CHARA TECHNICAL REPORT

NO. 2 15 APR 1994

Visibility, Optical Tolerances, and Error Budget

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1. INTRODUCTION

Many effects combine to reduce the detected fringe visibility. Obtaining accurate visibility values thus requires correction and calibration. Some calibrations can be carried out from internal information (the measured optics aberrations, the tilt and OPLE servo errors), some can be estimated (atmospheric dispersion), some can be reduced by temporal averaging (photon noise) and some can only be compensated by observing a known source in alternation with the source of interest (atmospheric turbulence). Fortunately, experience with prototype interferometers has shown that visibility calibration to of order 1% is possible (Mozurkewich et al, 1991; Foresto et al. 1992).

It is expected that the performance of an interferometer will be limited by the atmosphere at some level. A reasonable criterion for instrumental optical quality is to not significantly degrade the atmosphere-limited potential. The objective of the CHARA array is to achieve this performance without relying on possible optical corrections from adaptive optics.

The more insidious impact of low visibility is reduced effectiveness of the fringe tracking scheme, thus a reduced sensitivity and limiting magnitude for the array. Correspondingly, any technique which enhances the detectability of fringes will extend the sensitivity of the array to fainter sources. The conditions for optimum fringe sensitivity are therefore of great interest, as are the possible contributions of adaptive optics to enhanced performance.

These issues will be discussed from the point of view of a visibility budget, which aids understanding of an interferometer much as an error budget aids in the understanding of other measurement systems.

The observing model will require fringe tracking only within the coherence envelope. Fringe locking, that is adjusting the optical path difference fast enough to effectively freeze the OPD, has many attractive features, but it will not be possible to employ this technique with faint sources. The most important observing technique is expected to employ short exposure data acquisition, with subsequent OPD compensation for rapid OPD variations based on OPD errors recorded with the fringe tracking system.

This analysis makes various simplifying assumptions and non-rigourous approximations. Where comparison with more elaborate models is possible the conclusions derived here hold up well.

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2. COMPONENTS OF THE VISIBILITY BUDGET

Reductions in the visibility will be introduced by wavefront degradation, in the general sense, including piston error (uncompensated differences in optical path), tilt error (differential wavefront tilt in the plane of recombination), and of course higher order wavefront error introduced by the atmosphere or by the optical components of the telescope and instruments.

In the discussion below, we will introduce the concept of a visibility transfer factor, which can be used to characterize the expected loss in raw observed visibility due to various components of the system and the observing process. We will discuss the optimization of interferometric signal-to-noise, and show that in some cases the greatest interferometric sensitivity may occur for relatively low instrumental visibility. Note that these various factors are interrelated, and a global numerical optimization should be carried out to include the additional interactions which are omitted here. However, with that caveat, our procedure will be as follows. For a baseline B, wind velocity v, seeing described by $r_{\circ}$, and coherence length computed from spectral resolution R, we will deduce the optimum time constant $\tau_{\rm coh}$ for updating the fringe envelope tracking. We will then find the optimum exposure time $\tau_{\rm opd}$ required to freeze the optical path fluctuation, and the optimum tilt servo time constant $\tau_{\rm tilt}$. We will find the optimum aperture size (when that size is smaller than the full available aperture) when limited by the atmosphere. We will also examine the fixed optical aberrations and establish suitable tolerances for the CHARA Array, and estimate the wavefront errors allowed by the residual aberrations. We will combine all of the foregoing to describe the visibility budget for several sets of operating conditions. Finally, we will discuss the impact of spatial filtering of the beams prior to detection.

2.1. The Interferogram

The following description of an interferogram captures enough of the dependence on OPD to characterize the major visibility losses. For the analytical description of interferograms, it is convenient to describe wavelength by its inverse, wavenumber, here represented by ν. Ignoring phase information, a two-beam interferogram, corresponding to a rectangular bandpass of width Δ_ν wavenumbers, and visibility V may be written,

$$I(x) = I_0[1 + V\cos(2\pi\nu_0 x)\mathrm{sinc}(\pi\Delta_\nu \mathrm{x})] \tag{1}$$

where x is the optical path difference.

The cos term describes fringe oscillations at the mean wavenumber ν_0, and the sinc term describes the envelope corresponding to the coherence length associated with spectral bandpass Δ_ν. A variety of effects ascribable to atmosphere, telescopes, instruments and observing procedure will depress the amplitude of the observed fringes. These effects can individually and collectively be viewed as a visibility transfer factor, which will be designated T.

