Array Configuration

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B.1. INTRODUCTION

The telescope array configuration chosen must enable the scientific goals of the interferometer to be met. Especially when imaging is required, an adequate $u-v$ plane coverage is essential. The cost and complexity of the array increases with the number of telescopes, and a trade-off must be made between the cost and $u-v$ coverage (imaging fidelity), and observing efficiency. Among the many issues that must be addressed are:

1. What is the basic array geometry? Should the array be linear or two-dimensional?
2. Should there be a small number of moveable[1] telescopes or a larger number of fixed ones?
3. If fixed telescopes are used, how many telescopes are necessary?
4. What are the best configurations for a given number of telescopes?
5. Is it cost-effective to make some telescopes repositionable[1] in order to give different $u-v$ coverages for, say, the visual and IR wavebands?

B.2. BASIC ARRAY GEOMETRY

The so-called $u-v$ plane coverage is determined by the projection of a baseline from two telescopes of an array up to the celestial sphere. Two geometrical transformations are needed to relate the (x, y, z) coordinates on the ground to the $u-v$ coverage in the sky. The first transformation is between the (x', y', z') coordinate system on the ground and a system in which the North Celestial Pole is straight up (x,y,z).

In the (x', y', z') system, x' is directed east, y', north, and z', the altitude, toward zenith; δ is the declination. In the standard array coordinate system (Thompson *et al.* 1986, p. 83) y is directed due east, x south toward the intersection of the meridian and the Celestial Equator, and z is directed toward the North Celestial Pole (NCP).

This transformation may be summarized by:

$$\begin{vmatrix} x \\ y \\ z \end{vmatrix} = \begin{vmatrix} 0 \\ 1 \\ 0 \end{vmatrix} \begin{vmatrix} -\sin\delta & \cos\delta & x' \\ 0 & 0 & y' \\ \cos\delta & \sin\delta & z' \end{vmatrix} \tag{B.1}$$

[1] Note the distinction between these terms. By *moveable* telescopes, we mean telescopes which can be relocated to different observing stations on a rapid (i.e. minutes to hours) timescale. By *repositionable* telescopes, on the other hand, we mean telescopes which would remain fixed for fairly long observing periods and only moved on timescales of weeks to months.

or $X = A_1 X'$ in vector notation.

A further transformation must be made to the (u, v, w) system in which w is directed toward the star, v is directed perpendicular to w along a great circle from the star to the NCP, and u is directed perpendicular to the other two axes. Equivalently, u and v can be represented as vectors directed from the observer: v is on the great circle containing the star and the NCP at an angle of δ from the NCP, and u is directed along the celestial equator at $-H$ (hour angle) from the east direction. (Figures 4.5 and 4.6 from Thompson *et al.* may be helpful.)

This transformation is thus:

$$\begin{vmatrix} \mathrm{u} \\ \mathrm{v} \\ \mathrm{w} \end{vmatrix} = \begin{vmatrix} \sin H & \cos H & 0 \\ -\sin\delta\cos H & \sin\delta\sin H & \cos\delta \\ \cos\delta\cos H & -\cos\delta\sin H & \sin\delta \end{vmatrix} \begin{vmatrix} x' \\ y' \\ z' \end{vmatrix} \tag{B.2}$$

or

$$U = A_2 X = A_1 A_2 X'.$$

From Equation B.2 it can be shown that

$$u^2 + (v - zcos\delta)/sin\delta)^2 = x^2 + y^2 \tag{B.3}$$

Thus, for a given baseline at a given declination, the $u - v$ coverage (trajectory) is an ellipse centered at $(u, v) = (0, z\cos\delta)$ with a semimajor axis of $(x^2 + y^2)^{1/2}$.

B.3. MOVEABLE VS. FIXED TELESCOPES, AN EXAMPLE

A number of investigations have been carried out by CHARA aimed at answering the interrelated questions posed in the introduction. One of these analyses considered the first question, that of the fixed versus moveable telescopes. Two systems were considered – a fixed system of 7 telescopes versus a system of three moveable telescopes. Three moveable telescopes provide one closure phase, so this is the minimum system that could still provide imaging. That comparison was overwhelmingly in favor of the fixed telescopes. We consider here a perhaps fairer comparison of the seven fixed telescopes to four moveable telescopes, which would provide six simultaneous baselines and three closure phases.

Table B.1 gives the results of this comparison. The criteria used are costs (construction and operating), construction time, performance, and other factors. Of these, the construction costs are pretty well known for telescopes, but only approximately for a transportation system. Telescope costs from our lowest-cost vendors turned out to be roughly \$400K. The moveable rail system was estimated to cost some \$750K plus three telescope *pickers* of \$100K [one for each baseline], as well as \$150K in R&D costs for a total of about 1.2M\$. Basically, including the OPLEs, the costs of the systems are roughly comparable, although there would be some additional cost savings in the four-telescope system due to the possibility of using mirrors instead of fiber couplers for the visible beam combining system.

