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Wobbler Servo Control Requirements

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1. INTRODUCTION AND GENERAL INFORMATION

No fringes will be detected or measured from an astronomical interferometer unless there is a tip/tilt, or "wobbler," servo functioning on each input beam. The wobbler system consists of three components: the detector and control parts (to be described in this report), and the tip/tilt mirrors. In the case of the CHARA Array, the telescope secondary mirrors will be used to implement tip/tilt corrections. If we assume that the faintest objects will be observed using passive fringe tracking, the wobbler servo will determine the magnitude limit for the entire array. When tip/tilt tracking fails the entire array fails.

The detectors for the wobbler system are placed on an optical table within the beam combining lab (BCL) together with the visible imaging system. These two system share 50% (one polarization) of the light which has been split from the incoming beams using polarizing beam splitters. After the polarization split, 30% of this light is used for the wobbler detection system, and the remaining light is allocated to the visible imaging system. I will assume, however, that the visible imaging system will remain 'on hold' and that the entire amount of light in a single polarization will be available for tip/tilt measurement. Figure 1 shows a schematic of the detector optical layout.

2. FUNCTIONAL DESCRIPTION

The wobbler servo control systems must be capable of measuring the image position for each input beam, create a running mean of this position over an operator specified time period, use this new image position to calculate a new mirror position and send this mirror position out to each of the telescope control computers. All this must be performed within one sample time, which is at this time nominally 1 millisecond.

When viewed from the direction of the incoming beam, the quadrants are labeled as shown in Figure 2. Each quadrant counts the number of photon events received during a single sample time, and the raw counts in sample k in quadrant A will we denoted by $N_{A,k}$ with similar expressions for the other three quadrants. The control computer reads these raw counts each cycle, calculates a new value for the running mean for each quadrant and multiplies by a weighting factor to correct the count for sensitivity differences between the

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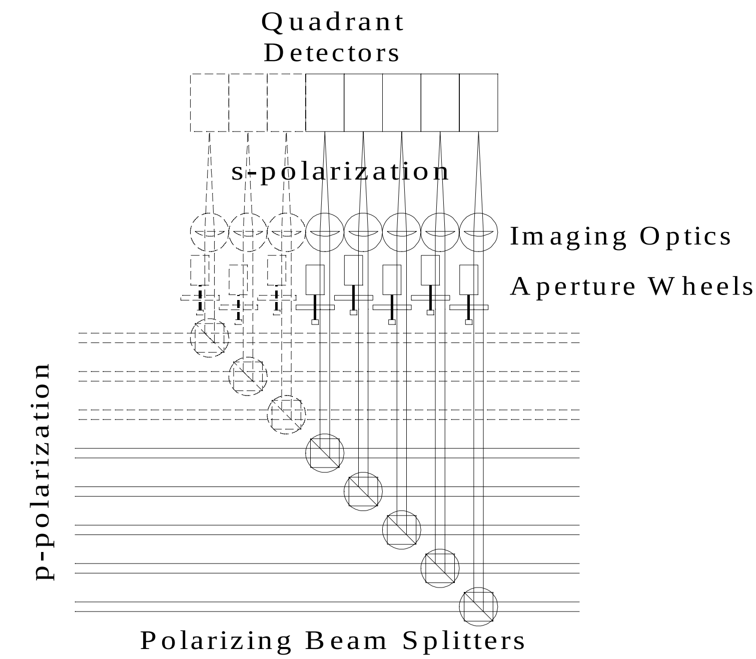

FIGURE 1. Schematic of the Wobbler optical layout.

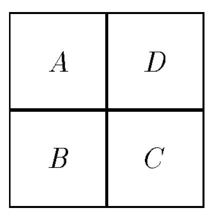

FIGURE 2. Definition of quadrant detector channels as viewed from the front of the quadrant detector.

quadrants. The final value for the photon count in quadrant A will then be given by

$$\mathcal{N}_{A,k} = W_A \times \frac{T}{T_d} \sum_{j=1}^{T_d/T} N_{A,k-j+1} \quad . \tag{1}$$

Here T is the sample period, T_d is the length of the running mean, and W_A is the weighting factor.

