CHARA TECHNICAL REPORT

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Wind Effects vs. Optical Path Length Changes for the Telescopes – A Preliminary Rationale

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1. INTRODUCTION

The current telescope design uses an off-center azimuth mount. Kibblewhite noted that this mount style is susceptible to optical path distance (OPD) variations when:

1. the telescope is horizon-pointing

2. disturbances (e.g. wind) arise that cause rotation around the azimuth axis

This appears to be true from considering how the optics move along the optical axis when a small angular change (θ) occurs about the azimuth axis (see Figure 1). Motion of the primary (δ) produces a 2δ change in the OPD. This is offset by a similar motion of the secondary, but motion of the tertiary is *not* offset when the telescope is horizon-pointing. An OPD error $\simeq \delta$ results.

A similar effect will occur in a conventional "on-center" yoke mount if the yoke arms oscillate back and forth in the direction of the optical axis (i.e., a front-to-back movement of the yoke arms).

The rationale to follow examines whether these effects are serious and what could be done to cope with them.

An estimate will be made first of the tolerable error in the OPD.

The atmosphere causes OPD changes that have been measured by Davis et al. (1995) for baseline distances up to 80 m, the approximate separation that may exist between the CHARA telescopes.

A table of measured values of the standard deviation (RMS) values of the OPD fluctuations is extracted from the Davis paper and shown in Table 1.

Since the tabulated values are the RMS variations in OPD, the peak-to-peak variations will be about three times greater. A reasonable goal for the telescope then is to limit peak-to-peak OPD variations caused by the telescope to the RMS values listed above. Thus, a "budget" value of $11.4\,\mu$m will be used in the remainder of this examination.

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TABLE 1. Standard deviation of optical path length fluctuations ($\sigma_{OPD}(x)$) averaged from 50 s long data sets for three baselines.

Baseline (m)	$\sigma_{OPD}(x)$ (μm)	Source	Number of Data sets
5	4.6 ± 0.6	α CMa	24
20	8.4 ± 0.6	α Vir	10
80	11.4 ± 1.8	α Vir	9

The principal source of unwanted telescope motion will be the wind. It is reasonable to assume that the telescope azimuth drive will control (i.e. eliminate) most of the motions occurring at frequencies within the servo bandwidth.[2] Motions at frequencies outside the bandwidth can be caused by higher frequency wind gusts (or vibrations of the yoke not detectable by the azimuth drive encoders).

2. AN ESTIMATE OF THE FORCE PRODUCED BY HIGHER FREQUENCY WIND GUSTS

The Gemini Project estimates the wind velocity spectral density for Mauna Kea to be as shown in Figure 2 (extracted from Critical Design Review Report dated March 1994 — assumptions need to be reviewed).

Estimating from the graph, in the frequency region from about 0.1 to 100 Hz, the equation of the curve is:

$$V(f) \approx 0.15 f^{-5/3}$$

where units are m^2s^2 and f is the frequency. If one integrates this equation over the frequency region of interest, then finds the square root, the RMS wind velocity for that region may be determined. The peak wind velocity will be about three times larger assuming sinusoidal variation.

$$\begin{aligned}
V_{max} &\equiv \text{peak wind velocity (in region above } f_1) \\
&= 3\left[\int_{f_1}^{\infty} 0.15 f^{-5/3} df\right]^{1/2} \\
&= 1.423 \cdot f_1^{-1/3}
\end{aligned}$$

where f_1 is the lower bound of the frequency range.

The force acting on the telescope may be estimated by assuming the peak velocity in the higher frequency range of interest is effectively acting as a steady-state wind on the telescope. This is conservative since the actual force produced will be the integrated effect of all higher frequency wind components which all have lower velocities.

[2] For an off-center azimuth yoke. Not true for the on-axis yoke.