The construction time is perhaps the least known factor. One argument against the moveable system is that the design of the transportation system is a task comparable in complexity to designing the telescope, and therefore will certainly use a lot of our limited R&D budget and time.

The performance criterion contains several rough estimates of *performance factors.* For example, the much greater variety of baselines available on a given night with the fixed telescopes should result in better scheduling, as well as the collection of visibility data from 21 baselines at a time, instead of 6. For imaging, 15 closure phases are available with the fixed telescopes, versus 3 for the moveable ones. A possible advantage of the four moveable telescopes is a greater throughput from mirrors instead of fiber couplers in the beam combining system. But, even taking favorable factors into account, a performance factor of 0.3 for the moveable telescope system is generous. Finally, the telescopes can be used for spectroscopy or other programs (primarily on nights of poor seeing). The seven telescope fixed array is equivalent to a single telescope of 2.6 m diameter, and is clearly superior for these purposes.

Thus, Table B.1 shows that the cost/benefit ratio is at least 3:1 in favor of the fixed telescopes. The construction time, R&D effort, and site adaptability also weigh in favor of the fixed telescope concept.

B.4. LINEAR VS. TWO–DIMENSIONAL ARRAYS

The University of Sydney group has concentrated on building a single 640 m north–south baseline for SUSI. The primary reason is that observations of accurate star diameters and limb darkening effects can be done with one-dimensional visibility data. In our original proposal we did an analysis of three possible one-dimensional baselines: N-S (North-South), E-W and 45° slant NW–SE. Six stations were assumed, three each on either side of the combining house at staggered intervals. The approximate coverage in the $u - v$ plane was determined for stars of declinations 50°, 20°, and 0° for baselines at latitude 35°. A limit of 50° in zenith distance was assumed.

Figure B.1 shows the $u - v$ plane coverage for the three baselines for the three star declinations (δ). At higher declinations, the coverage approaches 40% of the whole $u - v$ plane for the E–W and slant cases and about a third for the N–S. At lower declinations, the N–S baseline tends to have more complete coverage. At the equator the E–W coverage is reduced to a line. On the other hand, the maximum projected baseline for the N–S case is foreshortened for stars with high or low δ by a factor of cos (1 - δ).

Basically, given the gaps in $u - v$ coverage, linear configurations seemed adequate for stellar diameter or limb-darkening measurements, marginal for orbits of binaries, and completely inadequate for imaging. Given that it costs little more for full 2-dimensional coverage at our preferred site, it did not seem worthwhile to consider the possibility of a linear array further.

B.5. OPTIMAL TWO–DIMENSIONAL ARRAYS (FIXED TELESCOPES)

Although useful for stellar diameter measurements, a one–dimensional array is inadequate even for the relatively simple task of localizing the true position angle and separation of double stars. Consider the case of a double star at declination 20°, with a position angle of 90° or 270° (*i.e.,* along the E–W baseline). Even at the greatest allowable hour angles one has only 34% of the maximum baseline with which to resolve the stars. Another problem with depending on diurnal motion for better $u - v$ plane coverage is that up to 6–8 hours may

TABLE B.1. Fixed vs Moveable Telescopes

Criteria	**Fixed (7 telescopes)**	**Moveable (4 telescope)**
Costs:		
Constructions Costs:		
Telescopes+OPLEs	7 tel. @ \$400K = \$2.8M	4 tel @ \$450K + 4 extra fixed sites @ \$50K = \$2.00M
	7 OPLEs @ \$100K =\$0.7M	4 OPLEs @ \$100K =\$0.4M
Transportation System	none	\$1.5M x 50% (donations) = \$750K for rail system + 3 telescope pickers = \$300K R&D \$150K, total = \$1.2M
Subtotal	\$3.5M	\$3.6M
Operating Costs:		
Maintenance (Tels)	\$25K	\$14K
Utilities	\$15K	\$15K
Transportation System		
Crew (Move Tels)		\$75K
Observer/Operator(s)	\$75K	\$75K
Subtotal	\$115K	\$201K
Construction Time:	extra time for telescope construction and mirror fabrication	extra time for R&D of transp. system (prob. 6 mon. longer)
Performance:		
Baselines Available		
(visibility)	21	6
(closure phases)	15	3
Performance Factor	1.0	≈0.3
Beam Comb. Thruput P.F.	1.0	1.50
Mapability of System P.F.	1.0	0.75
Extra telescope Aperture (spectroscopy)	+20% (2.6 m telescope equivalent from 7 tel)	+12% (2.0 m tel equiv.)
Total Performance Factor	1.2	0.45
Other Factors:		
Local Seeing	easier to raise tel. higher off ground, thereby improving the seeing	
Sites	can have 10° slope	limited to 3° slope
Environmental Impact	more benign	tree removal for rails
Expansion to longer baseline	relocate existing telescopes	easier w/o losing existing capability
Aesthetic	7 tel. Y or spiderweb configuration attractive	railroad
Operation: breakdown	better for breakdown (redundancy)	added problem with tel. mover breakdown scheduling vulnerable, down time
Weather		more susceptible to snow

