

The celestial sphere and coordinate systems

In the night sky the stars appear as bright points on a dark spherical surface. No such surface really exists, of course, but the concept of a celestial sphere is a useful one that goes back thousands of years. Ptolemy described it and so did Pythagoras and many others. Today we no longer have to worry about the reality of that sphere, and so we eliminate the need for speculation on its composition, radius, thickness and so forth. And we don't have to worry about what is on the other side of it. (Fig. 1.1. shows a drawing of an astronomer with his head sticking

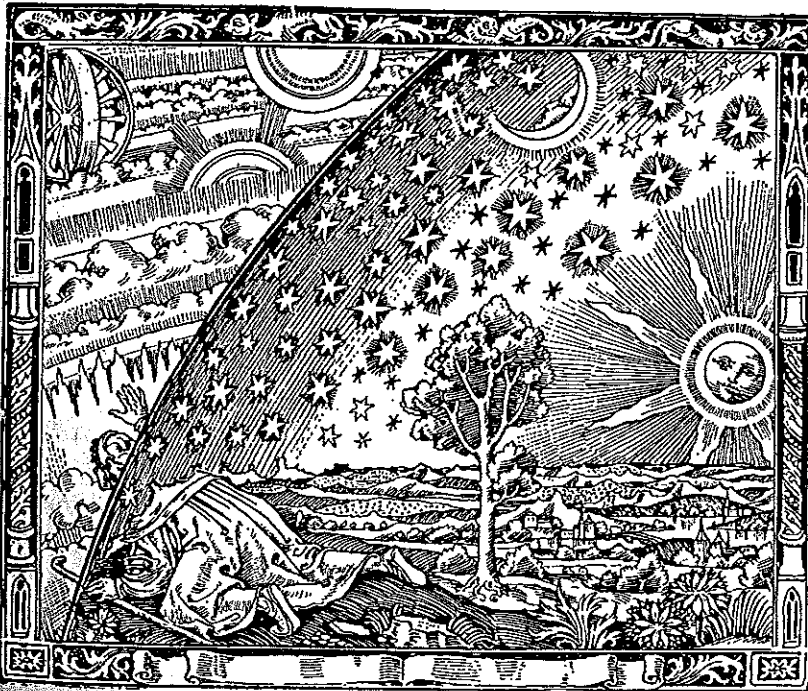


Fig. 1.1. An early concept of the celestial sphere. The careful reader may see that there is a serious flaw in the artist's knowledge of the moon.

through the sphere.) On the other hand, even though the celestial sphere is not a physical entity, we have many practical uses for the concept. The observer is always at the center of it, and the direction from the observer to any star may be considered to be a radius of the celestial sphere.

Coordinate systems

The most fundamental application of the concept of a celestial sphere is in the determination of the coordinates of objects which appear in the sky (or perhaps, on the sky). We may approach this problem of coordinates in a very general way and see first of all just what is involved in specifying the location of a point on the surface of any sphere. Assume, to begin with, that the sphere is rotating. This requires the existence of an axis which must pass through the center of the sphere as in Fig. 1.2. The axis is thus defined by the rotation, and the axis defines, in turn, two points – the poles. Following the convention established for the earth, we designate these as the ‘north’ pole and the ‘south’ pole. Now consider a plane passed through the sphere in such a way that it is perpendicular to the axis and includes the center of the sphere. In the case of the earth the plane defined in this way is actually the plane of the equator. In the more general case this plane is referred to as the ‘fundamental’ plane, and it takes on a more specific name in each of several systems of coordinates. Imagine now that we wish to define in a specific way the location of the point A on the sphere shown in Fig. 1.3(a). Let us first pass a plane through the sphere in such a way that the

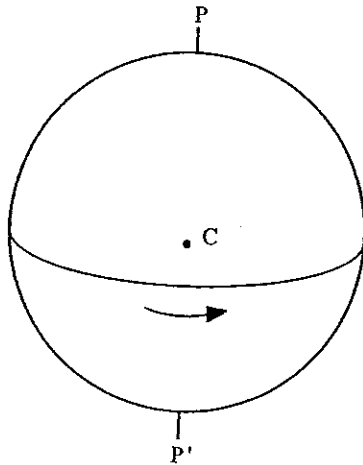


Fig. 1.2. A rotating sphere. The poles and the equator are defined by the rotation.

plane includes both the axis of the sphere and Point A. In Fig. 1.3(a) this plane has been indicated by the curve passing through the two poles and Point A. Points B and C have also been indicated. Point C is at the center of the sphere, and point B is the intersection of PA with the equator. It should now be obvious that the angle BCA defines the angular distance of A from the fundamental plane. On the earth an angle similar to BCA is referred to as the 'latitude'. We do not go to the center of the earth in order to determine the latitude of a place, but it is actually this angle that we are talking about when we use the term.