2.2. Optical Path Difference

It is important to distinguish two time constants associated with OPD tracking. The exposure time for a single data sample, $\tau_{\rm opd}$, is limited by the requirement to minimize blurring of the fringes due to OPD change. The time allowed to update the OPD tracking, $\tau_{\rm coh}$, is limited by the coherence length associated with the resolution of the fringe detection technique.

2.2.1. Staying within the Coherence Length

According to the Kolmogoroff theory of atmospheric turbulence, the RMS OPD difference in waves between two telescopes separated by a distance B much less than the outer scale of turbulence will be (Beckers 1993),

$$E_{\rm opd} = 0.42(\frac{B}{r_\circ})^{\frac{5}{6}} \tag{2}$$

where $r_\circ$ is the (wavelength dependent) Fried parameter. A conservative assumption is that the outer scale exceeds the telescope separation and this equation applies.

This expression will be the basis for deriving estimates of the update time for the fringe tracker, $\tau_{\rm coh}$, and the frame time $\tau_{\rm opd}$ for the fringe detector. Since in general $\tau_{\rm opd} << \tau_{\rm coh}$, many exposures of the fringe signal can be used in computing each update of the fringe tracker.

If we designate a time constant by

$$\tau_{\rm B} = B/v \tag{3}$$

the typical rate of change of the OPD without fringe tracking will be

$$\frac{{\rm d}E_{\rm opd}}{{\rm d}t} \approx \frac{E_{\rm opd}}{\tau_{\rm B}} = \frac{0.42v}{B^{\frac{1}{6}} r_\circ^{\frac{5}{6}}} \tag{4}$$

Then an estimate of the OPD change in time $\tau_{\rm coh}$ will be,

$$\Delta_x \approx \frac{E_{\rm opd}\tau_{\rm coh}}{\tau_{\rm B}\nu} \tag{5}$$

For typical array dimensions $B \approx 100m$, the time constant τ_B will be of order ten seconds. It is important to understand that this is the characteristic time for which the OPD will wander over the range associated with baseline B. This is much longer than the exposure time (few milliseconds) required to freeze the fringes in a single data frame, which will be considered separately.

For a bright source, the fringe tracking can operate in locked mode, stabilizing the OPD to a fraction of a fringe. In this case, the depression of the fringe amplitude due to the coherence length will be negligible, since the OPD will remain locked to less than a wavelength. For faint sources, the fringe tracker can operate in envelope tracking mode, merely keeping the OPD within the coherence length, and requiring short integrations on the science detector. In this case the OPD may vary sufficiently to reduce the fringe amplitude.

The visibility transfer factor for loss of coherence will depend on the spectral passband. For a rectangular passband of width Δ_ν with an OPD of $\Delta_{\rm x}$ the visibility can be described by,

$$T_{\rm coh} = {\rm sinc}(\pi\Delta_\nu\Delta_{\rm x}) \tag{6}$$

In the limiting case of fringe tracking on a faint source, it will be important to maximize the signal-to-noise of the fringe detection. We can distinguish two cases: detector noise limited detection and photon limited detection. For photon limited detection, the quantity $\sqrt{\tau_{\rm coh}}T_{\rm coh}$ should be maximized. This condition corresponds to solving,

$$\tan(\frac{\pi E_{\rm opd}}{R\tau_{\rm B}}\tau_{\rm coh}) = 2(\frac{\pi E_{\rm opd}}{R\tau_{\rm B}}\tau_{\rm coh}) \tag{7}$$

for $\tau_{\rm coh}$ where $R = \Delta_\nu/\nu$ is the spectral resolution of the fringe detection. The solution, which may be found numerically, is,

$$\tau_{\rm coh} \approx 0.69 \frac{R\tau_{\rm B}}{\pi E_{\rm opd}} \tag{8}$$

and the corresponding visibility transfer factor is $T_{\rm coh} = 0.92$.