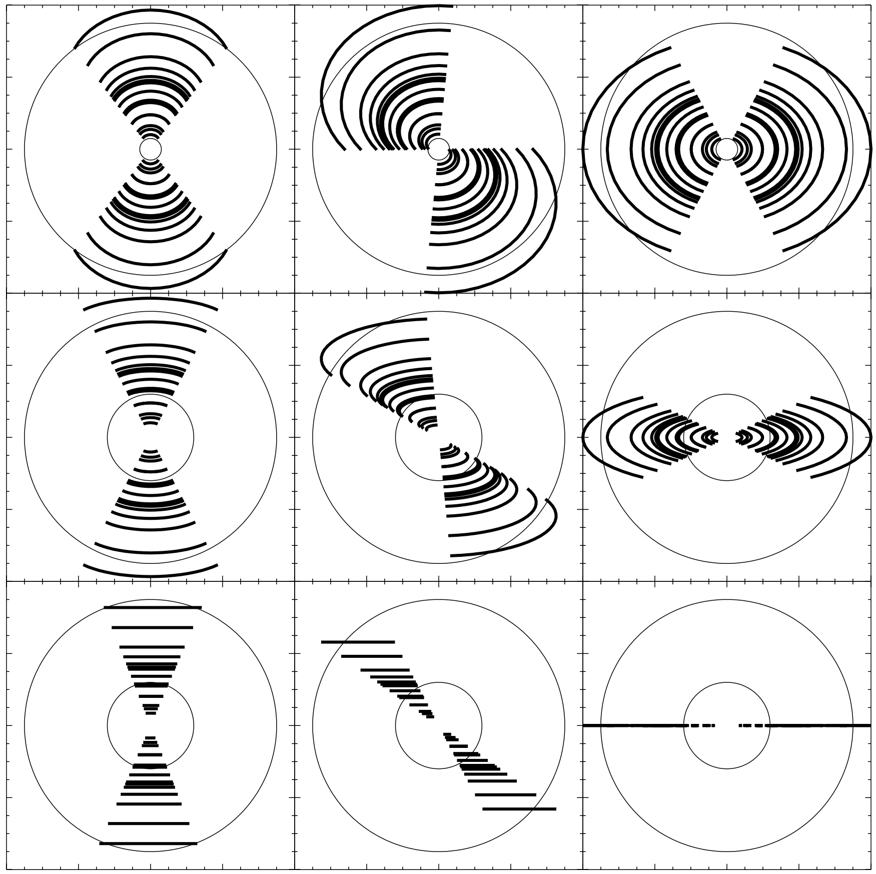


FIGURE B.1. The $u - v$ coverages for three linear arrays with orientations (left to right) of N–S, NW–SE, and E–W are shown for several declinations (top to bottom): 50°, 20°, and 0°.

elapse between observations, which may be too long for short period binaries. Of course, the main reason for choosing two-dimensional arrays is for imaging more complex objects than double stars, a potential that is greatly enhanced by the extensive $u - v$ coverage of 2-d arrays.

For the above reasons, therefore, we have considered only two-dimensional arrays for our interferometer. One conclusion from the one-dimensional study that also applies to the two-dimensional case is that each telescope should have its own tube to the central combining house [unlike SUSI]. Although optimal solutions to the one–dimensional array problem exist, no general solution to the two-dimensional case has been found. A method described below, based on the one-dimensional case in a paper by Seielstad *et al.* (1979) was used to find optimal arrays. For a given number of telescopes, the $u - v$ plane coverage was optimized for a number of trial configurations for both a Y shaped array and a more general *cobweb* array by means of the following procedure:

- For 3 to 9 telescopes the coverage of an array was calculated for latitude 35°, a limit of 40° elevation, and for declinations of 20°, 50°, and 0°. The coverage was also calculated for a single meridian observation at 20° declination.
- The Y arrays were defined by axes going out from the central combining house at

position angles of 0° (North), 120° (East), and 240° (West). For each run the initial three telescopes are located at ρ and θ = (195 m, 0°), (200, 120°), and (205, 240°). The slight asymmetry aides the search procedure. Additional telescope positions are chosen at random in cyclic order so that the fourth telescope is added on the North line, the fifth on the East line, and so on. The radial position of these telescopes is randomly selected between 0 and 200 meters.

