CHARA TECHNICAL REPORT

No. 39 18 November 1996

The Telescope Secondary as a Tip/Tilt mirror

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1. INTRODUCTION AND GENERAL INFORMATION

The CHARA Array will employ five 1-m size, alt-azimuth style telescopes at a site on Mount Wilson in southern California. The telescopes will be housed separately and operated remotely from a central laboratory. Light from each telescope will be directed by subsequent flat mirrors through vacuum pipes to additional optics and instrumentation at the central laboratory.

In order for the light to reach the central laboratory and for the fringes created to remain stable, or indeed to be measured at all, the average wavefront gradient introduced by the atmosphere, or tip and tilt, of the incoming beams needs to be corrected. Since the design of the telescope secondary includes three PZT actuators it is hoped that the secondaries themselves can be used as Tip/Tilt actuators. In this report I develop a simple analytical model for the secondaries and PZT drivers and show that the potential bandwidth meets the requirements set out in technical report 19.

2. SIMPLIFIED SECONDARY MODEL

The secondary mount system consists of several parts and is, unfortunately for this analysis, rather complex. Please refer to the telescope design drawings for a detailed description of the various parts and how they fit together.

In this analysis we shall use the physical model shown in figure 1, where the mirror and mount system are all included in one single cylindrical assembly held up by a rod connected to a large spring. This 'mirror' consists of the mirror glass, the mirror cell, and the mirror support. The assembly is attached to the rest of the secondary mount via the "Flex pin", which, as the name implies, will bend allowing the mirror and mount system to move about. The flex pin also provides part of the spring force necessary to bring the mirror back to the central position when the PZTs are turned off. The rest of the returning force is supplied by the large spring at the top of the assembly.

For most of this analysis we will assume the telescope is pointing to zenith, and therefore the weight of the entire mirror assembly will be acting against the spring.

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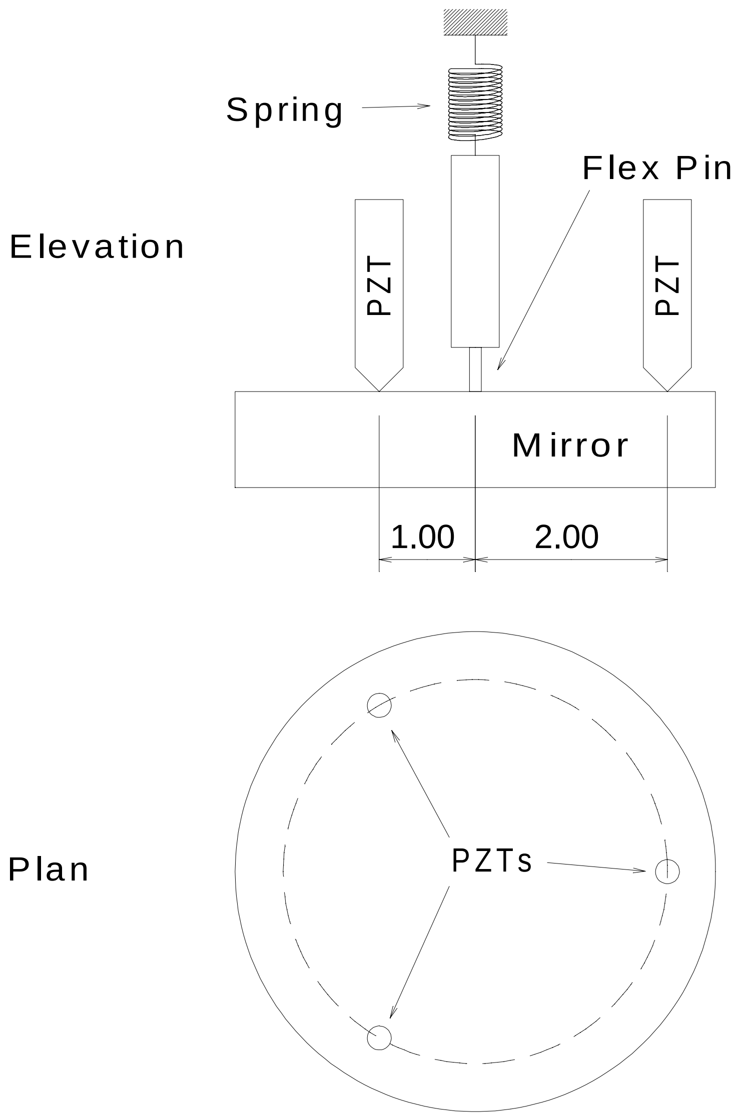

FIGURE 1. Plan and elevation of secondary model (Dimensions in inches).