With the new weighted mean, the normalized position of the image can be calculated for the horizontal and the vertical positions using

$$\begin{aligned} \Theta_{h,k} &= \frac{(\mathcal{N}_{A,k} + \mathcal{N}_{B,k}) - (\mathcal{N}_{C,k} + \mathcal{N}_{D,k})}{(\mathcal{N}_{A,k} + \mathcal{N}_{B,k} + \mathcal{N}_{C,k} + \mathcal{N}_{D,k})} \text{ and} \\ \Theta_{v,k} &= \frac{(\mathcal{N}_{A,k} + \mathcal{N}_{D,k}) - (\mathcal{N}_{B,k} + \mathcal{N}_{C,k})}{(\mathcal{N}_{A,k} + \mathcal{N}_{B,k} + \mathcal{N}_{C,k} + \mathcal{N}_{D,k})} \quad . \end{aligned} \tag{2}$$

Both $\Theta_{h,k}$ and $\Theta_{v,k}$ will be restricted to values between -1 and $+1$. If there are no photons detected, the position is defined to be zero in both axes.

When the position of the stellar image is close to the origin the response of the detector is very linear. The linearity of the function decreases as the image moves farther away from the origin. When the servo is working, one must assume the image remains close to the origin, thus the normalized detected position is given by

$$\theta_v = K_d \Theta_v \quad , \tag{3}$$

where θ_v is the vertical beam tilt in arc-seconds and K_d is a calibration constant. Under the assumption of linearity, the calibration constant can be approximated using (Tyler & Fried 1982)

$$K_d = \frac{3\pi}{16}\frac{\lambda}{D} \quad , \tag{4}$$

where D is the beam diameter and λ is the wavelength.

The measurement of θ_v and θ_h represent the error signal for the wobbler servo control loop. Since the horizontal and vertical channels will have essentially the same behavior, we shall only consider a single channel and write the stellar position during the k^{th} sample period as θ_k.

We shall use a simple proportional/damping feedback system to control the mirror position, leaving the integral term to the wobbler/telescope servo loop. The algorithm to be used in the control computer, using C_1 as the constant of proportionality and C_2 as the damping constant, is

$$\underbrace{o_k - o_{k-1}}_{Change\,in\,output} \quad = \quad \underbrace{C_1\,\theta_{k-j}}_{Proportional\,term} \quad - \quad \underbrace{C_2\,(o_{k-1} - o_{k-2})}_{Damping\,term} \quad . \tag{5}$$

This control algorithm yields a new mirror position for each telescope secondary that will center the stellar images in each beam. Before this new position can be used, however, the field rotation due to the skew reflections in the light pipe beams and the Alt/Az mount of the telescope needs to be taken into account. To do this we first consider a reflection of a single light beam on a single mirror and define two unit vectors, $\hat{\mathrm{l}}_0$ for the incoming ray and $\hat{\mathrm{l}}_1$ for the out-going ray. The unit vector that describes the mirror position required to reflect $\hat{\mathrm{l}}_0$ into $\hat{\mathrm{l}}_1$ is the normalized vector average of the two, given by

$$\hat{\mathrm{m}} = \frac{\hat{\mathrm{l}}_1 - \hat{\mathrm{l}}_0}{\sqrt{2(1 - \hat{\mathrm{l}}_0 \cdot \hat{\mathrm{l}}_1)}} \quad . \tag{6}$$

This can be inverted to yield an expression for $\hat{\mathrm{l}}_1$ in terms of $\hat{\mathrm{l}}_0$ and $\hat{\mathrm{m}}$ resulting in the reflection formula

$$\hat{\mathrm{l}}_1 = \hat{\mathrm{l}}_0 - 2\,(\hat{\mathrm{m}} \cdot \hat{\mathrm{l}}_0)\,\hat{\mathrm{m}} \quad . \tag{7}$$