For the case of a detector noise limited fringe detector, it is necessary to recall that the signal will consist of multiple rapid reads of the fringe signal, so the signal-to-noise of the fringe will be proportional to,

$$(\frac{\tau_{\rm coh}}{R})^{\frac{1}{2}} T_{\rm coh} = (\frac{\tau_{\rm coh}}{R})^{\frac{1}{2}} {\rm sinc}(\frac{\pi {\rm E_{opd}}}{{\rm R}\tau_{\rm B}}\tau_{\rm coh}) \tag{9}$$

which has a maximum for the same condition as the photon noise limited case,

$$\tau_{\rm coh} \approx 0.69 \frac{R\tau_{\rm B}}{\pi E_{\rm opd}} \tag{10}$$

with a visibility transfer factor $T_{\rm coh} = 0.92$.

2.2.2. The Fringe Tracking Exposure Time

The exposure time, $\tau_{\rm opd}$, is determined in virtually the same way. The visibility transfer factor due to averaging an exposure over a time significant with respect to the atmospheric time constant, $\tau = r_\circ/v$, is (Lawson, 1993)

$$T_{\rm opd} = {\rm sinc}(\pi\nu\Delta_{\rm x}) \tag{11}$$

Note that the wavenumber, rather than the bandwidth, appears in the argument of the sinc function, so the expression does not depend on spectral resolution.

In this case the condition for a peak depends on the limiting noise. For the photon limited case, the maximum of $\sqrt{\tau_{\rm opd}}T_{\rm opd}$ is found for,

$$\tau_{\rm opd} \approx \frac{0.69\tau}{\pi E_{\rm opd}} = 0.52\tau \tag{12}$$

where now $E_{\rm opd}$ has been evaluated for $B = r_\circ$. Again, the visibility transfer factor for this condition will be

$$T_{\rm opd} \approx 0.92 \tag{13}$$

For the case of detector noise limited detection, assuming a fixed resolution R, we find the maximum signal-visibility product $\tau_{\rm opd}T_{\rm opd}$ for,

$$\tau_{\rm opd} = \frac{\tau}{2E_{\rm opd}} = 1.19\tau \tag{14}$$

with a visibility transfer factor of,

$$T_{\rm opd} = 0.64 \tag{15}$$

In practice there may be a number of contributions to the OPD error, which will be reflected in reductions of the visibility transfer factor. These will vary as the cos or sinc of the error in radians. Sources of OPD error include: fringe measurement error, both systematic and due to limited source flux; servo effects such as processor lag; metrology error; and possible coupling of tilt-correction and path difference. Note that the effect of finite pixel size, integrating over bandwidth, has already been taken into account implicitly in the foregoing discussion through the dependence of visibility on spectral resolution.

For the case of fringe locking, limited to relatively bright sources, the exposure time must be less than τ_{opd} by an amount dependent on the servo characteristics but probably about $10\times$ shorter.

2.3. Wavefront Tilt

At recombination, the beams should be precisely overlapped and the wavefronts should lie in the same plane. Considering these requirements separately instead of together will give a more stringent, but adequate, criterion for each.

Consider recombination in the pupil plane. For a small displacement of δ_{r} for a pupil of diameter D, the loss in visibility will be

$$T_{\mathrm{dis}} \approx \frac{8\delta_{\mathrm{r}}}{\pi D} \tag{16}$$

For recombination of source images the loss will be considerably smaller (the PSF effectively apodizing the images) and loss of visibility due to displacement will be ignored here.

For a tilt of one beam with respect to the other, the modulation will vary across the beam, depressing the observed visibility amplitude. For a relative tilt (angle at recombination) of α, the visibility will be reduced by the factor

$$T_{\mathrm{tilt}} = 2\frac{J_1(\pi\alpha\nu D)}{\pi\alpha\nu D} \tag{17}$$

Several factors will contribute to the value of α. The most important will the the time constant of the tilt correction, which will allow the tilt to move away from the desired value. This component will increase with time constant. The second important factor, at least for faint sources, will be the error in tilt measurement due to limited source flux. This component will decrease with the time constant, as more signal can be collected.