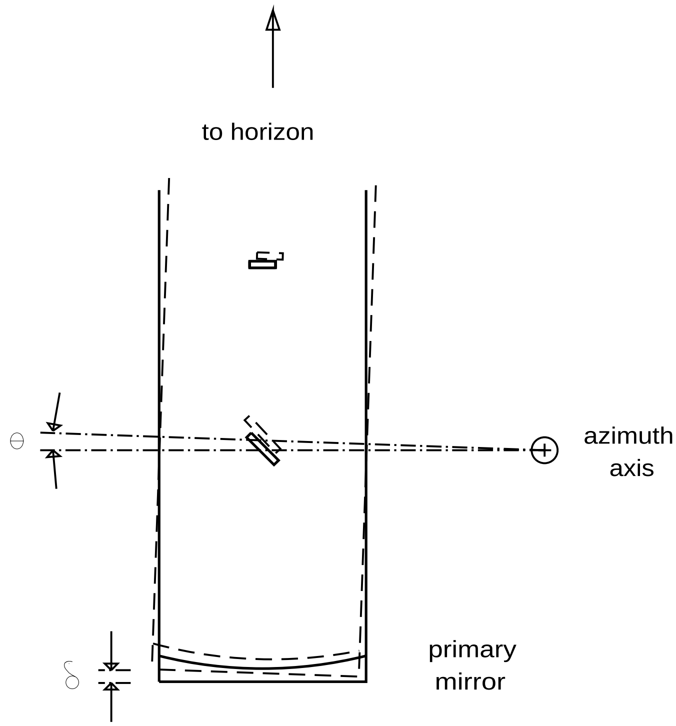

FIGURE 1. Path length differences due to small angular changes about the azimuth axis.

The standard equation for the force on an object due to wind with velocity, V_{max}, is:

$$
\begin{aligned}
F_{\text{wind}} &= 1/2 C_D \cdot \sigma \cdot V_{max}^2 \cdot A \\[1em]
C_D &= \text{drag coefficient} \\
&\equiv 1.0 \text{ for this study} \\[1em]
\sigma &= \text{air mass density} \\
&= \frac{0.063}{g}\,\text{lb/ft}^3 \text{ at } 6500\,\text{ft elevation (std. atmosphere)} \\
&= \frac{0.014}{g}\,\text{kg/m}^3 \text{ at } 1980\,\text{m elevation} \\[1em]
A &= \text{area of the object normal to the wind} \\
&\quad \text{(i.e. the “wind cross – section”)} \\[1em]
F_{\text{wind}} &= \frac{1/2(1.0)(1.014\,\text{kg/m}^3)}{9.8\,\text{m/s}^2 \cdot V_{max}^2 \cdot A} \\
&= 0.052\, V_{max}^2 \cdot A
\end{aligned}
$$

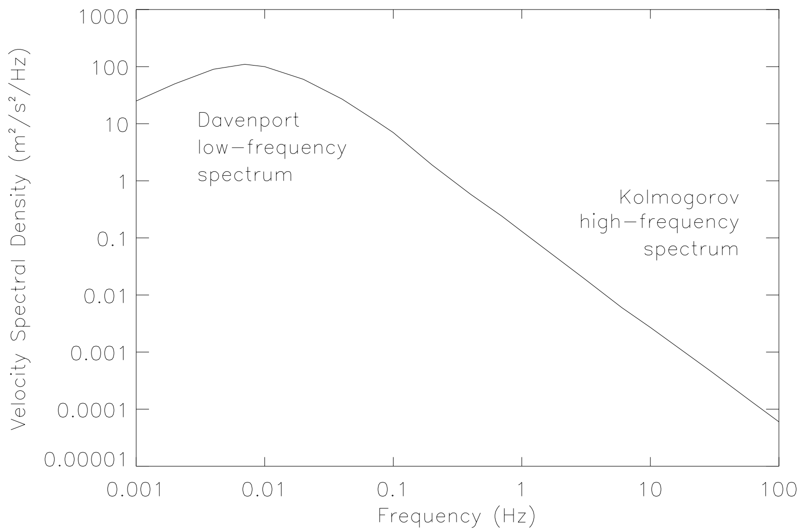

FIGURE 2. Typical wind velocity spectrum.

for A in m^2 and V_{max} in m/s.