The transform from $\hat{\mathrm{l}}_0$ to $\hat{\mathrm{l}}_1$ can also be written in the matrix form

$$\hat{\mathrm{l}}_1 = \mathcal{R}\,\hat{\mathrm{l}}_0 \quad , \tag{8}$$

where

$$\mathcal{R}_{i,j} = \mathcal{I}_{i,j} - 2 \times m_i m_j \tag{9}$$

and $\mathcal{I}$ is the 3×3 unit matrix

$$\mathcal{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (10)$$

With these two equations and the analysis presented in TR 48, it is possible to calculate the rotation of the field of view as the Alt/Az axis change. Table 2 contains a list of the ray vectors, starting with $\hat{\mathrm{l}}_0$, the ray from the stellar target to the primary mirror of the telescope, through to $\hat{\mathrm{l}}_{21}$, the ray reaching the quadrant detectors in the BCL. One or two more reflections may be used to get the beam onto the quadrant cell, but as the exact form of these reflections is unknown at this time I will stop at this ray. Also note that the results presented here are bound to have an odd number of -1 factors incorrect. In this table, the ray directions leading in and out of the OPLE carts and beam reducing telescope have been set to angles measured from the drawings of the OPLE carts. I have used the convention of Azimuth being zero due north and increasing towards the east, with the y axis due north, the x axis due east and the z axis straight up. Altitude angles are measured from horizontal at zero to straight up at 90 degrees.

TABLE 1. Light ray unit vectors.

$\hat{\mathrm{l}}_0$	$[-\cos(\mathrm{Alt})\sin(\mathrm{Az}), -\cos(\mathrm{Alt})\cos(\mathrm{Az}), -\sin(\mathrm{Alt})]$
$\hat{\mathrm{l}}_1$	$-\hat{\mathrm{l}}_0$
$\hat{\mathrm{l}}_2$	$\hat{\mathrm{l}}_0$
$\hat{\mathrm{l}}_3$	$[\cos(\mathrm{Az}), -\sin(\mathrm{Az}), 0]$
$\hat{\mathrm{l}}_4$	$[\ 0.0000,\ 0.0000, -1.0000]$
$\hat{\mathrm{l}}_5$	$-\hat{\mathrm{l}}_3$
$\hat{\mathrm{l}}_6$	$[\ 0.0000,\ 0.0000, -1.0000]$
$\hat{\mathrm{l}}_7$	$[-0.8269, -0.5602,\ 0.0490]$
$\hat{\mathrm{l}}_8$	$[\ 0.9816, -0.1600,\ 0.1040]$
$\hat{\mathrm{l}}_9$	$[-0.1537,\ 0.9879,\ 0.0189]$
$\hat{\mathrm{l}}_{10}$	$[\ 1.0000,\ 0.0000,\ 0.0000]$
$\hat{\mathrm{l}}_{11}$	$[\ 0.0000,\ 1.0000,\ 0.0000]$
$\hat{\mathrm{l}}_{12}$	$[-1.0000,\ 0.0000,\ 0.0000]$
$\hat{\mathrm{l}}_{13}$	$[\ 0.0000,\ 0.0000,\ 1.0000]$
$\hat{\mathrm{l}}_{14}$	$[\ 1.0000,\ 0.0000,\ 0.0000]$
$\hat{\mathrm{l}}_{15}$	$[-0.9969,\ 0.0786,\ 0.0000]$
$\hat{\mathrm{l}}_{16}$	$[\ 0.9969,\ 0.0786,\ 0.0000]$
$\hat{\mathrm{l}}_{17}$	$[-1.0000,\ 0.0000,\ 0.0000]$
$\hat{\mathrm{l}}_{18}$	$[\ 0.9969,\ 0.0786,\ 0.0000]$
$\hat{\mathrm{l}}_{19}$	$[-1.0000,\ 0.0000,\ 0.0000]$
$\hat{\mathrm{l}}_{20}$	$[\ 0.0000, -1.0000,\ 0.0000]$
$\hat{\mathrm{l}}_{21}$	$[-1.0000,\ 0.0000,\ 0.0000]$

Using Equation 6 and the vector positions in Table 2 it is possible to calculate the mirror unit vectors. The first six mirrors in the system move around with the telescope and so their effect on the image reaching the BCL must be calculated for each Alt/Az position, with one calculation about every minute probably yielding enough precision for the tip/tilt servo to operate. The remaining mirrors, $\hat{m}_6$ through $\hat{m}_{21}$ are fixed and the unit vectors for these mirrors are listed in Table 2. The combined effect of this set of mirrors can be calculated

TABLE 2. Mirror unit vectors.