Now let us pass another plane through the sphere. Let this one be parallel to the fundamental plane, and let it pass through A (Fig. 1.3(b)). Notice that the radius of this circle is smaller than that of the fundamental circle. At this point we must introduce two definitions. First, a 'great circle' is the intersection of a plane and a sphere in any case in which the plane passes through the center of the sphere. Thus, the fundamental circle is a great circle, and the arc PAP' is half of a great circle. Second, a 'small circle' is the intersection of a plane and a sphere in any case in which the plane does not pass through the center of the sphere. In the case shown in Fig. 1.3(b) the small circle happens to be parallel to the fundamental plane, but a small circle may actually have any orientation.

Returning to Fig. 1.3(b) we note now that all points on the small circle are at the same angular distance from the fundamental circle. For the earth we would say that all points on this small circle had the same latitude. In order to be precise about the location of A, therefore, we must specify in some way which of all of the possible great circles through the poles is the one which passes through A. We may do this by means of a second angle measured this time in the fundamental plane. We must first select some arbitrary point as our zero point, and in Fig. 1.3(c) such a point has been indicated as D. Now the angle DCB quite specifically defines the great circle through A. Again with reference to the earth the circle PDP' represents the meridian of Greenwich, England, and the angle, DCB represents the longitude of point A.

The above discussion is intended to show that the location of a point on a sphere can always be specified by means of two angles. One angle is measured at right angles to the fundamental plane, and the second angle is measured in the fundamental plane itself. Astronomers have found it convenient to make use of several coordinate systems, and we shall discuss these below. The individual systems differ most notably in that the positions of the stars are referred to a different fundamental plane in each one.

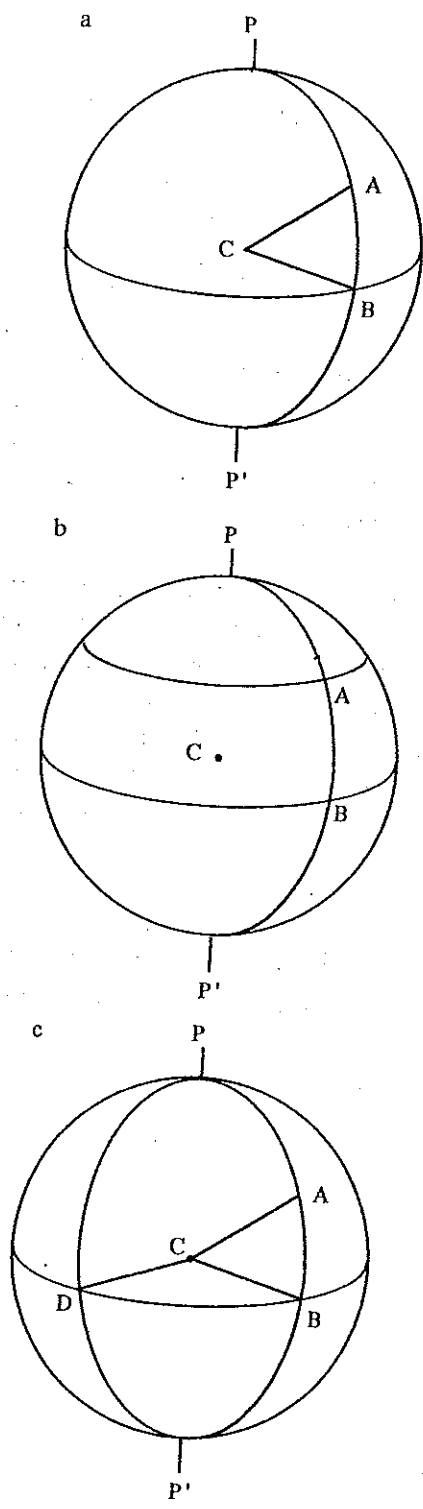


Fig. 1.3. (a) The angle between two points on a sphere (b) A small circle through point A (c) Two angles define the location of point A with respect to the equator and an arbitrarily chosen point D.

The horizon system is perhaps the one which would seem to be the most obvious when we first begin to discuss the positions of stars. Here the fundamental plane is the plane of the observer's horizon, and the pole is the zenith which may be defined as the point directly overhead. The true horizon is by definition ninety degrees from the zenith. The first coordinate is now referred to as the 'altitude'. From our general discussion one should be able to guess that altitude is the angle which the observer would measure from the horizon to an object in the sky and that this angle is measured along a great-circle arc which passes through the zenith and the object. Altitude may vary from zero for an object on the horizon to ninety degrees for an object at the zenith. An object below the horizon would not be visible, but it may be considered to have a negative altitude.