For this estimate, the telescope tube is regarded as the object acted on by the wind. Approximate dimensions are shown in Figure 3. Assume that 50% of the area represented is acted on by the wind, or:

$$A \approx 0.5(3\,\mathrm{m})(1\,\mathrm{m}) \approx 1.5\,\mathrm{m}^2$$

Then:

$$F_{\mathrm{wind}} \approx 0.078\ V_{max}^2\ (\text{in kg}_\mathrm{f})$$

This will be taken as the force tending to rotate the telescope about the AZ axis (off-center design).

3. WIND FORCE ACTING ON THE OFF-CENTER AZ MOUNT

For the off-center design, the telescope tube c.g. will be about 1 m from the azimuth axis and will weigh about 700 kg (1500+ lb.) We will assume that all of the mass is concentrated at the c.g. and that the wind force acts at the c.g. This simplistic situation then appears as shown in Figure 4.

The rotation angle (θ) can be found from:

$$\begin{aligned} \theta &= 1/2\ \alpha t^2 \\ t &= \text{elapsed time (s)} \\ &= 1/f,\ \text{for } f \text{ in cycles/s} \end{aligned}$$

where α = angular acceleration due to F_{wind} and is defined from:

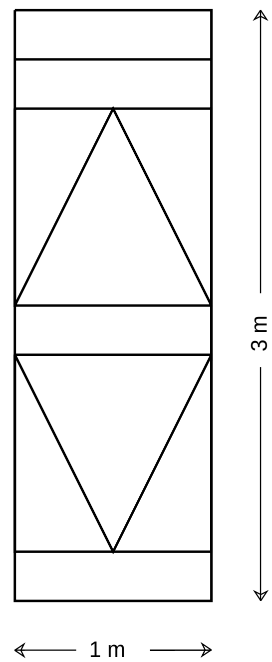

FIGURE 3. Approximate dimensions of the telescope tube.

$$
\begin{aligned}
T &= I \cdot \alpha \\
T &= F_{\text{wind}} \cdot r = F_{\text{wind}} \cdot 1\,\text{m} \\
I &= \text{torsional moment of inertia about AZ axis} \\
&= W/g \cdot r^2 \\
&= \frac{700\,\text{kg}}{9.8\,\text{m/s}^2}(1\ \text{m})^2 = 71.4\,\text{kg}\,\text{m}\,\text{s}^2 \\
\theta &= 1/2 \cdot T/I \cdot 1/f^2 \\
&= 1/2 \frac{F_{\text{wind}}(1\ \text{m})}{(71.4\ \text{kg}\,\text{m}\,\text{s}^2)} \cdot 1/f^2 \\
&= \frac{0.007 \cdot F_{\text{wind}}}{f^2} \\
&= 5.46 \times 10^{-4}\ V_{max}^2/f^2 \\
F_{\text{wind}}\ \text{in kg} &= 0.078\ V_{max}^2
\end{aligned}
$$

Here f is in cps and V_{max} in m/s.

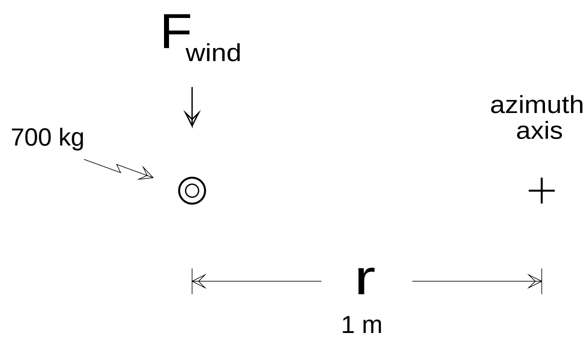

FIGURE 4. Simplified view of wind force acting on the off-center azimuth mount.