The RMS tilt error due to the atmosphere (without tilt correction) is expected to be (reference Parenti and Sasiela, 1993)

$$E_{\mathrm{tilt}} = \frac{0.42}{\nu D^{\frac{1}{6}} r_{\mathrm{o}}^{\frac{5}{6}}} \tag{18}$$

in radians on the sky. If we designate a time constant associated with tilt,

$$\tau_D = \frac{v}{D} \tag{19}$$

the typical tilt error accumulated in a time τ_{tilt} will be

$$E_{\mathrm{tilt}}\frac{\tau_{\mathrm{tilt}}}{\tau_{\mathrm{D}}} \approx \frac{0.42 v \tau_{\mathrm{tilt}}}{\nu D^{\frac{7}{6}} r_{\mathrm{o}}^{\frac{5}{6}}} = \xi\tau_{\mathrm{tilt}} \tag{20}$$

The quantity $\tau_{\rm tilt}$ is approximately the tilt servo time constant.

The parametric dependence of tilt detection on aperture and atmosphere can be estimated based on discussions of tilt detection (Appendix O and Parenti and Sasiela 1993). The RMS tilt measurement error $\sigma_{\rm tilt}$ can be written

$$\sigma_{\rm tilt} \;=\; \frac{\zeta}{\tau_{\rm tilt}^{\epsilon}} \tag{21}$$

where ϵ is 1/2 for the photon limited case and 1 for the detector noise limited case. The factor ζ includes geometrical factors and depends on the source flux available for tilt detection (e.g., see Equation O.10 in the CHARA Array Report.

The total tilt error from these two sources will be

$$\Delta_\alpha \;=\; \sqrt{\xi^2\tau_{\rm tilt}^2 + (\frac{\zeta}{\tau_{\rm tilt}^{\epsilon}})^2} \tag{22}$$

We wish to find the maximum value for

$$T_{\rm tilt} = \frac{2J_1(\pi\nu D\Delta_\alpha)}{\pi\nu D\Delta_\alpha} \tag{23}$$

This can be seen by inspection to occur for $\Delta_\alpha \;=\; 0$, which is not physically realistic. In the case of a finite tilt detector integration time, the optimum value for $T_{\rm tilt}$ will occur for the minimum value of the argument. For the case of detector limited tilt detection, this occurs for,

$$\tau_{\rm tilt} \;=\; \sqrt{\frac{\zeta}{\xi}} \tag{24}$$

For the case of photon limited tilt detection, the maximum corresponds to

$$\tau_{\rm tilt} \;=\; (\frac{\zeta^2}{2\xi^2})^{\frac{1}{3}} \tag{25}$$

The actual values clearly depend on the details of the tilt detector and noise sources. We note that fringe detection requires more flux than tilt detection, so commonly tilt detection will not be the limiting consideration. For this general discussion we assume that tilt is reduced by a factor of 10× relative to its uncompensated value $E_{\rm tilt}$. For a rather extreme example of $\frac{D}{r_\circ} \;=\; 4$, this implies a visibility transfer factor of $T_{\rm tilt} \;=\; 0.98$, so this source of visibility reduction is not an important issue.

There are numerous sources of differential wavefront tilt which must be considered in addition to the obvious residual tilt from the atmospheric tilt and the tilt corrector. There may be systematic tilt due to calibration of the tilt correcting system, and systematic refraction differences between the tilt detector wavelength and the science wavelength. There will be residual tilt error due to the finite bandwidth and limited stellar reference signal.

2.4. Higher Order Atmospheric Wavefront Errors

If tilt correction is employed without higher order adaptive optics, the residual higher order wavefront aberrations due to the atmosphere will be

$$\sigma_{\rm atmos}^2 = 0.134(D/r_\circ)^{\frac{5}{3}} \tag{26}$$

where σ here refers to mean square wavefront error in radians^2. Using the Maréchal approximation, the visibility transfer factor will be

$$T_{\rm atmos} \approx \exp(-\sigma^2_{\rm atmos}) \tag{27}$$

If adaptive optics are implemented, then there will still be a residual high order wavefront error, but it will be reduced. The amount of residual wavefront error will be a complex function of the AO system and observing conditions. In very rough approximation, we can use the Noll expression for just terms higher than Zernike polynomial order n,

$$\sigma^2_{\rm atmos} \approx 0.2944 n^{\frac{\sqrt{3}}{2}} (\frac{D}{r_\circ})^{\frac{5}{3}} \tag{28}$$

Improved estimates of the residual wavefront error can be obtained from a generic model of adaptive correction (Appendix S), and when a detailed AO concept is adopted, modeling can give even better values (e.g. Appendix Q).

