or the visibility transfer factor, \( T = 1 - \eta \). The visibility transfer factor,
which also equals the visibility of a point source as degraded by the system, is equal to the
Strehl ratio, $S$, produced by the system (ten Brummelaar et al, 1995).

As an afocal interferometric system, the CHARA Array wavefront quality is most conve-
niently formulated in terms of the RMS deviation of the wavefront. For large Strehl ratios,
hopefully including all values of interest in this discussion, the Strehl ratio can be simply
related to the wavefront RMS by the extended Maréchal approximation,

$$S = \exp(-\sigma^2).$$

(1)

A reasonable benchmark for the Array performance is the Strehl delivered by the atmosphere
under typical conditions.

Depending on source flux and detector characteristics, the beam combining system will
normally operate with $D/r_o \leq 3$, where $D$ is the effective aperture diameter (which may
be intentionally reduced from the nominal maximum value), and $r_o$ is the Fried parameter
characterizing atmospheric turbulence. Typical values are shown in Table 1.

<table>
<thead>
<tr>
<th>$D/r_o$</th>
<th>Strehl</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.96</td>
</tr>
<tr>
<td>1.0</td>
<td>0.87</td>
</tr>
<tr>
<td>1.5</td>
<td>0.71</td>
</tr>
<tr>
<td>2.0</td>
<td>0.65</td>
</tr>
<tr>
<td>2.5</td>
<td>0.54</td>
</tr>
<tr>
<td>3.0</td>
<td>0.43</td>
</tr>
</tbody>
</table>

As a specific example of $D/r_o$ values, consider the case of exceptionally good atmospheric
seeing. Seeing with $r_o$ values as high as 20 cm (at 0.5 μm) over several nights has been
reported at Mt. Wilson. If the full 1 m aperture is utilized, the corresponding value of $D/r_o$
at 0.75 μm will be 3.1, while at 2.2 μm the value would be 0.84.

The science objectives of the CHARA Array do not lead directly to a unique sensitivity
requirement. Therefore the sensitivity achieved will be determined in part by physics, and
in part by cost tradeoffs in the design. Therefore, somewhat arbitrarily, we propose that the
CHARA Array will have a required system Strehl (not including atmosphere) of $S \geq 0.75$
for the full aperture at 0.63 μm.

4. A TOP-DOWN DERIVATION OF A PRIMARY MIRROR
SURFACE REQUIREMENT

Several factors in addition to optical quality will enter into the instrumental visibility.
In Table 2, $T_{coh}$ refers to visibility loss due to drift of the optical path difference from
zero, $T_{opt}$ describes visibility loss due to optical path change during an integration, $T_{tilt}$
describes visibility loss due to imperfect tilt correction, and $T_{diff}$ describes visibility loss
due to imbalance in diffraction effects in beams. For current purposes, Strehl and visibility
transfer factor, $T$, may be considered interchangeable. Note that the goal budget pushes
all other parts of the array to near perfect operation.
The optical error budget should be allocated among the major optical subsystems. Table 3 describes such an allocation.

Table 3 indicates a proposed telescope requirement of $S = 0.86$. Now this should be allocated to the optical elements and their support and alignment. Table 4 suggests a distribution of the Strehl allocation for these. Note that all other image quality related features of the telescope are pushed to near perfection.

Table 5 indicates a required Strehl of 0.95 for residual aberrations in the primary mirror, that is the aberrations permitted in the specifications. This is a stringent specification. From the extended Maréchal approximation, this corresponds to a wavefront quality of 0.23 radians RMS, or 0.036 waves RMS. Dividing by a factor of two (reflecting from a mirror) the specification for the primary would be 0.018 waves RMS surface error. This is substantially better than the criterion of “diffraction limited”, which is customarily applied to systems with a Strehl of 0.8 or 0.076 waves RMS (Schroeder 1987). A mirror surface quality of 0.018 waves RMS is not impossible, but it is considerably better than is normally produced by conventional techniques for a mirror of this size.
TABLE 5. Primary mirror Strehl budget allocation.