Since $V_{max} = 1.423\, f_1^{-1/3}$ from the earlier calculation,

$$
\begin{aligned}
\theta &= 5.46 \times 10^{-4} \frac{(1.423 f_1^{-1/3})^2}{f^2} \\
&= 11.1 \times 10^{-4} f_1^{-8/3} \\
\text{if } f &\equiv f_1
\end{aligned}
$$

The maximum allowable value for θ is set by the "budget" OPD variation ($11.4\,\mu$m) discussed earlier, or $\theta_{max} = 11.4\,\mu$rad, as seen in Figure 5.

The lower bound of the allowable, uncorrected higher frequency wind gusting is then:

$$
\begin{aligned}
f_1 &= \left[\frac{11.1 \times 10^{-4}}{\theta_{max}}\right]^{3/8} \\
&= \left[\frac{11.1 \times 10^{-4}}{11.4 \times 10^{-6}}\right]^{3/8} \\
&= 5.6\,\text{cycles/s}
\end{aligned}
$$

Conclusion: The off-center mount will require an azimuth drive able to correct rotational disturbances occurring at frequencies ≤ 5.6 Hz. Erring on the safe side, one should use 6 Hz as the minimum acceptable servo bandwidth with a goal of 10 Hz.

4. WIND EFFECTS ON THE ON-AXIS AZIMUTH MOUNT

The force produced by the wind acting on the telescope tube will tend to bend the yoke arms backward which will cause the tube to translate along the optical axis (horizon pointing situation — see Figure 6).

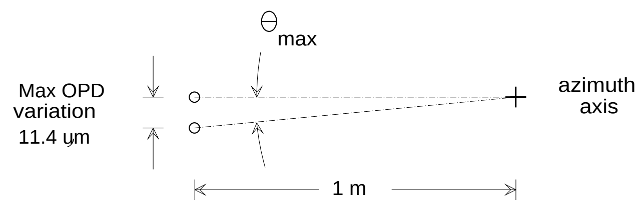

FIGURE 5. Maximum variation in the optical path length distance.

If the yoke is symmetric about its azimuth axis, there is little tendency for it to rotate. The OPD variation will arise mainly from yoke arm bending, cross-arm torsional wind-up, and deflections associated with the azimuth bearing and axle.

Unlike the off-center design, the on-axis design cannot rely on the drives to correct for any of the wind-induced OPD variation. Thus, the uncorrected wind force will be 100% of the horizontal component.

For this study, it will be assumed that the present tube design will be retained, which means the yoke arms will be about 1.5 m long. Two arms will resist the wind force. The cross-arm connecting the two vertical arms will tend to twist about its central axis. The bearing compliance is difficult to estimate but it can be preloaded to reduce internal clearances. For this study, 75% of the OPD budget will be assigned to the bearing.

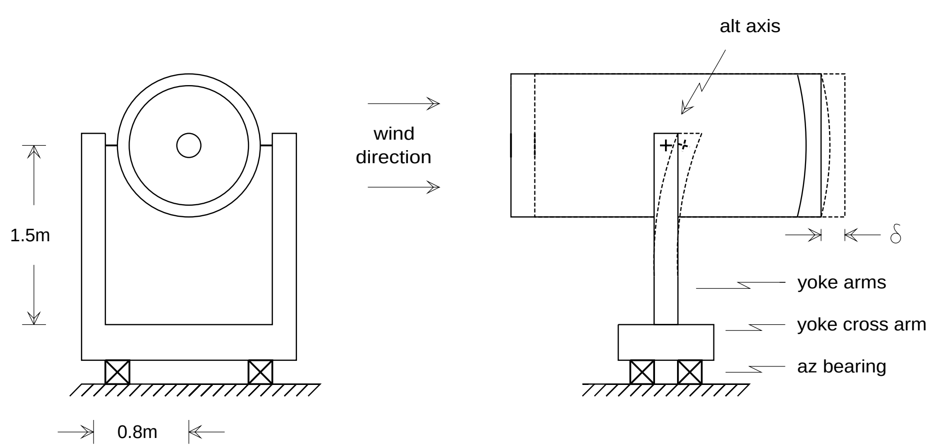

FIGURE 6. Bending of the yoke arms due to wind forces, for a telescope pointed at the horizon.