The determination of the second coordinate requires the previous selection of a zero point on the horizon, and here this point has been selected as the north point. The 'azimuth' may now be defined as the angle measured in the plane of the horizon from the north point to the point at which the arc from the zenith through the object crosses the horizon. By convention the azimuth increases in the direction towards the east. Fig. 1.4 illustrates altitude and azimuth measured with respect to the horizon.

This coordinate system is variously referred to as the horizon system, the altitude-azimuth system or as the altaz system. The instrument shown in Fig. 1.5 is a theodolite routinely used by surveyors to measure altitude and azimuth. Since the earth rotates continually toward the east, all celestial objects appear to move from east to west across the sky. This means that in the horizon system the coordinates of all objects are con-

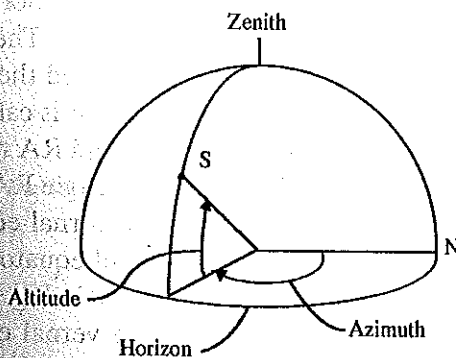


Fig. 1.4. Altitude and azimuth in the horizon system. S is the position of the star.

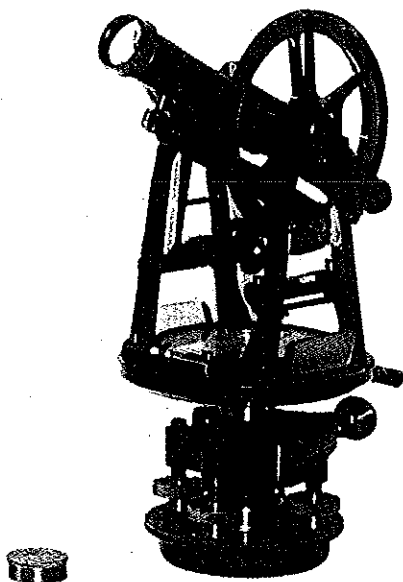


Fig. 1.5. The theodolite, an instrument for measuring altitude and azimuth (Wellesley College photography).

tinually changing. Such a system does have certain applications as we shall see, but it cannot be used to provide permanent descriptions of the locations of stars on the celestial sphere.

A coordinate system with more useful properties in astronomy is the equatorial system in which the fundamental plane is now the celestial equator. If one imagines a very small spherical earth at the center of a very large spherical sky, then the extension of the earth's axis defines the celestial poles, and the extension of the plane of the earth's equator defines the celestial equator. In the equatorial system, then, coordinates of stars are defined with respect to the celestial equator. The angle measured northward or southward from the equator is called the 'declination', and the angle measured in the plane of the equator is called the 'right ascension'. These two angles are indicated as Dec and RA respectively in Fig. 1.6(a). A zero-point for the measurement of angle RA must, of course, be selected, and this has been chosen as the vernal equinox, the point on the sky at which the sun crosses the celestial equator on or near March 21 each spring. This point has been marked V in Fig. 1.6(a). There is no obvious way in which one can identify the vernal equinox when looking at the sky, but later in Chapter 4 we shall show that this point can be located quite specifically from very simple observations.

Later in this chapter we shall also describe the methods by which the astronomer determines the declinations and right ascensions of stars.

It is conventional that declination be considered to be positive for objects in the northern hemisphere and negative for objects in the southern. Right ascension increases in the eastward direction on the celestial sphere, and is measured in hours, minutes and seconds.