$\hat{m}_7$	[−0.5709, −0.3868, 0.7242]
$\hat{m}_8$	[0.9759, 0.2160, 0.0297]
$\hat{m}_9$	[−0.7022, 0.7100, −0.0526]
$\hat{m}_{10}$	[0.7595, −0.6504, −0.0124]
$\hat{m}_{11}$	[−0.7071, 0.7071, 0.0000]
$\hat{m}_{12}$	[−0.7071, −0.7071, 0.0000]
$\hat{m}_{13}$	[0.7071, 0.0000, 0.7071]
$\hat{m}_{14}$	[0.7071, 0.0000, −0.7071]
$\hat{m}_{15}$	[−0.9992, 0.0393, 0.0000]
$\hat{m}_{16}$	[1.0000, 0.0000, 0.0000]
$\hat{m}_{17}$	[−0.9992, −0.0393, 0.0000]
$\hat{m}_{18}$	[0.9992, 0.0393, 0.0000]
$\hat{m}_{19}$	[−0.9992, −0.0393, 0.0000]
$\hat{m}_{20}$	[0.7071, −0.7071, 0.0000]
$\hat{m}_{21}$	[−0.7071, 0.7071, 0.0000]

using Equation 9 for each mirror in Table 2 and multiplying the resulting matrices together to yield

$$\mathcal{R}_{7..21} = \mathcal{R}_{21}\mathcal{R}_{20}\cdots\mathcal{R}_{7} = \begin{bmatrix} 0.8268 & 0.5601 & -0.0490 \\ 0.5622 & -0.8209 & 0.0998 \\ 0.0157 & -0.1101 & -0.9938 \end{bmatrix}. \tag{11}$$

A similar matrix $\mathcal{R}_{1..6}$ can be calculated for the first six mirrors for any given Alt/Az position. With these two matrices it is possible to calculate the transform between the axis of the telescope and the axis of the quadrant detectors. The two unit vectors describing the coordinate frame of the image in the telescope are given by

$$\hat{u} = [\cos(\mathrm{Az}), -\sin(\mathrm{Az}), 0] \tag{12}$$

and

$$\hat{v} = [-\sin(\mathrm{Az})\sin(\mathrm{Alt}), -\cos(\mathrm{Az})\sin(\mathrm{Alt}), \cos(\mathrm{Alt})] \ , \tag{13}$$

where $\hat{u}$ is the unit vector in the azimuth direction and $\hat{v}$ is the unit vector in the altitude direction. After passing through the optical chain these unit vectors are mapped to

$$\hat{u}' = \mathcal{R}_{7..21}\,\mathcal{R}_{1..6}\,\hat{u} \tag{14}$$

and

$$\hat{v}' = \mathcal{R}_{7..21}\,\mathcal{R}_{1..6}\,\hat{v} \tag{15}$$

and these two vectors will lie in the YZ plane defined by the unit vectors $\hat{\jmath}$ and $\hat{k}$, the same plane as the quadrant detectors themselves. The field rotation, not including some sign