5. PEAK WIND VELOCITY ACTING ON THE ON-AXIS YOKE

The wind velocity spectrum used earlier is valid for the on-axis mount except that the low frequency components of the wind must be included.

The low frequency region from f = 0 to 0.01 Hz will be approximated by a constant = $100\,\mathrm{m}^2/\mathrm{s}^2/\mathrm{Hz}$, which assumes a steady wind below 0.01 Hz.

$$V(f) = 100 + 0.15\ f^{-5/3}$$

with units in $\mathrm{m}^2/\mathrm{s}^2/\mathrm{Hz}$.

Proceeding as before:

$$\begin{aligned}
V_{max} &= 3\left[\int V(f)df\right]^{1/2} \\
&= 3\left[\int_0^{0.01} 100df + \int_{0.05}^{\infty} 0.15f^{-5/3}df\right]^{1/2} \\
&= 3\left[100(0.01-0.)+0.225(0.01)^{-2/3}\right]^{1/2} \\
&= 3[1.0+4.85]^{1/2} = 7.3\,\mathrm{m/s}
\end{aligned}$$

6. WIND FORCE ACTING ON THE ON-AXIS MOUNT

The on-axis mount will experience a larger uncorrected wind force than the off-center style because:

1. The azimuth drive cannot correct front-to-back motions
2. The area of the yoke arms will also experience the wind in the front-to-back direction.

The maximum wind velocity was estimated to be ~7.3 m/s. A frontal area of $2\,\mathrm{m}^2$ (instead of $1.5\,\mathrm{m}^2$ for the off-center design) will be used. The wind force equation then becomes:

$$\begin{aligned}
F_{\mathrm{wind}} &= 0.078\ V_{max}^2\left(\frac{2.0\,\mathrm{m}^2}{0.5\,\mathrm{m}^2}\right) \\
&= 0.104\ V_{max}^2 \\
&\approx 5.6\,\mathrm{kg_f}\ (= 12.2\,\mathrm{lb})
\end{aligned}$$

Deflection at the altitude bearings will be the sum of the bending of arms and the deflection due to torsional twist of the crossarm, or:

$$\delta = \left(\frac{\frac{F_{\mathrm{wind}}}{2}\cdot l_1^3}{3EI}\right) + \left(l_1\cdot\frac{T_{\mathrm{wind}}\cdot l_2}{JG}\right)$$

where

$$\begin{aligned}
E &= \text{Young's modulus} \\
G &= \text{shear modulus} = 0.4\,E\ \text{(for steel)} \\
T_{\mathrm{wind}} &= F\mathrm{wind}\cdot l_1 \\
I &= \text{moment of inertia} \\
J &= \text{torsional moment of inertia}
\end{aligned}$$