- For a given set of telescope positions the $u-v$ plane coverage is calculated at intervals of 50 hour angle locations from the maximum to minimum hour angles consistent with the 40° elevation limit. The points in the $u-v$ plane are *quantized* to the nearest grid points of a 32×32 pixel grid in which the maximum $u-v$ plane separation of 354 m corresponds to a radius of ten pixels (35 m/pixel). Thus, points within a distance of about 20 m from the $u-v$ plane trajectories are "covered". (A circle equal area has a 20 m radius.) If an off–axis alt–az telescope design is used, points up to 3 m from the trajectory are covered, but coverage of points within 20 m is a reasonable goal consistent with the targeted science. For the instantaneous meridian case a 3×3 pixel region is covered for each point along the trajectory, corresponding to a 60 m radius.

- Points were also linearly weighted with a weight of 2.0 at the center and 1.0 at the edge. In our study of linear arrays the points were effectively weighted as 1/r, but it was thought that points with larger baselines will ultimately prove more valuable than this. Points with $r < 30\,m$ or $r > 200\,m$ were not weighted. (Very short baselines can be observed with conventional speckle techniques.)

- The iterative optimization process for an initial array configuration is a grid search with a single step in radius for each telescope moved successively. The initial step size was 50 m and was reduced by a factor of 0.7 after each cycle. The search was terminated when the step size dropped below 1 m.

- Runs were also made with cobweb arrays in which the points other than the first three locations were dropped in randomly within the 200 m radius. The telescope locations were moved in a grid search in two dimensions using similar procedure as in the Y array case. For both the Y and cobweb twenty starting locations were used and the best fit for Dec=20° were selected.

Table B.2 shows the results of this optimization, listing the coverage for the two array configuration schemes that were considered. An overall weighted figure of merit was determined by weighting the first and last of these by 2 and the second and third by 1. Figures B.2 and B.3 show the coverages for three to nine telescopes for the 4 test cases for the Y arrays. The coverage for the general cobweb array is qualitatively similar and is not shown separately in a figure.

The coverages for the three declinations are indicated by shaded areas, the maximum coverage by a circle. Notice that the coverage increases monotonically from 3 to 9 telescopes. In general, the 50° declination cases have the best coverage and the 0° declination the least. (At δ=0° the ellipses of the coverage trajectories collapse into straight lines, thereby losing a lot of coverage).

If the cost of the entire interferometer were strictly proportional to the number of telescopes, then the best cost–benefit ratio would occur at 7 telescopes, the point at the greatest slope in the cost–benefit curve in Figure B.4 (left). For a more realistic approximation, the project cost can be regarded as the sum of a fixed cost and a variable cost proportional to the