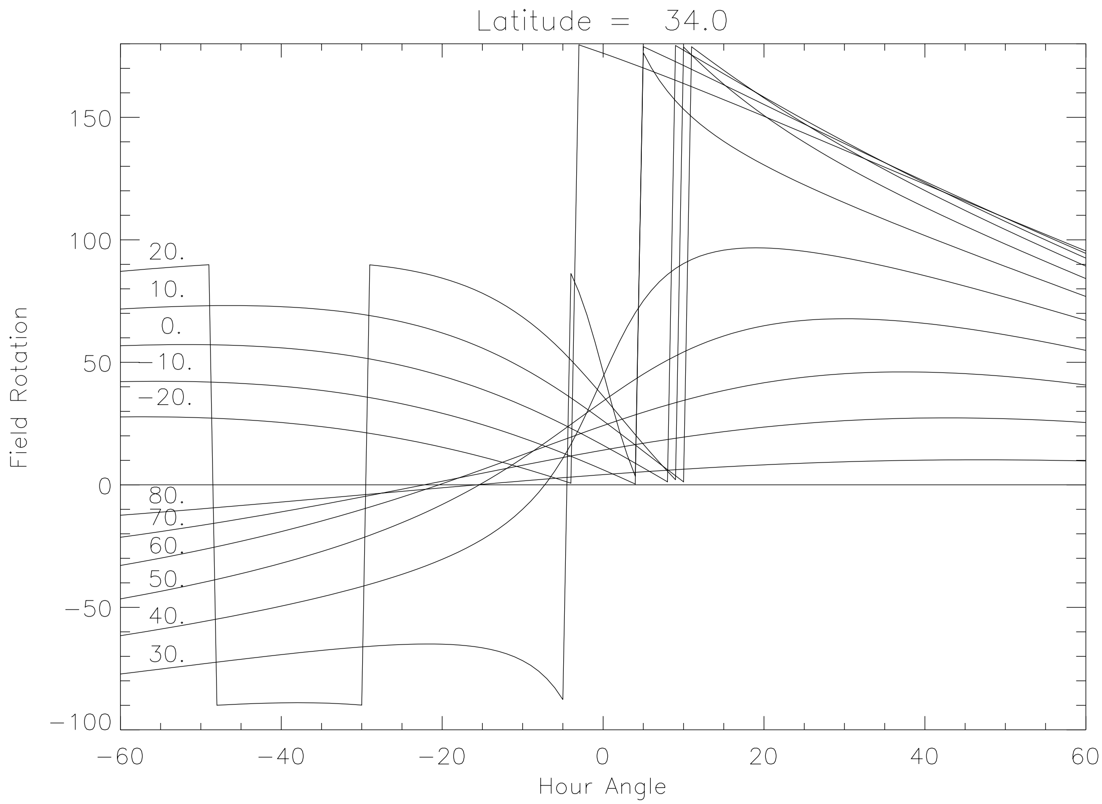

FIGURE 3. Rotation of telescope axes on quadrant detector for a range of declinations.

changes, can then be found by comparing $\hat{u}'$ and $\hat{v}'$ to $\hat{j}$ and $\hat{k}$. It may be beneficial to rotate the quadrant detectors themselves to be aligned with the permanent rotation due to mirrors 7 through 21.

A plot of the total field rotation for various declinations and a range of hour angles is given in Figure 3. It remains to be decided if this rotation calculation is performed by the tip/tilt computer in the BCL or by the computer running the secondary in the telescope domes. Since all telescopes will have to be pointing in the same direction, only one matrix needs to be calculated; therefore it may be best to do this in the wobbler control computer. On the other hand, each telescope control computer knows the current Alt/Az position, while the tip/tilt computer needs to get these data from the outside.

Once they have been converted to the Alt/Az coordinate system, the new secondary positions must be transformed into signals for the three piezo actuators that move the mirror itself. This final calculation may be performed either in the wobbler control computer, or by a computer in the telescope domes. Figure 4 shows the layout of the piezos behind the secondary. The equation relating the vertical and horizontal coordinates (V, H) with the three piezo positions (X, Y, Z) is

$$X = -\frac{1}{3}V - \frac{\sqrt{3}}{2}H$$

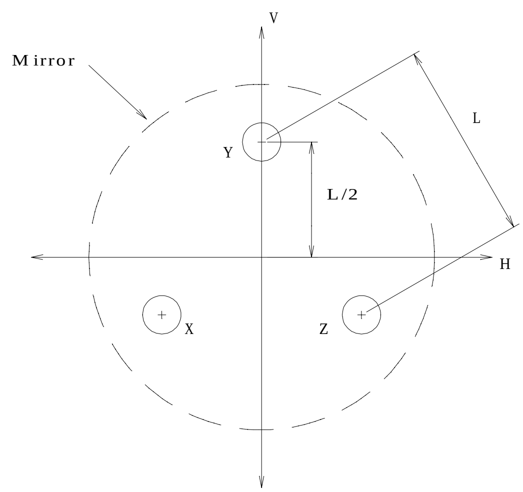

FIGURE 4. The axes used to define mirror response. The position of the mirror can be represented as a vertical and horizontal tilt (V and H) or as the positions of the three actuators (X, Y and Z).

$$\begin{aligned} Y &= \frac{2}{3}V \\ Z &= -\frac{1}{3}V + \frac{\sqrt{3}}{2}H \quad . \end{aligned} \tag{16}$$

These equations are correct for an infinitely thin mirror and may have to be modified somewhat to ensure that the secondary introduces no path modulations as it tracks the wobbler signal.