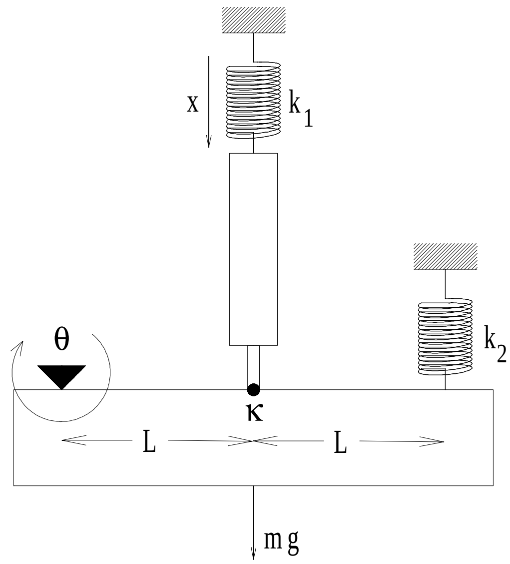

FIGURE 2. Dynamic model for mirror assembly.

Figure 1 shows that there are two PZTs acting on one side of the mirror while there is only one on the other side. However, the two PZTs are half the distance away from the pivot point. Thus the two PZTs will act as a single PZT at the same distance as the single PZT on the other side. None of the PZTs are actually attached to the mirror and so can not 'pull' on the mirror. However since there are pushing forces on both sides we will model the PZTs as a single simple spring on one side of the mirror. The fulcrum will be placed on the opposite side. The dynamic model of the system is shown in figure 2.

2.1. Mirror Mass and Moment of Inertia

The total mass of the secondary assembly, including the vertical rod connecting the secondary mirror to the rest of the mount is estimated to be about 8 pounds or 3.63 kg.

The total mass of the 'mirror' itself is the sum of the masses of the glass, the mirror cell and the mirror support. The glass can be modeled as a 1 inch thick cylinder of diameter 5 inches. Using the density of quartz silica of 2.2 grams per cubic centimeter results in a mirror mass of

$$m_{\mathrm{Mir}} \approx 2.54\pi \times (2.5 \times 2.54)^2 \times 0.0022\,\mathrm{kg} \quad (1)$$
$$= 0.71\,\mathrm{kg}. \quad (2)$$

The mass of the two metal components can be approximated by using the density for iron of 7 grams per cubic centimeter.

The mirror cell is a cylinder of diameter 5 inches and thickness of 0.38 inches, so it's mass is

$$m_{\mathrm{Cell}} \approx 2.54\pi \times 0.38 \times (2.5 \times 2.54)^2 \times 0.007\,\mathrm{kg} \quad (3)$$
$$= 0.86\,\mathrm{kg}. \quad (4)$$

The mass of the mirror support can be approximated by the sum of a large flat cylinder (thickness 0.29 inches and diameter of 5 inches) and a long hollow cylinder (length 1 inch, diameter 0.5 inches and central hole diameter of 0.25 inches). It's mass will then be

$$m_{\text{Sup}} \approx 2.54^3\pi\left[0.25^2 - 0.125^2 + 0.29\left(\frac{2.94}{2}\right)^2\right] \times 0.007\,\text{kg} \quad (5)$$

$$= 0.24\,\text{kg}. \quad (6)$$

The total mass of the mirror assembly is then

$$m = m_{\text{Mir}} + m_{\text{Cell}} + m_{\text{Sup}} = 0.71 + 0.86 + 0.24 = 1.81\,\text{kg}. \quad (7)$$

With an approximation of the mass we can now calculate the moment of inertia for the system. The moment of inertia of a system about a given axis is written

$$I = \int r^2 dm \quad (8)$$

where r is the distance from the axis, dm is in units of mass and the integral is taken over the entire body. For a disk rotating about a chord of the circular a distance d from the center of symmetry we have

$$I = \frac{2m}{\pi R^2}\int_{-R}^{R} dx \int_0^{\sqrt{R^2-x^2}} (x+d)^2\,dy \quad (9)$$

$$= \frac{2m}{\pi R^2}\int_{-R}^{R} \sqrt{R^2 - x^2}(x+d)^2\,dx \quad (10)$$

where R is the disk radius and m is it's mass. By the use of Mathematica we find that

$$\int_0^R \sqrt{R^2 - x^2}(x+d)^2\,dx \approx \sqrt{\pi}R^2(0.89d^2 + 0.22R^2) \quad (11)$$

and so

$$I = 2.99 \times 10^{-3}\,\text{kgm}^{-2}. \quad (12)$$

2.2. PZT Moment

The PZT will be modeled as a simple spring. The specifications for the PZT to be used in the secondary system are set out in Table 1.