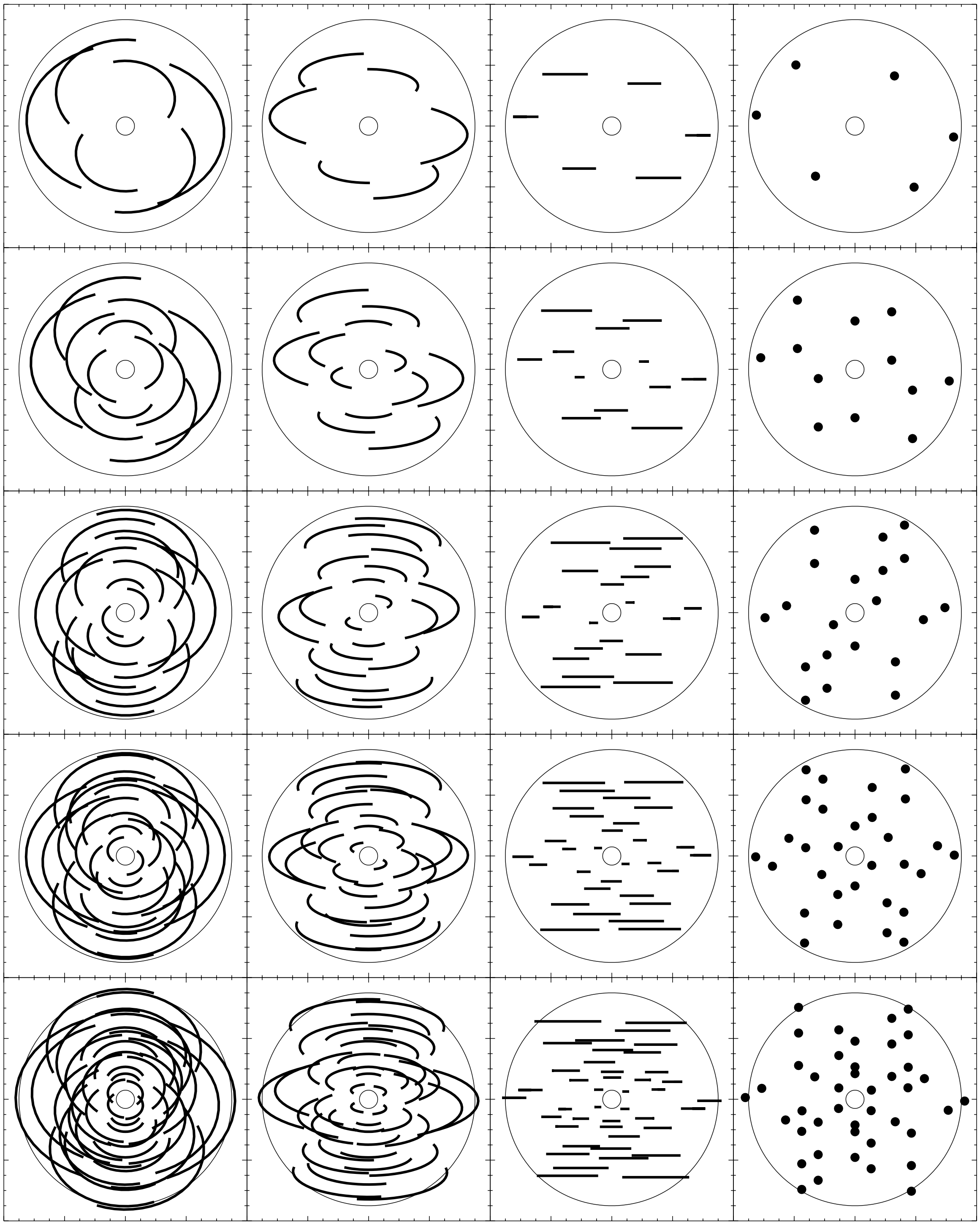


FIGURE B.2. The $u-v$ coverages of three declinations (left to right: 50°, 20°, 0°) following tracking to within 50° of the zenith, as well as a meridian snapshot for a star of 20° declination are shown for arrays (top to bottom) of three to seven telescopes.

TABLE B.2. $U - V$ Coverage with Y vs. Cobweb Configurations

		50°	20°	0°	20° Merid	Weighted Coverage	Cob − Y
3	Cobweb	0.2390	0.2098	0.0784	0.1235	0.1640	−0.0150
	Y	0.2526	0.2037	0.1011	0.1565	0.1790	
4	Cobweb	0.5257	0.4052	0.1897	0.3122	0.3584	+0.0249
	Y	0.4680	0.3962	0.1484	0.2960	0.3335	
5	Cobweb	0.6229	0.6190	0.2615	0.4757	0.5123	+0.0276
	Y	0.6145	0.5679	0.2882	0.4349	0.4847	
6	Cobweb	0.7972	0.7832	0.3472	0.6585	0.6713	+0.0417
	Y	0.7811	0.7418	0.3733	0.5697	0.6296	
7	Cobweb	0.8730	0.8945	0.5063	0.6834	0.7558	+0.0279
	Y	0.8232	0.8775	0.4258	0.6818	0.7279	
8	Cobweb	0.9122	0.9667	0.6179	0.8153	0.8490	+0.0095
	Y	0.9365	0.9530	0.5953	0.7997	0.8395	