2.5. Optical Aberrations

The CHARA optical system will have optical aberrations, most fixed and some varying slowly with telescope pointing, temperature, etc. Optical aberrations will contribute to a visibility transfer factor

$$T_{\rm opt} \approx \exp(-\sigma^2_{\rm opt}) \tag{29}$$

where $\sigma_{\rm opt}$ is the wavefront error in radians due to optical aberrations.

The quantity usually specified for small optics is the peak-to-valley in waves, $E_{\rm pv}$, which is related by,

$$\sigma^2_{\rm opt} \approx (\frac{2\pi E_{\rm pv}}{3.47})^2 \tag{30}$$

As of this writing, we expect for CHARA to specify most optics to normal, good research grade tolerances, both for cost-effectiveness, and because this quality will be sufficient for the highest priority CHARA science objectives. However, higher optical quality will have a role. This will be discussed below, but first we will discuss the implications of standard quality components.

We will require a wavefront quality from the telescope and beam compressors of 1/5 wave, and from the catseyes of 1/8 wave (in single pass). The requirement for all flats will be 1/20 wave. (On reflection from a flat at normal incidence, the wavefront quality will be 1/10 wave - doubling on reflection). On reflection at 45° incidence, the wavefront error will be further increased by $\sqrt{2}$ (projection effect). For transmissive elements, the wavefront error will be reduced relative to the surface error by $n-1$, where n is the refractive index of the material (here we take $n = 1.5$). Therefore a requirement of 1/10 wave flatness for each transmissive surface will suffice to keep the wavefront error contribution of these elements comparable to the other components.

The array will have approximately 44 optical surfaces, with the following contributions to to the wavefront error of the system. Surface qualities are described in waves peak-to-valley at 0.6328 micron.

The array will probably operate initially with tilt correction, but without adaptive optics. In that case, for short wavelengths and/or poor seeing, it will be advantageous to employ

TABLE 1. Error budget for the fixed optical aberrations in the CHARA Array. The column "spec" gives the specification for optical surface quality in waves peak-to-valley at 0.63 νm.

Component	Number Contributing	Spec (waves)	Contribution (waves)
Telescope:	1	0.20	0.20
Catseyes:	2 (dble pass)	0.13	0.18
Beam compressor:	1	0.20	0.20
Transmissive elements:	10 surfaces	0.10	0.16
Flats at normal incidence:	14	0.20	0.19
Flats at 45 deg incidence:	10	0.20	0.22
Root Sum Square Wavefront Error			0.47

Spec on all flats is 0.05, not 0.20

a reduced aperture for the interferometric combination. The optimum aperture size will depend on seeing, but in addition will depend on calibration issues which will be determined with experience. Here we will assume that an aperture diameter of 1.5 r_o will be utilized (up to 100 cm maximum, of course). For aperture diameters smaller than 100 cm, the wavefront error contributed by optical aberrations will scale approximately as the square of the aperture diameter, since figure errors in small optics are dominated by low order, and especially spherical, error. (The contribution of high order aberrations (irregularities) to the surface figure will be a significant parameter in the optical specification.)

This leads to the following table which describes the optical wavefront errors and the optical and atmospheric Strehls. For the atmospheric error, we assume that tilt power is reduced by 98%, hence the residual wavefront square error is given by $0.152(\frac{D}{r_o})^{\frac{5}{3}}$. This is for good seeing, r_o (0.5 μm) = 20 cm, which will be more demanding of optical tolerances than poorer seeing.

TABLE 2. Aperture, wavefront quality and Strehls, limiting the aperture D to no more than 1.5 r_o . For r_o = 20 cm at 0.5 micron.

Wavelength (microns)	Aperture D(cm)	D/r_o	Optics Wavefront (waves p-v) at λ	Strehl (optics)	Strehl (atmos)	Strehl (o&a)
0.5	30	1.5	0.053	0.99	0.74	0.73
0.63	40	1.5	0.074	0.98	0.74	0.73
0.8	53	1.5	0.103	0.97	0.74	0.72
1.0	69	1.5	0.141	0.94	0.74	0.70
1.6	100	1.2	0.19	0.89	0.80	0.71
2.2	100	0.85	0.13	0.94	0.89	0.84

This shows that relatively standard optical tolerances, which can be obtained at moderate cost and can be easily tested, will suffice to maintain a high optical Strehl for 1.5 r_o apertures. This is consistent with the assumption in Appendix R, so these optics will be adequate for achieving the most important CHARA science objectives.