In Fig. 1.6(b) we have shown both the equatorial and the horizon systems in the same sketch. We remind the reader that as the sky appears to rotate around the north celestial pole, the altitude and azimuth will both be changing continuously. The declination and right ascension will, of course, remain fixed. In the diagram the angle ZPS has been marked H , and it is this angle that is commonly referred to as the 'hour angle'. The hour angle decreases progressively when a star is east of the meridian and becomes progressively larger after the star has crossed the meridian into the western sky. It is conventional to regard the hour angle

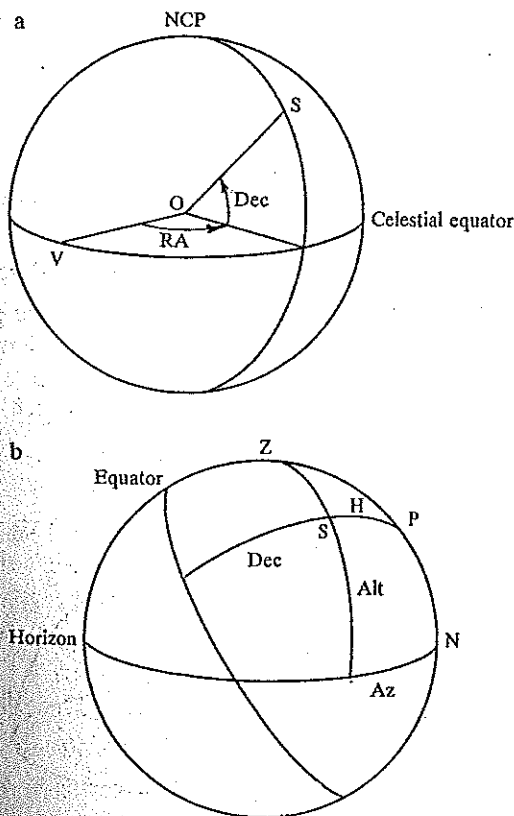


Fig. 1.6. (a) The equatorial coordinate system (b) The equatorial system superimposed on the horizon system.

as being negative when a star is in the east and positive when a star is in the west. If we are to try to point a telescope toward a particular star, we must first determine the hour angle of the star. The hour angle will depend upon the time at which an observation is being made and the right ascension of the star. We shall return to this subject in Chapter 4 where we shall see how the hour angle is routinely determined.

Another system of coordinates was of considerable use among astronomers several hundred years ago and is used today by those who are engaged in the study of the motions of the planets. In this system the fundamental plane is the ecliptic, the apparent path of the sun around the sky. The zero-point on the ecliptic is again the vernal equinox, and the two coordinates are known as celestial latitude, β , and celestial longitude, λ . Fig. 1.7 illustrates the ecliptic system of coordinates.

Finally, in studies of our galaxy, astronomers make use of a system of coordinates in which the fundamental plane is the galactic equator, a great circle which closely approximates the center line of the Milky Way on the sky. Astronomers have carefully chosen the right ascension and declination of the galactic north pole in such a way that the galactic equator is precisely defined. As illustrated in Fig. 1.8, the two coordinates are known as galactic latitude, b , and galactic longitude, l , and the zero-point on the galactic equator is a point in the constellation Sagittarius, in which we look toward the center of our galaxy. It is worthy of mention here that galactic coordinates cannot be determined by direct observation. They are calculated from the equatorial coordinates of the star, of the galactic pole and of the direction of the center of the galaxy. Details of this computation will be presented in Chapter 4.

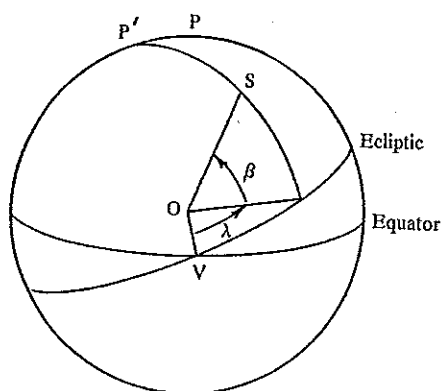


Fig. 1.7. The ecliptic system superimposed on the equatorial system. P' is the pole of the ecliptic.

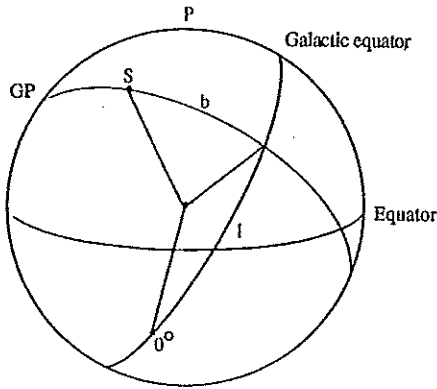


Fig. 1.8. The galactic coordinate system. GP is the galactic pole.