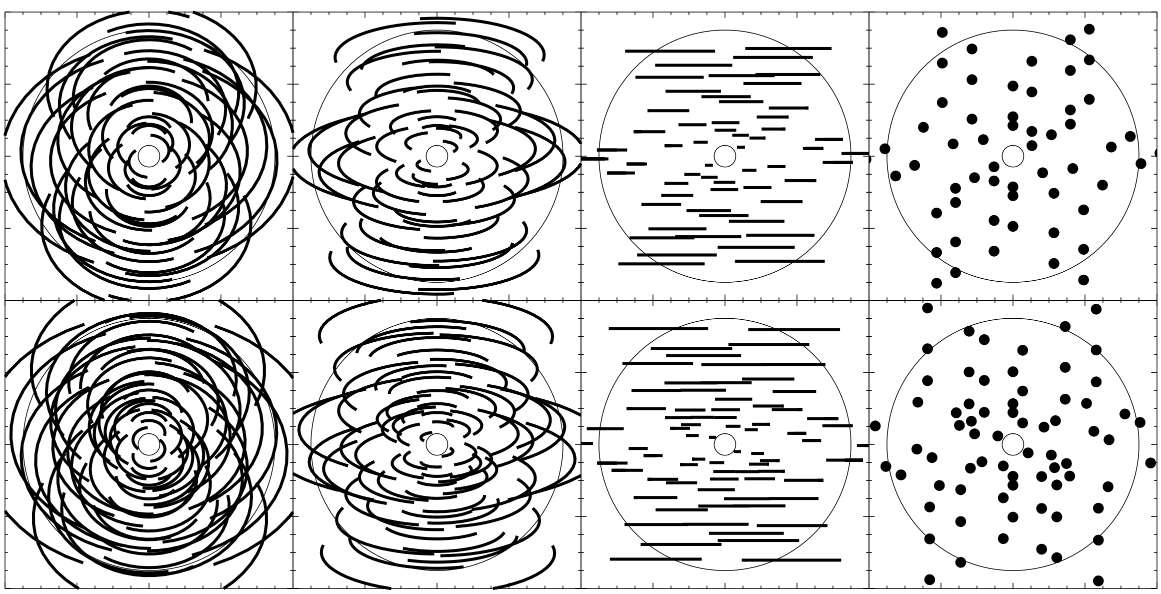
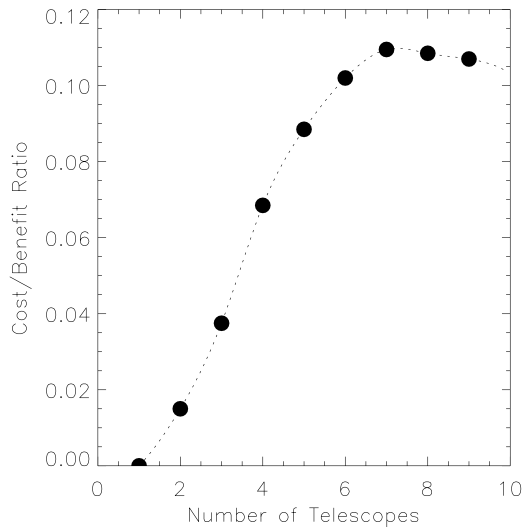


FIGURE B.3. The $u - v$ coverages as in Figure B.2 are shown for arrays (top to bottom) of eight to nine telescopes.



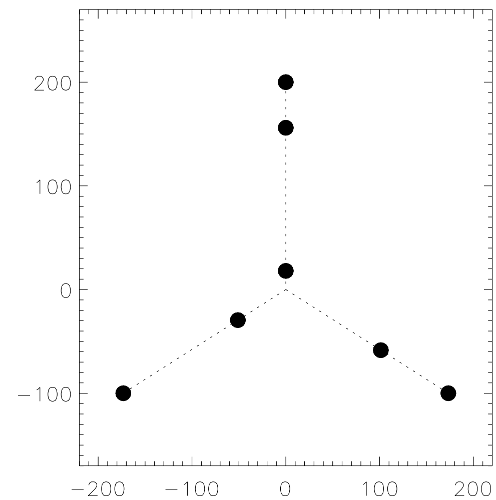


FIGURE B.4. (left) The cost–benefit ratio for an increasing number of telescopes in an array shows a maximum benefit for seven telescopes. (right) The optimum layout for a Y-shaped array of seven telescopes is shown.

number of telescopes. (The latter would include not only telescopes and domes, but beam tubes, OPLEs, correcting servos, and at least a portion of the detectors.) The variable costs including telescopes are estimated to be \$3.5 M and \$5.0 M, *i.e.*, between 1/2 and 2/3 of the cost; the fixed costs (including R&D) thus represents 1/3 to 1/2 the project cost. Figure B.4 (left) shows that the cost–benefit ratio (CBR) rises up to 7 telescopes and then levels off or slowly declines. Although various approximations have been used to obtain this result, it appears that 6 to 9 telescopes is a reasonable selection, thus answering the question of *how many telescopes*. Figure B.4 (right) also shows the optimum layout found for a 7–telescope array.

Table B.2 shows that *cobwebbing* is worth roughly 1/4 of a telescope (*i.e.*, about \$110 K). This must be weighed against the greater cost of plinths, surveying, road access, *etc.* The present baseline design does not include cobwebbing. The biggest objection to cobwebbing is that a Y-shaped array allows a solution to the polarization problem at the cost of only two more reflections (see Appendix D).

Finally, in Appendix Q on Array Performance Limits, we describe a procedure for producing reconstructed images of an object and a given array configuration via interpolation of complex visibilities in the $u-v$ plane. This procedure can be used for qualitative comparisions of how good given array configurations are at providing image reconstruction. In Figure B.5, we compare two arrays with good $u-v$ coverage in terms of reconstructing the image of a resolved binary star (based on the O-binary 29 CMa, described in Appendix Q). The performance of the seven-element array is clearly qualitatively better.