In the foregoing, an aperture diameter $1.5\,r_\circ$ was selected in order to maintain relatively high atmospheric Strehl. When the CHARA array is operated to its ultimate sensitivity limit, in the sense of observing the faintest sources for which fringe tracking will function, a somewhat different strategy may be preferred. Specifically, the optimum aperture size will be the size which maximizes the product of flux and Strehl ratio, FS, where the detected flux will be proportional to D^2. Again assuming 98% reduction in tilt power, the aperture diameter D which maximizes D^2S is found to be $D = 3.48\,r_\circ$. (Note that this gives a relatively low atmospheric Strehl, $S = 0.30$.)

Again we take the case of $r_\circ = 20$ cm at $0.5\,\mu$m in order to explore the more stringent case of good seeing. For these assumptions, we find the following Strehl values.

TABLE 3. Aperture, wavefront quality and Strehls, limiting the aperture D to no more than $3.48\,r_\circ$. For $r_\circ = 20$ cm at 0.5 micron.

Wavelength (microns)	Aperture D(cm)	D/r$_\circ$	Optics Wavefront (waves p-v) at λ	Strehl (optics)	Strehl (atmos)
0.5	70	3.48	0.29	0.76	0.30
0.63	92	3.48	0.40	0.60	0.30
0.8	100	2.84	0.37	0.64	0.42
1.0	100	2.18	0.30	0.75	0.57
1.6	100	1.23	0.19	0.89	0.80
2.2	100	0.84	0.14	0.94	0.89

This table shows that under conditions of a relatively large aperture D, the Strehl loss due to optics quality with standard tolerances will be modest but significant. An improvement in optics tolerances by a factor of 2× would improve the optics Strehl to at least 90% at all wavelengths.

The strategy for achieving improved optical tolerances will have several parts. First, the CHARA goal will be to obtain optics 2× better than the minimum requirement whenever possible – we will use the leverage of our relatively large optics purchases to negotiate with vendors for the highest optical quality possible within the budget. Second, we will measure the fixed aberrations associated with each telescope-OPLE unit, and experiment with corrective phase plates to reduce these aberrations. This technique was used successfully with the Monte Porzio interferometer prototype (Tango, 1979). With the very narrow field-of-view of the interferometric system, this technique should be very effective. Finally, when adaptive optics are added to the array, the first benefit will be to compensate the remaining optical aberrations.

Another use for this kind of optimization is in finding the best combination of parameters for fringe tracking. Assuming that the initial implementation of the CHARA Array has optics and atmosphere as described above, the combined visibility transfer will be

$$T_{\rm atmos}T_{\rm opt} \;=\; \exp[-0.152(\frac{D}{R_\circ})^{\frac{5}{3}}]\exp[-\sigma^2_{\rm opt}D^4] \qquad (31)$$

where $r_\circ$ and $\sigma_{\rm opt}$ are adjusted to the fringe tracking wavelength. Optimum sensitivity to fringe detection should occur for the maximum value of $D^2T_{\rm atmos}T_{\rm opt}$ and $DT_{\rm atmos}T_{\rm opt}$ for detector noise and photon noise limited detection, respectively. A useful wavelength for fringe tracking is likely to be 0.8 μm . Adopting $r_\circ(0.5\mu\text{m}) = 0.1\,m$ and scaling σ_{opt} from 0.63 μm to 0.8 μm , the optimum aperture diameter can be found.

For the detector noise limited case, the optimum aperture is $D = 0.85\,m$, for $D/r_\circ = 2.4$ and $T_{\text{atmos}}T_{\text{opt}} = 0.41$, and for the photon limited case, $D = 0.65\,m$, for $D/r_\circ = 1.85$ and $T_{\text{atmos}}T_{\text{opt}} = 0.60$,

These results will be useful in estimating the limiting sensitivity for detecting fringes and fringe tracking.