TABLE 1. PZT Characteristics

Maximum Expansion	20μm
Stiffness	5.5 N/μm
Resonant Frequency	11 kHz
Maximum push	150N
Maximum pull	30N

We shall therefore use the spring constant

$$k_2 = 5.5 \times 10^6\,\text{Nm}^{-1} \quad (13)$$

Note that the maximum angular deviation possible will be when the two PZTs on one side are fully off and the single PZT on the other side is at maximum expansion, resulting in an angle of

$$\theta_{\max} = \frac{20 \times 10^{-6}}{3 \times 0.0254}\,\mathrm{rad} \approx 3 \times 10^{-4}\,\mathrm{rad} = 54\,\mathrm{arcsec}. \tag{14}$$

Since the telescope is really an eight times beam reducer this angle projects to 6.75 arcsec on the sky, which should be enough to cover any seeing quality good enough to allow interferometry.

2.3. Main Spring and Flex Pin Spring Constants

The main spring is rated as having a spring constant of

$$k_1 = 8.3\,\mathrm{lb/in} = 1455\,\mathrm{Nm}^{-1} \tag{15}$$

In section 3.1 of the notes accompanying the telescope design drawings Larry Barr states that the "moment required to bend the flex pin through 1 degree is estimated to be less than 0.7 in-lb". This means that the "moment spring constant" for the flex pin will have a maximum value of

$$\begin{aligned} \kappa_{\max} &= 0.7 \times 0.0254 \times 4.45 \times \frac{180}{\pi}\,\mathrm{mN\,rad}^{-1} & (16) \\ &= 4.531\,\mathrm{mN\,rad}^{-1}. & (17) \end{aligned}$$

3. DYNAMIC MODEL OF MIRROR SYSTEM

We are now in a position to derive the equations of motion for the system and search for a resonance frequency. We will use the x and θ parameters shown in figure 2 to describe the system.

The kinetic energy of the system is given by

$$V = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}I\dot{\theta}^2 \tag{18}$$

while the potential energy of the system is

$$T = -xmg + \frac{1}{2}k_1x^2 + 2k_2x^2 + \frac{1}{2}\kappa\theta^2 \tag{19}$$

where we have used the fact that for any angle θ, the PZT spring will have to be compressed twice as much as the main spring.

Note that the two parameters x and θ are related via

$$\theta \approx \frac{x}{L} \tag{20}$$

and so

$$\begin{aligned} V &= \frac{1}{2}\left(m + \frac{I}{L^2}\right)\dot{x}^2 & (21) \\ &= \frac{1}{2}A\dot{x}^2 & (22) \end{aligned}$$

and

$$T = -xmg + \frac{1}{2}\left(k_1 + 4k_2 + \frac{\kappa}{L^2}\right)x^2 \quad (23)$$

$$= -xmg + \frac{1}{2}Bx^2. \quad (24)$$

The Lagrangian of the system is therefore

$$L = T - V = -xmg + \frac{1}{2}Bx^2 - \frac{1}{2}A\dot{x}^2. \quad (25)$$

The equation of motion can then be found by using the relation

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = \frac{\partial L}{\partial x} \quad (26)$$

which results in

$$-A\ddot{x} = -Mg + Bx. \quad (27)$$

This is a simple harmonic oscillator (surprise!!!!) with the resonance frequency

$$\omega_0 = \sqrt{\frac{B}{A}} \quad (28)$$

$$= \sqrt{\frac{k_1 + 4k_2 + \frac{\kappa}{L^2}}{m + \frac{I}{L^2}}}. \quad (29)$$

The enumerator is dominated by the PZT spring constant k_2 while the denominator is dominated by the mass of the assembly. Substituting the values derived above for the various parameters we find that the resonance frequency is

$$f_0 = 340\,\text{Hz}. \quad (30)$$

If we calibrate the system performance so that the 0 Hz response is set to 0 dB we can write the response of the system as

$$R(\omega) = \frac{\omega_0^2}{\omega_o^2 - \omega^2} \quad (31)$$

which will go to infinity when $\omega = \omega_0$. Obviously any real system will be damped, but we do not have enough information to estimate the damping coefficient. Instead, we re-write equation (31) in the damped harmonic oscillator form

$$R(\omega) = \frac{\omega_0^2}{\sqrt{(\omega_o^2 - \omega^2)^2 + (\omega\omega_0/Q)^2}}. \quad (32)$$

Figure 3 contains a plot of the mirror response function for various values of Q.