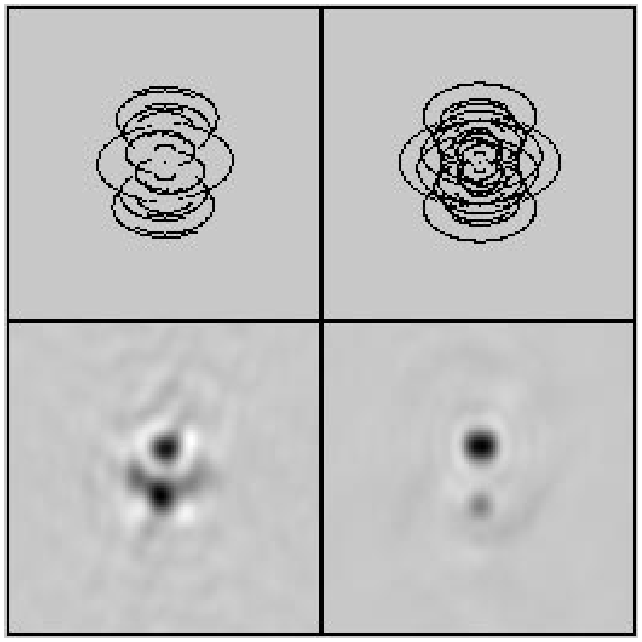


FIGURE B.5. A comparison between representative five and seven-element arrays. *Top:* The $u - v$ coverages for (right) five- and (left) seven-element arrays. *Bottom:* Resulting images from complex visibility interpolation.

B.6. OPTIMAL TWO–DIMENSIONAL ARRAYS (REPOSITIONABLE TELESCOPES)

Finally, we have examined the possibility of making some of the telescopes in a seven-telescope array repositionable, in order to provide more than one possible $u - v$ plane coverage. Our scientific goals (Appendix A) seem to naturally fall into two main areas in our parameter space. One set of objects, such as stellar diameters and close binary stars, requires very high resolution in the visible; these are relatively simple objects, however, and thus require limited imaging capability. The other set of objects, such as young stellar objects (YSOs) and extended photospheres of giant stars, requires less angular resolution but denser $u - v$ plane coverage for more full–fledged imaging, mainly in the K-band IR.

In order to increase the flexibility of the array, therefore, we have opted to allow the outermost telescope along each leg of the "Y" to be repositionable at a shorter baseline, and have optimized telescope positions for two possible $u - v$ coverage options: a "wide" option covering the regime 30 m to 350 m (to give higher resolution but a sparser $u - v$ coverage) and a "close" option covering 10 m to 120 m (for lower resolution but denser $u - v$ coverage). The routine used is similar to that described in the section above:

- Telescopes are confined to a Y-shaped array, and the outermost telescopes are fixed at 200 m from the origin for the "wide" option.
- Other telescope positions along the Y are chosen using a random number generator, with the constraint the telescopes along a given arm are separated by at least 10 m. The three repositionable telescopes are further constrained to lie within 100 m of the

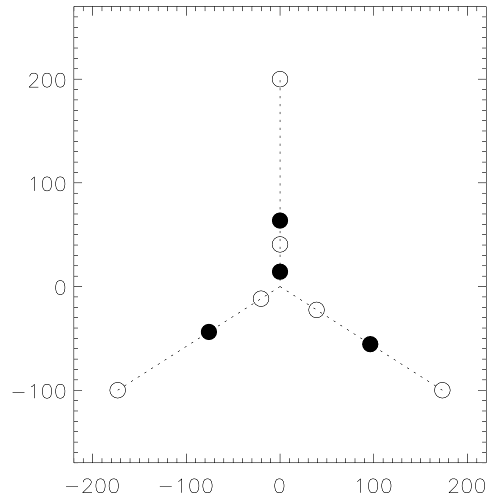


FIGURE B.6. Optimum array configuration, with four fixed telescopes and three telescopes which may be relocated for "close" and "wide" configurations. The fixed telescopes are shown as filled circles; the moveable telescopes as shown as open circles in both their possible locations.

origin.

- A star of declination 20° is observed to a maximum zenith distance of 50° and fractional $u - v$ coverage determined over both "wide" and "close" regimes.
- Weighted averages of the two fractional $u - v$ coverages are derived ("close"/"wide" weightings of 1/1, 3/2, and 2/1, reflecting differing opinions of imaging over visibility measurements).
- The procedure is repeated for numerous telescope positionings, in order to derive an optimum coverage for a given weighting.

Figure B.6 shows the optimum coverage resulting from 8,000 random array configurations, assuming either 1/1 or 3/2 relative weight for imaging (the optimum "2/1" coverage was found to give insufficiently uniform coverage in the "wide" configuration and only marginally better coverage in the "close" configuration). Figure B.7 shows the resulting $u - v$ coverage for stars at several declinations tracked over the maximum allowed range of zenith distance, as well as the "snapshot" of a star at δ=20° observed at the meridian. Coverages are shown for both "close" and "wide" telescope configurations, as well as the maximum possible coverage resulting from selectively moving individual telescopes rather than all three at once.