2.6. Diffraction and Propagation

The issue of diffraction has been discussed extensively in Appendix E. There are basically two effects. For an ideal wavefront, there will be diffractive loss of flux owing to the telescope aperture. This loss will be a function of $ML\frac{D}{\lambda}$, where M is the magnification of the optical system (beam reduction factor) and L the propagation distance. The second loss occurs due to Fresnel diffraction of the turbulence induced wavefront structure, which will be a function of $ML\frac{D}{r_\circ}$. As shown in Appendix E, the actual effect is a complex interplay between these factors and the differential propagation distance for multiple beams. We do not have a simple approximation for diffractive visibility losses, but test calculations indicate that a visibility transfer factor, T_{diff}, will have a value in the vicinity of 0.9-1.0

2.7. Polarization

Mixing of polarizations can cause visibility errors for polarized sources, which are expected to be common among YSO's and any sources with circumstellar shell or disk structures. Polarization mixing can occur owing to differences in optical configuration from one optical beam to another, resulting in different angles of incidence on mirrors. Polarization mixing can also occur in transmissive elements as a result of inhomogeneous crystalline structure, possibly induced by strain. In CHARA, these effects will be kept low by employing nominally identical numbers and angles of reflection in all beams (Appendix D), and by choice of materials and tolerances for windows (Appendix F). The residual can be calibrated by observing sources whose polarization is minimal or understood. For observation of polarized sources, it will probably be necessary to insert polarizers as early as possible in the optical train.

3. EXAMPLE VISIBILITY BUDGETS

It will be clear from the foregoing discussion that the raw visibility will depend in a complex way on a number of parameters. We have not carried out an end-to-end calculation of visibility taking into account all effects and the interactions between them. The raw visibility can be estimated from the transfer factors noted above. These estimates should be conservative because considering each factor independently over estimates the visibility reduction.

For examples, we will take relatively favorable parameters, since the performance limits of the array under good conditions will be the most important scientifically. We take $r_\circ = 20\,cm$, $v = 5\,m/sec$, $B = 100m$, and resolution $R = 100$. Estimates of T_{diff} are taken from the most nearly comparable example in Appendix E.

Two cases will be shown as examples. For the case of a source at least 1 magnitude brighter than the faint limit, we will make the reasonable assumptions that $\tau_{\rm coh} = 0.1\tau_B$, and $\tau_{\rm opd} = 0.5\tau_D$. The expected visibility factors are collected in Table 4. The next to last column shows the product of the instrumental factors, designated $T_{\rm array}$, which should give a conservative estimate of the imaging efficiency of the facility. As discussed in Appendix S, the interferometric efficiency will be equal to the Strehl ratio, if the same Strehl applies to each beam, thus the expected interferometric efficiency is equal to $T_{\rm array}$, and not the square of this quantity. This may be compared to the $T_{\rm atmos}$, in the final column. It is clear from this comparison that in bright source operation both array and atmosphere will allow high Strehl and high efficiency operation.

TABLE 4. Visibility budget for a bright source under favorable conditions.

Wavelength	$T_{\rm coh}$	$T_{\rm opd}$	$T_{\rm tilt}$	$T_{\rm opt}$	$T_{\rm diff}$	$T_{\rm array}$	$T_{\rm atmos}$
0.5	0.99	0.93	0.98	0.99	0.96	0.86	0.74
2.2	0.99	0.93	0.98	0.94	0.98	0.83	0.89

A second case shown is for a faint source, at the limit of observation, for which the optimizations discussed will apply. In particular, at short wavelengths a larger aperture is utilized, as discussed in reference to Table 3. Here, the low value of $T_{\rm opd}$ is a consequence of optimization to reach the faint limit hence is the result of a tradeoff. In the infrared case, a more effective observing strategy, which will often be appropriate, will be to fringe track in the visible where lower noise detectors are available, keeping $T_{\rm opd}$ at values nearer 1.0.

The low $T_{\rm atmos}$ for the visible case illustrates the potential value of adaptive optics for the CHARA array. Clearly an immediate gain of more than a magnitude will be achieved here. (In fact, the aperture can be opened even larger than assumed in this illustration, gaining at least another magnitude.)

TABLE 5. Visibility budget for a faint source under favorable conditions.