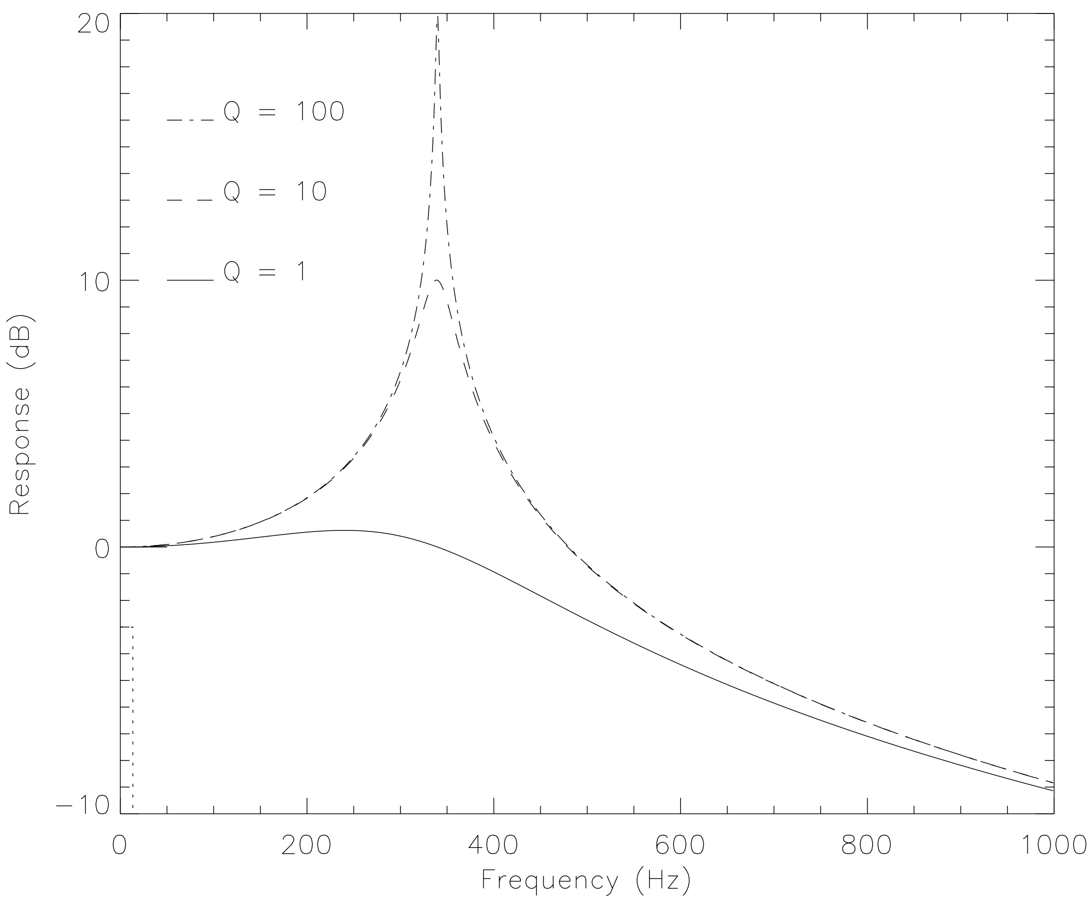

FIGURE 3. Mirror system response function for various Q values.

4. SYSTEM CHATTER

As discussed above, the PZTs are not connected to the mirror assembly, and the return force is almost completely supplied by the main spring. If the PZT retracts at a faster rate than the spring is able to move the mirror the system will 'chatter', something we would like to avoid. It is necessary therefore to estimate at which frequency this will start to occur.

With the telescope at zenith the spring is acting against the gravitation force of the mirror assembly. The telescope documentation specifies that the spring is extended approximately half an inch further than is required to hold the mirror up. The force acting upwards will therefore be

$$F = k_1 \times 0.0254 \times 0.5 = 18.5\,\mathrm{N}. \tag{33}$$

When the telescope is pointing away from zenith this force will be greater, but we can use this figure as a worst case. The torque acting on the mirror will therefore be

$$\tau = LF = I\ddot{\theta} \tag{34}$$

resulting in an angular acceleration of

$$\ddot{\theta} = \frac{LF}{I} = 156\,\mathrm{rad\,s^{-2}} \tag{35}$$

In section 2.2 we derived the maximum angular movement to be approximately 3×10^{-4} radians and so, assuming the mirror is being driven in a sine-wave, the motion of the mirror will be

$$\theta(t) = 3 \times 10^{-4} \cos \omega t \tag{36}$$

which has a maximum angular acceleration of

$$\ddot{\theta}_{\max} = 3 \times 10^{-4} \omega^2. \tag{37}$$

Combining equations (35) and (37) we find that chattering will start to occur when $\omega = 721$ which is at a frequency of 115Hz. This represents the maximum frequency we can expect to reliably drive the mirror.

5. CONCLUSION

In technical report 19 it is stated that the resonance frequency of the Tip/Tilt mirror should be greater than 100Hz and it's high frequency throw should be 10 arcsec. As far as the maximum throw is concerned the current design comes very close and in combination with the micrometer motors easily exceeds the requirement. In terms of bandwidth this analysis shows that the secondary exceeds the requirement, and the frequency at which 'chatter' begins meets the requirement.

We can therefore conclude that, assuming it will not excite resonances in the telescope mount itself, the secondary mirrors can be used for Tip/Tilt correction and it will not be necessary to buy additional Tip/Tilt mirrors.