<table>
<thead>
<tr>
<th>Budget</th>
<th>$S_{primary}$</th>
<th>Surface Irregularity</th>
<th>Radius Error</th>
<th>Support</th>
</tr>
</thead>
<tbody>
<tr>
<td>Goal</td>
<td>0.93</td>
<td>0.95</td>
<td>0.99</td>
<td>0.99</td>
</tr>
</tbody>
</table>

5. DESCRIPTION IN TERMS OF A STRUCTURE FUNCTION

A weakness of the approach in the previous section was the failure to consider the relative contribution of low and high spatial frequency wavefront errors to the Strehl. This is important because both the atmosphere and optical polishing techniques tend to impose more aberration power at low spatial frequencies than at high spatial frequencies. The resulting Strehl ratio is actually tolerant of this kind of wavefront error. Large spatial scale errors such as spherical aberration and coma can have relatively larger amplitudes than high frequency errors. This information is lost in the peak to valley description.

An alternate specification for the wavefront quality is in terms of the structure function of the wavefront. The application of the structure function to optical tolerancing has been described by Hill (1990).

The tilt-corrected atmospheric structure function can be described by,

$$\delta^2(x) = \left(\frac{\lambda}{2\pi}\right)^2 6.88 \left(\frac{x}{r_o}\right)^{5/3} \left[1.0 - 0.975 \left(\frac{x}{D}\right)^{1/3}\right],$$  (2)

where $\delta(x)$ is the RMS wavefront difference between two points on the wavefront separated by $x$. This is actually independent of $\lambda$ because $r_o \propto \lambda^{6/5}$.

A requirement for $S = 0.95$ corresponds to a wavefront error of 0.23 radians RMS. The atmosphere, with tilt removed (Noll, 1976), will give the required Strehl for $D/r_o$ which satisfies,

$$0.134 \left(\frac{D}{r_o}\right)^{5/3} = (0.23)^2,$$  (3)

or, $D/r_o = 0.57$, where $r_o$ describes the effective $r_o$ of just the primary mirror surface. For our case, $D=1$ m gives $r_o = 1.75$ m. Evaluating Equation 2 for these parameters and for $\lambda = 632$ nm, we obtain the desired wavefront structure function attributed to the mirror surface, $\delta^2(x)$, and the mirror surface difference function will be $0.5 \delta(x)$. This function is shown in Table 6.

Table 6 shows how somewhat larger surface error amplitudes can be tolerated on larger spatial scales.

6. SCATTERING EFFECTS

At the smallest scales, less than about 1 cm, diffraction effects from surface irregularities produce scattering. The fractional loss due to scattering on scales much smaller than $r_o$ can be described by (Hill et al. 1990),
TABLE 6. Primary mirror surface difference function.

<table>
<thead>
<tr>
<th>x(m)</th>
<th>0.5 δ(x) (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>5</td>
</tr>
<tr>
<td>0.1</td>
<td>9</td>
</tr>
<tr>
<td>0.2</td>
<td>14</td>
</tr>
<tr>
<td>0.4</td>
<td>20</td>
</tr>
<tr>
<td>0.6</td>
<td>23</td>
</tr>
<tr>
<td>0.8</td>
<td>21</td>
</tr>
<tr>
<td>1.0</td>
<td>13</td>
</tr>
</tbody>
</table>

\[
\text{loss} = \left( \frac{2 \pi \sigma}{\lambda} \right)^2 ,
\]

where \( \sigma \) is the RMS surface deviation from the mean. It is tempting to specify \( \sigma \) so that scattering losses will be small, like 5% at 0.5 \( \mu \text{m} \). However, in fact opticians find it difficult to obtain values of \( \sigma \) sufficiently small. So in practice it is not realistic to impose a stringent specification on the smallest scale.

7. COMPARISON OF SPECIFICATIONS

It is common to specify the surface quality of telescope mirrors with reference to peak errors, for example the peak to valley error, or the peak difference on various spatial scales. However, since the Array performance is best related to the RMS wavefront character, this is the preferred specification for interferometry.

In fact, placing emphasis on the peak error could be disadvantageous for the CHARA Array. Extensive efforts to remove mirror peaks (probably with a small tool) could actually produce an increase in the RMS. In practice, the most likely mirror defects which may be permitted by the RMS description would be localized errors, such as a turned edge, support point print-through, or actual damage.