Wavelength	$T_{\rm coh}$	$T_{\rm opd}$	$T_{\rm tilt}$	$T_{\rm opt}$	$T_{\rm diff}$	$T_{\rm array}$	$T_{\rm atmos}$
0.5	0.92	0.64	0.98	0.76	0.96	0.42	0.30
2.2	0.92	0.64	0.98	0.94	0.98	0.53	0.89

The example described in Table 4 (Bright source) should correspond approximately to the case evaluated in Appendix Q on performance limits. The parameters used for the illustration in Table 5 (Faint source) should reach fainter sources. Predicting the gain requires detailed modeling of the fringe tracking algorithm for multiple low signal-to-noise data sets. This is not trivial, involving pattern recognition issues. In very general terms, we note that fringe tracking within the coherence envelope allows the use of approximately $\tau_{\rm coh}/\tau_{\rm opd}$ data sets. This might suggest a potential gain in integrated signal-to-noise of

$$Gain = \sqrt{\frac{\tau_{\rm coh}}{\tau_{\rm opd}}} T_{\rm coh} T_{\rm opd} = 0.76\sqrt{\frac{RB}{D}} T_{\rm array} T_{\rm atmos} \tag{32}$$

Here, the T visibility factors allow for the reduction in visibility associated with optimizing $\tau_{\rm coh}$ for the faint limit, relative to the higher values appropriate for bright sources.

For large baselines and high resolution the potential gain is clearly large. For the parameters discussed above ($R = 100$, $B = 100$ m, and $D = 1$ m, and for the detector limited, faint object optimized case this predicts a gain of 48×, or 4 magnitudes at 2.2 μm . This gain would be in additional performance beyond the predictions of Appendix Q. At 0.5 μm , the predicted gains range from 18× without adaptive optics, to 44× with adaptive optics.

Simulations of fringe tracking (Buscher, 1988; Lawson, 1993) have so far shown significantly smaller gains than these. However, they have taken the approach of simply summing groups of channel spectrum power spectra. It may be that a pattern recognition approach could extend the applicability.

The amplitude of the potential increases in limiting sensitivity are clearly of great importance, justifying a substantial effort to realize fringe envelope tracking with low signal-to-noise. Of course it should be understood that at the faintest limit, only visibility information will be available (though with the spectral resolution R of course). At somewhat increased brightness, it will be possible to reconstruct the OPD variation (perhaps intermittently), and to post-correct the beam combiner data to extract phase information.

4. THE IMPACT OF SPATIAL FILTERING

The visibility transfer factors can be considered instrumental, even though the atmosphere is involved, since even the atmospheric effects depend on instrumental performance. The visibility loss factors fall into two groups. The first group is associated with optical path, and includes $T_{\rm coh}$ and $T_{\rm opd}$. Once the beams are combined nothing can be done to change these. The other groups of factors, including $T_{\rm tilt}$, $T_{\rm opt}$ and $T_{\rm diff}$ impact primarily the efficiency of overlap of the beams. This can be modified, in a sense, even after the beams are combined. By transmitting the beams through a spatial filter the spatially incoherent part of the wavefront can be rejected. When carrying this out after beam combination, the parameters $T_{\rm tilt}$, $T_{\rm opt}$ and $T_{\rm diff}$ will acquire a different significance. Rather than describing a depression of visibility, they will describe a transmission factor for the spatial filter, which will not transmit the corresponding incoherent fraction of the flux. The atmospheric induced wavefront distortions will be treated similarly.

As a result of spatial filtering, the transmitted light of each beam will be spatially coherent, allowing a clean measurement of the relative coherence between beams. However, the transmittance of the spatial filter will be substantially less than unity. Also, since the atmospheric turbulence is variable, the transmittance of the spatial filter will be variable as well.

In order to obtain correctly normalized visibilities, it is necessary to measure the flux of each beam after the spatial filter. In some beam combination schemes the beams are detected in combinations which allow the determination of each beam flux from a system of simultaneous equations. In schemes which combine all beams, this is not possible. In this case, it will be necessary to separately monitor the beam fluxes in order to calibrate visibilities.

Once the relative fluxes in the beams are known, the visibilities can be calibrated accurately for flux variations. Then only the visibility losses due to $T_{\rm coh}$ and $T_{\rm opd}$ remain to be considered. In the visibility budgets above, these losses correspond to 0.94 (bright source case) and 0.59 (faint case). These can be calibrated by reference to the fringe tracking error signal.

4.1. Summary

An examination of the major sources of visibility degradation shows that the array will be successful in preserving the intrinsic wavefront quality of the atmosphere. The required quality of optical components is similar to conventional optical tolerances. The array sensitivity can probably be extended 2-4 magnitudes fainter than the more conservative estimates described elsewhere. Several examples show the importance of adaptive optics in optimizing the performance of the array for faint sources and high dynamic range.

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