B.7. SUMMARY

In terms of the basic questions presented in the Introduction it was found that:

(1) The basic array geometry should be two-dimensional rather than linear.

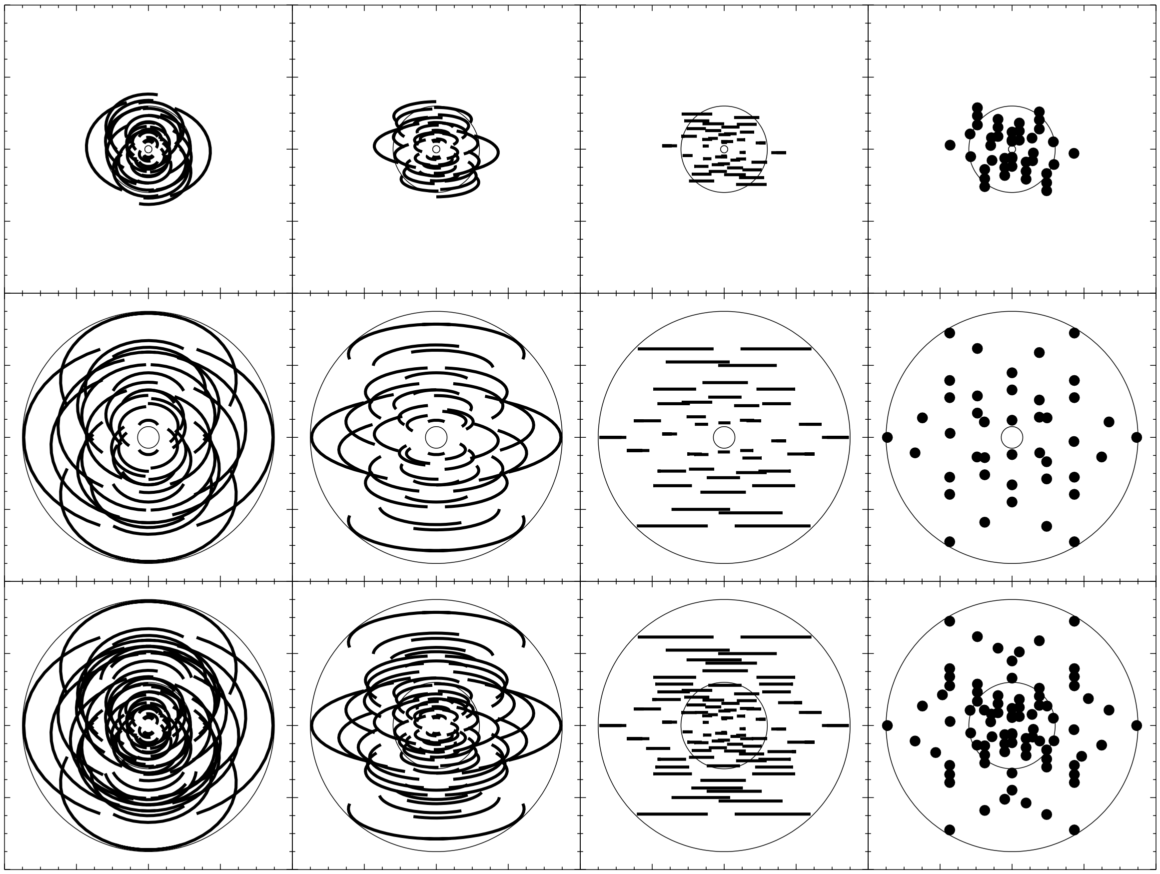


FIGURE B.7. The $u-v$ coverages for the array shown in Figure B.6 (top to bottom: "close" configuration, "wide" configuration, and all possible telescope combinations) are presented for several declinations (left to right: 50° , 20° , 0° , and a 20° "snapshot").

(2) A larger number of fixed rather than a smaller number of moveable telescopes should be used.

(3) It appears that seven telescopes is about optimal.

(4) A Y-shaped array appears to be a reasonable choice for our configuration. A more general cobweb design can provide a slightly better coverage at a slightly higher cost; however, the Y-shaped array has a simpler solution to minimize polarization effects.

(5) A seven-telescope array with three of the telescopes repositionable gives better flexibility to do both high-resolution imaging and very high resolution visibility measurement at a lower cost than possible with all fixed telescopes.

B.8. REFERENCES

Thompson, Moran, & Swenson, 1986, *Interferometry and Synthesis in Radio Astronomy*, (New York: John Wiley & Sons)

Seielstad, G. A., Swenson, G. W. Jr., & Webber, J. C. 1979, *Radio Sci*, **14**, 509