In full-aperture description of large mirrors, the ratio of peak-to-valley to RMS is typically on the order of 5. (Very different values may apply for different mirrors sizes or particular spatial scales.)

8. RECOMMENDED PRIMARY SURFACE TOLERANCE

The suggested goal of \( S = 0.75 \) at 0.63 \( \mu \text{m} \) for the full aperture leads to a difficult surface requirement. Soliciting bids based on this requirement would probably lead to a limited number of creditable bids, and the bids would probably be very high.

The approach recommended here is to adopt a conventional surface specification of 0.03 waves RMS over the full aperture, and a more stringent specification, 0.02 waves, over any 30 cm subaperture.
In addition, we propose to specify a peak-to-valley $5 \times$ larger, 0.15 waves. This will exclude large localized defects or damage, but not be so demanding as to encourage extensive work with a small tool.

A separate specification for the small scale (scattering) surface error is not proposed, since the RMS requirement over any 30 cm subaperture is already relatively demanding.

Note that the subaperture specification satisfies the requirement derived from a goal of $S=0.75$, described above, but the full aperture requirement does not. This ensures that the excess surface error will be in low order aberrations. If the CHARA experience shows that these surface errors can be consistently characterized (and that the telescope structure, alignment and mirror support will allow effective utilization of a better mirror surface), it will be possible to improve the mirrors further with the now well established technique of ion polishing. This process is likely to be no more expensive, and probably more satisfactory, then attempting to achieve the same tolerance by classical techniques.

9. PRIMARY MIRROR RADIUS

The primary mirror radius does not impact image quality.

The absolute value of the primary and secondary radii determine the separation of the mirrors and must conform to the dimensions and tolerance of the telescope structural design. A tolerance of several centimeters is reasonable.

The ratio of the radii of the primary and the secondary determines the diameter of the afocal beam. If two telescopes produce afocal beams which differ in diameter, then some beam combination techniques will yield a reduced Strehl. For equality of compressed beam diameters, we would like to have equal mirror radii.

For the CHARA primary radius of 5000 mm, it is relatively easy to measure accurate radii with a spherometer.

For the CHARA mirrors we propose the specification of $5000 \pm 25$ mm, and equality between the mirrors within $\pm 15$ mm. The maximum beam diameter difference between two telescope pairs will then be 0.5%. This is not a difficult requirement and is not expected to result in significant cost increase.

10. CONIC CONSTANT

The conic constant of a parabola is 1. Deviations from this value indicate a figure which is not a parabola. In some telescope systems, the conic constant of the primary, as fabricated, is measured and used to specify the conic constant of the secondary for an optimum match. In the case of the CHARA Array, the objective is to have interchangeable optics, and this approach will not be considered.

The optical analysis shows that the conic constant of the primary mirrors must have the value $1.0000 \pm 0.0005$ in order to function correctly with any secondary which meets its conic constant requirement.

The RMS and peak to valley specifications of the surface are measured with respect to an ideal parabola. Therefore the conic constant tolerance is partially redundant with the surface specification.

TR 15 – 6
The conic constant is a difficult parameter to measure. It is normally obtained from an interferometric measurement followed by analysis with sophisticated software. This software is not available to all potential vendors for the CHARA mirrors. Specifying the conic constant is not expected to change the requirement on the mirrors, but could limit the number of vendors or increase the measurement cost. Therefore, use of the conic constant tolerance in the specifications is not recommended.

11. SUMMARY OF PRIMARY MIRROR OPTICAL REQUIREMENTS

<table>
<thead>
<tr>
<th>TABLE 7. Primary mirror optical requirements.</th>
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<tbody>
<tr>
<td>Primary (vertex) radius of curvature:</td>
</tr>
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</tr>
<tr>
<td>(maximum difference, any two mirrors)</td>
</tr>
<tr>
<td>Surface deviation from parabola:</td>
</tr>
<tr>
<td>(full aperture)</td>
</tr>
<tr>
<td>Surface deviation from parabola:</td>
</tr>
<tr>
<td>(any 30 cm sub-aperture)</td>
</tr>
<tr>
<td>Maximum peak to valley error:</td>
</tr>
</tbody>
</table>

12. REFERENCES