Apart from closing servo loops, the wobbler control system will also need to produce spatial seeing estimates for calibration of the visibilities and for monitoring the seeing during an observational run. This is done by logging the mirror position. It will be necessary to calibrate the mirror movement so that the image position on the sky can be calculated. This can be done interferometrically by using the internal laser light source, creating fringes and moving the secondary around. With the secondary tilted, linear fringes should be produced in the aperture plane, and the spatial frequency of these fringes, ω, is related to the mirror tilt angle via

$$\theta_{\rm mirror} = \frac{\lambda\omega}{4} \quad . \tag{17}$$

The mirror position will need to be multiplied by 8 to project these angles onto the sky.

With this mirror calibration it is possible to obtain measurements of Fried's parameter r_0. The wobbler system tracks wavefront tilt, and it would even be possible to add in a correction for the residual errors measured on the detector system. With these data it will be relatively easy to measure the total power, or variance, of tilt in each axis, $\sigma^2_{\rm tilt}$, which is related to the coherence length (Noll, 1976) via

$$\sigma^2_{\rm tilt} = 0.183\,(D/r_0)^{\frac{5}{3}}\,(\lambda/D)^2\ {\rm rad}^2. \tag{18}$$

Thus an r_0 measurement can be made in each axis of each telescope. This will be useful for also measuring, and hopefully tracking down, internal seeing problems.

The final part of the wobbler servo system is the feedback loop with the telescope drives. If the secondary is moved too far from its central position while tracking image motion it is necessary to move the telescope itself to re-center the secondary. Thus the wobbler system acts as a tracking signal for the telescopes. Either a low-pass filter, or defined 'active areas' and 'dead-zones', or perhaps both, need to be implemented on the tip/tilt signal being sent to the telescope secondaries, which can then be used as an error signal for the telescope tracking software to use. Once again, this filtering operation can be done either by the wobbler control computer, the computer controlling the secondaries or the main telescope control computer. I think the latter is most appropriate, but we do not need to make a final decision on this for the time being.

3. HARDWARE LIST AND COST ESTIMATE

Figure 5 contains a schematic of the hardware required to run the tip/tilt detector system. These components are listed in Table 3 along with cost estimates and the source of these estimates. As for the telescope control system, many of the price quotes I have received seem very high, considering the small number of components the units contain (the amplifiers and discriminators, for example). Substantial savings could be made by making these components ourselves. Of course, no real savings may eventuate due to the manpower investment required. The decision between build and buy will have to made based on available manpower and budgetary constraints.

4. REFERENCES

Noll, R.J. "Zernike Polynomials and Atmospheric Turbulence", JOSA, **66**, 207-211, 1976

Tyler, G.A. & Fried, D.L., "Image-Position Error Associated with a Quadrant Detector", JOSA, **72**, 804-808, 1982

TABLE 3. Control system costs.

Item	Source	Unit Cost	Number	Total Cost
Polarizing beam splitters	(estimate)	$750	5	$3750
Beam splitter mount	Newport PO80BL	554	5	2770
Iris (manual control)	Newport ID1.5	100	5	500
Mount for iris	Newport Stem	30	5	150
Achromat lens	Newport PAC079	113	5	565
Lens mount and stem	Newport LH-150	51	5	255
Objective lens	Newport M-20X	100	5	500
Mount and stem	Newport Stem	30	5	150
Motorized XYZ stage	New Focus 8062	1900	5	9500
Motor driver	New Focus 8801	1350	1	1350
Quad cell photomultiplier	Hamamatsu H6600	677	5	3385
HV power supply	Hamamatsu 3350	2877	1	2877
Discriminators	Hamamatsu	755	20	15100
Mounts	Newport Stem	30	5	150
Industrial PC	Web searches	3500	1	3500
Multiport RS232	Web searches	500	1	500
10×16bit 7Meg counter card	National 776452-01	335	2	670
Connector block	National 776164-01	150	2	300
Cabling	(estimate)	2000	1	2000
Total				$47,972