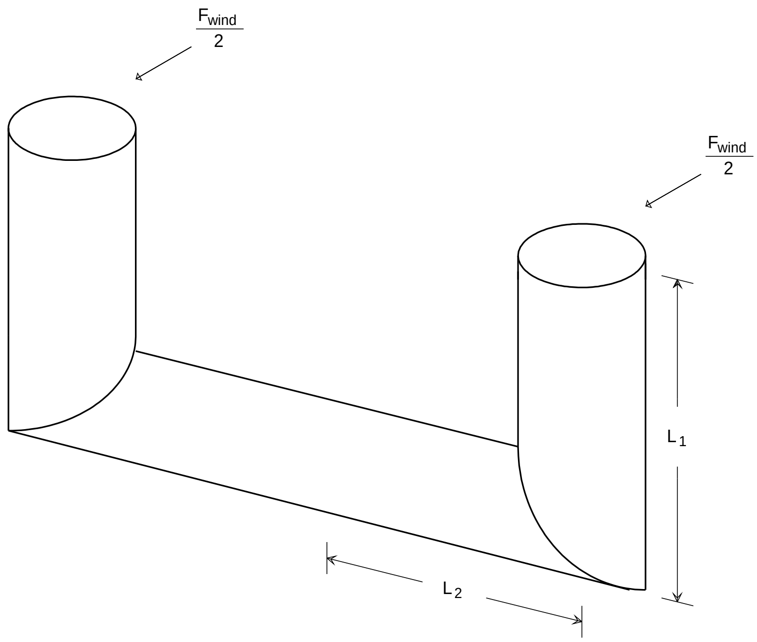

FIGURE 7. Wind forces acting upon the on-axis mount.

See Figure 7. If round tubes of equal size are used, J = 2I.
From the earlier discussion, $l_1 \approx 1.5\,\mathrm{m}$ and $l_2 \approx 0.8\,\mathrm{m} \approx 0.53\ l_1$, so

$$\begin{aligned}
\delta &= \frac{F_{\mathrm{wind}}}{2}\left[\frac{l_1^3}{3EI} + \frac{l_1^2 \cdot l_2}{2(0.4E)I}\right] \\
&= \frac{F_{\mathrm{wind}} \cdot l_1^3}{2EI}\left[1/3 + \frac{l_2}{0.8 l_1}\right] \\
&\approx \frac{F_{\mathrm{wind}} \cdot l_1^3}{2EI}
\end{aligned}$$

$$\begin{aligned}
E &= 30 \times 10^6\,\mathrm{lb/in^2}\ \text{(steel)} \\
&= 30 \times 10^6 \left(\frac{\mathrm{kg}}{2.2\,\mathrm{lb}}\right)\left(\frac{39.4\,\mathrm{in}}{\mathrm{m}}\right)^2 \\
&= 21169 \times 10^6\,\mathrm{kg/m^2}
\end{aligned}$$

This gives

$$\begin{aligned}
\delta &= \frac{(5.6\,\mathrm{kg})(1.5\,\mathrm{m})^2}{2(21169 \times 10^6\,\mathrm{kg/m^2})I} \\
&= 0.4464 \times 10^{-9}\,\mathrm{m^5}/I \\
\text{or}\quad I &= 0.4464 \times 10^{-9}\,\mathrm{m^5}/\delta \quad .
\end{aligned}$$

If δ is taken to be 25% of the allowable OPD variation of $11.4\,\mu$m, then

$$
\begin{aligned}
I_{\mathrm{req'd}} &= \frac{0.4464 \times 10^{-9}\,\mathrm{m}^5}{(0.25)(11.4 \times 10^{-6}\,\mathrm{m})} \\
&= 1.57 \times 10^4\,\mathrm{m}^4 \times \left(\frac{39.4\,\mathrm{in}}{1\,\mathrm{m}}\right)^4 \\
&= 377\,\mathrm{in}^4 \text{ in English units}
\end{aligned}
$$

For round tubes:

$$I = \frac{\pi}{64}\left(d_o^4 - d_i^4\right)$$

e.g., a standard 12 in diameter "extra strong" structural pipe (O.D. = 12.75 in, I.D. = 11.75 in) has I = $362\,\mathrm{in}^4$, which is close to the required value.

Conclusion: It is possible to limit yoke arm deflections to the desired range of OPD variation using structural elements (tubes, etc.) of practical sizes.

- This must be confirmed by FEA of the actual structure.
- Bearing deflections are still TBD.

7. REFERENCES

Davis, J., Lawson, P.R., Booth, A.J., Tango, W.J., & Thorvalson, E.D., "Atmospheric path variations for baselines up to 80 m measured with the Sidney University Stellar Interferometer", 1995, MNRAS, in press (18 Jan)