Signal to Noise Issues in Quadrant Detectors

T. A. ten Brummelaar
CHARA, Georgia State University, Atlanta, GA 30303

O.1. INTRODUCTION

A quadrant detector splits a focused stellar image into four parts or quadrants, with the light in each quadrant being separately detected. The image position is by definition centered on the detector when these four signals are equal in intensity, while an imbalance implies a centering error. The number of photon events registered in each quadrant is counted, and latched by electronics, and read every cycle. From these the image position can be calculated, and the appropriate signals sent to adaptive mirrors to re-center the image. Quadrant detector behaviour can be used to study the wavefront tilt correction servo as well as a higher order adaptive optics system.

O.2. THEORETICAL RESPONSE

When viewed from the direction of the incoming beam the quadrants are labeled as shown in Figure O.1. We therefore define the normalized image position as

$$\begin{aligned} \theta_h &= \frac{(\mathcal{N_A}+\mathcal{N_B})-(\mathcal{N_C}+\mathcal{N_D})}{(\mathcal{N_A}+\mathcal{N_B}+\mathcal{N_C}+\mathcal{N_D})} \\ \theta_v &= \frac{(\mathcal{N_A}+\mathcal{N_D})-(\mathcal{N_B}+\mathcal{N_C})}{(\mathcal{N_A}+\mathcal{N_B}+\mathcal{N_C}+\mathcal{N_D})} \end{aligned} \tag{O.1}$$

where $\mathcal{N}_{\mathcal{A}\ldots\mathcal{D}}$ are the weighted number of photon events detected in the four quadrants during the last sample period, and θ_h and θ_v are the normalized horizontal and vertical image positions. If there are no photons detected, the position is defined to be zero in both axes.

As this detector system has identical geometry in both the vertical and horizontal axes, we shall only consider the response of the vertical axis. The results for the horizontal follow in an identical manner. After passing through a defining aperture of radius R, the beam is focused onto the quadrant detector. Treating the star as a point source, so Fraunhofer diffraction conditions will apply, results in an Airy disk on the prism surface. Following Born & Wolf (1987), we write the intensity distribution of this Airy disk as

$$I(v) = I_0\left(\frac{2J_1(v)}{v}\right)^2 \tag{O.2}$$

where

$$I_0 = \left(\frac{\pi a^2|A|}{\lambda f^2}\right)^2 \tag{O.3}$$

and

$$v = k\frac{R}{f}r. \tag{O.4}$$

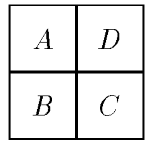

FIGURE O.1. Definition of quadrant detector channels as viewed from the front of the quadrant detector.

In this expression, $J_1(x)$ is the first-order Bessel function, k is the wavenumber $(\lambda/2\pi)$, f is the lens focal length, r is the distance from the optical axis, and A is the light amplitude in the lens plane. If there is no tilt in the beam this pattern should be centered on the quadrant detector, resulting in an output of zero. If the beam is tilted at an angle θ the center of the Airy disk will be displaced by an amount $f\theta$ which, after substituting for r in Equation O.4, results in

$$v = kR\theta. \tag{O.5}$$

Note that the position of the Airy disk is independent of the focal length of the lens. A long focal length should therefore be used to minimize aberrations due to refractive surfaces of high curvature. Taking the Airy disk center as (x_0, y_0) in the plane of the detector, in which the origin $(0, 0)$ is the detector center (i.e., $v = y_0$), the total intensity above the x-axis for a given value of y_0 will be

$$I_{\text{total}}(y_0) = \int_{-\infty}^{\infty} \int_{0}^{\infty} I_0 \frac{4J_1^2\left(\sqrt{x^2 + (y - y_0)^2}\right)}{x^2 + (y - y_0)^2} dx dy. \tag{O.6}$$

We now re-write the detector output θ_v defined in Equation O.1 as

$$\theta_v(\theta) = \frac{I_{\text{total}}(ka\theta) - I_{\text{total}}(0)}{I_{\text{total}}(0)}, \tag{O.7}$$

which is the theoretical detector response θ_v for a given wavefront tilt θ. Note that the intensity of light in the beam cancels, and the result depends only on the wavelength of the light, the aperture radius, and wavefront tilt. Unfortunately the integral defined in Equation O.6 cannot be easily performed analytically; however, for specific cases of aperture size, wavelength, and beam tilt, a numerical calculation can produce the desired result. Such a numerical calculation, based on the Romberg Integration method (see for example Press et al. 1990), was performed for a wavelength of 440 nm and for the 12.5 mm aperture size proposed for the CHARA Array. These calculations are displayed in Figure O.2.