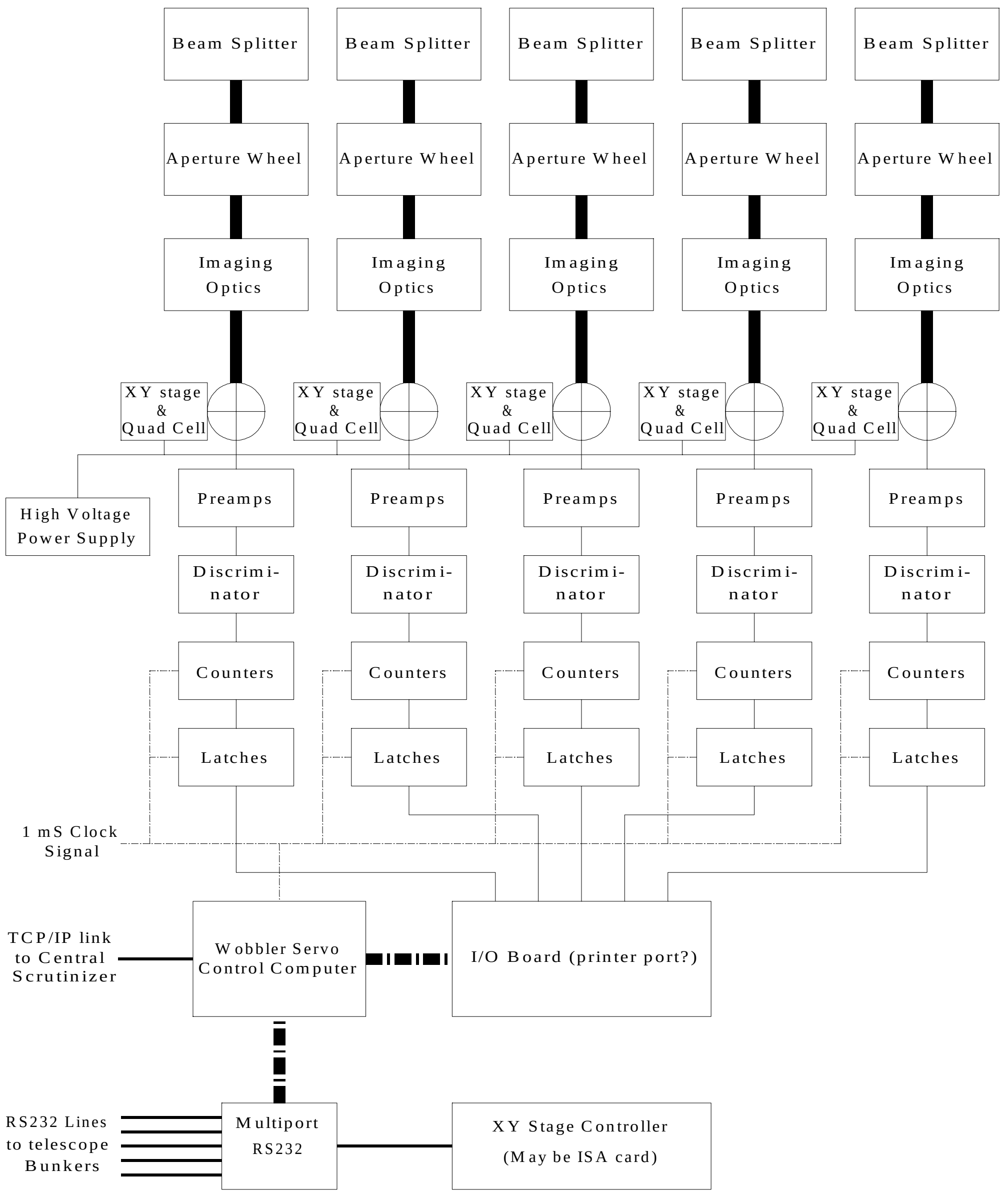

FIGURE 5. Schematic of the Wobbler servo control system hardware.