Figure O.2 clearly demonstrates that when the position of the star image is close to the origin, the response of the detector is very linear. The linearity of the function decreases as the image moves farther away from the origin. In the limit of large D/r_0 the image formed is a Gaussian rather than an Airy disk. These calculations were repeated for a Gaussian spot with very similar results. When the servo is working, one can assume the image must remain close to the origin, thus all calculations involving detected positions can be written in the form

$$\phi_v = K_d \theta_v \tag{O.8}$$

where ϕ_v is the beam tilt in arcseconds and K_d is a calibration constant. Under these conditions, the integrand in Equation O.6 can be expanded and approximated by (Tyler &

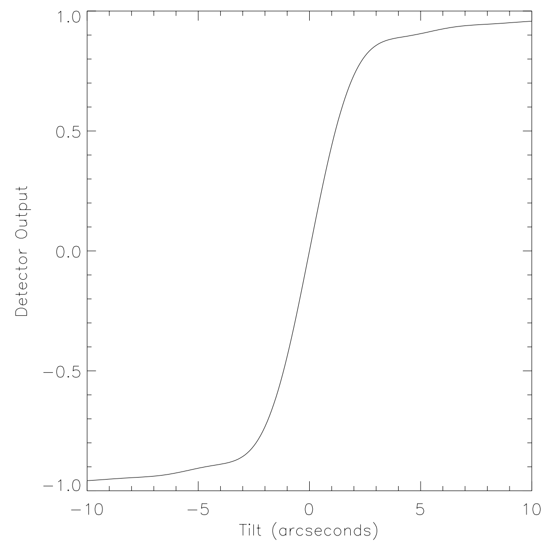

FIGURE O.2. The theoretical response of one axis of a 'perfect' quadrant detector for an aperture radius of 12.5mm. Inside the center of the range the response is very close to linear. It is this linear range near the origin that is used for the image position servo. It is also possible to see the effects of the inner rings of the Airy disk, as they cross the defining edge. The response was calculated by numerical methods using Equation O.7.

Fried 1982)

$$K_d = \frac{3\pi}{32}\frac{\lambda}{R}. \tag{O.9}$$

This equation yields the slope of the function plotted in Figure O.2 near the origin. Due to photon noise and inevitable servo errors, the detectors will not always be operating very close to the origin at all times. An estimate more appropriate for experimental situations can be found by performing a regression on the 'linear' part of the detector response curve. This results in a K_d value of 2.37 ± 0.02 arcseconds.

O.3. DETECTOR SIGNAL TO NOISE

The error associated with angular position measurements using quadrant detectors, as well as for other optical detectors used for adaptive optics, has been well studied (Tyler & Fried 1982, Dyson 1975, Walkup & Goodman 1973). The expression for the error term associated with the quadrant detector, as derived by Tyler & Fried (1982), is

$$\sigma_\phi = \pi \left[\left(\frac{3}{16}\right)^2 + \left(\frac{n}{8}\right)^2 \right]^{\frac{1}{2}} \frac{\left(\frac{\lambda}{D}\right)}{\mathrm{SNR}} \tag{O.10}$$

where n is the angular subtense of the object divided by the diffraction angle of the optical system, D is the aperture diameter, and SNR is the signal to noise ratio of the four detectors summed to act as a single detector. In the system under discussion here, the star is unresolved and we therefore say $n \ll 1$. The signal to noise ratio of the four detectors summed is primarily dependent upon the Poisson statistics of the photon events, so we can write the error in angular position measurement of the quadrant detectors as

$$\sigma_\phi = \frac{\pi \frac{3}{16} \left(\frac{\lambda}{D}\right)}{\sqrt{N}} \tag{O.11}$$

where N is the total number of counts received in all four quadrants.

For example, one way to specify the requirements of a wavefront tilt detector is to define the measurment error as a fraction of the Airy disk size $1.22\lambda/D$. In this way a minimum number of photons can be calculated for a given error ratio via

$$N = \left(\pi \frac{3}{16} \frac{1}{1.22} \frac{\sigma_\phi}{\theta_0}\right)^2 \sim \left(0.5 \frac{\sigma_\phi}{\theta_0}\right)^2 \tag{O.12}$$

where θ_0 is the size of the Airy disk of the system. This can be directly related to an error in visibility measurement using the rms visibility loss derived by Buscher (1988)

$$\eta = 1 - 1.8\langle(\theta/\theta_0)^2\rangle \tag{O.13}$$

where θ is the differential tilt error between two combining beams. Thus if the rms visibility loss due to noise in the quadrant detector is to be kept to 2%, the allowed differential tilt will be some 0.1 of the Airy disk size. This results in each beam requiring a tracking error of 0.07 of the Airy disk size. By equation (O.12) the minimum number of detected photon events required to achieve this is 45. We therefore say that any tip/tilt detector, be it for the tip/tilt servo or one subaperture of a higher order adaptive optics system, requires a minimum of 50 photon events detected each sample time.

O.4. REFERENCES

Allen, C. W., 1973, *Astrophysical Quantities*, (University of London Athlone Press, London)

Buscher, D. F., 1988, *Getting the most out of the COAST*, (PhD Thesis, Cambridge University)

Born, M. & Wolf, E., 1987, *Principles of Optics*, (Pergammon Press, Oxford), Section 8.5.2

Dyson, F. J., "Photon Noise and Atmospheric Noise in Active Optical Systems", 1975, *JOSA*, **65**, 551

Press, W. H., Flannery, B. P., Teukolsky, S. A., & Vetterlimg, W. T., 1990, *Numerical Recipes in C: The Art of Scientific Computing*, (Cambridge University Press, Cambridge)

Tyler, G. A. & Fried, D. L., 1982,"Image-Position Error Associated with a Quadrant Detector", *JOSA*, **72**, 804

Walkup, J. F. & Goodman, J. W., 1973, "Limitations of Fringe-Parameter Estimation at Low Light Levels", *JOSA*, **63**, 399