Measuring stellar coordinates

In Fig. 1.9(a) the celestial sphere has been drawn as it appears above an observer's horizon. The arc running from the north point on the horizon through the zenith and on to the south point on the horizon is the meridian of the observer at O. The zenith or point directly above O is marked Z. The celestial north pole is marked P. The angle NOP is equal to the latitude of the observer. In Fig. 1.9(b) a small circle has been

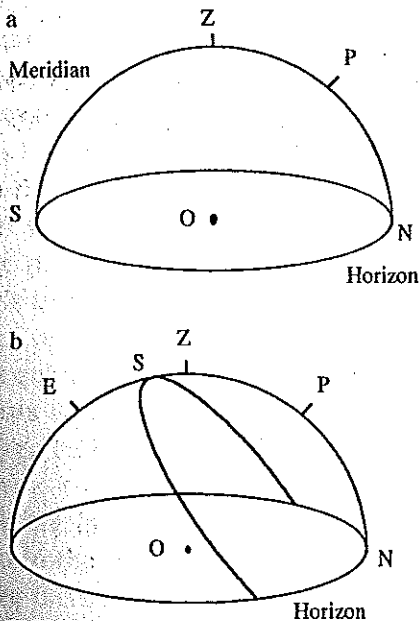


Fig. 1.9. (a) Zenith and pole as points on the meridian (b) A star crossing the meridian at S.

drawn to represent the path of a star as it rises in the east, crosses the meridian at S and sets in the west. From earlier statements it should be easily seen that angle SOE is equal to the declination of the star. We wish to find a precise, practical way of measuring this angle.

Suppose now that we construct a specialized telescope which is capable of moving only along the meridian. That is, the telescope is to be mounted on an axis oriented in the east-west direction as shown in Fig. 1.10. Such a telescope is known as a meridian telescope or a meridian circle. Small versions are referred to as transit telescopes because the passage of a star across the meridian is called a transit.

A meridian telescope is always equipped with a large and precisely graduated scale or circle so that the angle between successive pointings may be known as accurately as possible. Thus, if such a telescope is pointed first at P and then at S in Fig. 1.9(b), the angle POS will be known, and

$$90^\circ - \angle POS = \text{declination}$$

In principle, then, a star's declination may be found from rather simple measurements.

The graduated circle on the meridian telescope may be adjusted so that it reads 90 degrees when the telescope is pointed toward the north pole and zero degrees when pointed toward the celestial equator. The declination of a star may then be read directly from the scale after a transit. A special reticle is included in the optical path within a meridian telescope (see Fig. 1.11). The observer sees a horizontal line crossed by a series of

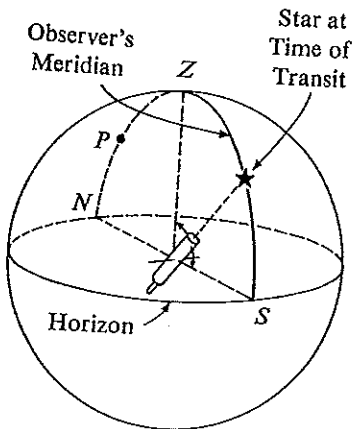


Fig. 1.10. The transit telescope is constrained to move only along the observer's celestial meridian.

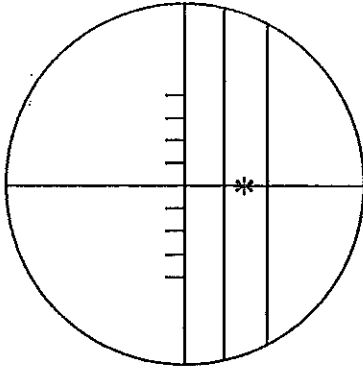


Fig. 1.11. A star in the field of a transit telescope.

vertical lines. The telescope is adjusted to the approximate declination in advance of the predicted time of transit. Then as the observer watches, a star will appear at one side of the field of view, move across the field and disappear on the opposite side. By pressing a key the observer attempts to record the exact moment of transit. Refinements in the declination may be made by noting the appropriate graduations on the vertical scale. Today the transits of stars are recorded photoelectrically and the judgment of the observer has been removed, but the general principles are the same.

The right ascension of a star may be found from the same meridian observation provided that a sidereal clock is available. Sidereal time will be discussed fully in the next chapter, but for the moment we may say that it is based on the rotation of the earth with respect to the stars rather than to the sun. This clock would be adjusted to read zero hours at the moment when the vernal equinox crosses the observer's meridian each day (see p. 19). Then if one records the sidereal time at the moment of the star's transit, that time is the star's right ascension. When the sidereal time is five hours, for example, five hours will have passed since the vernal equinox crossed the meridian and a star with a right ascension of five hours will now be crossing the meridian. This is indicated in Fig. 1.12. Fig. 1.13 is the polar view of the case depicted in the previous figure.

There are in the world only a small number of astronomers who are engaged in the determination of precise coordinates by methods such as those we have described, and these astronomers are only able to work with the brightest stars. The results of these observations are known as fundamental positions and are published in what are known as

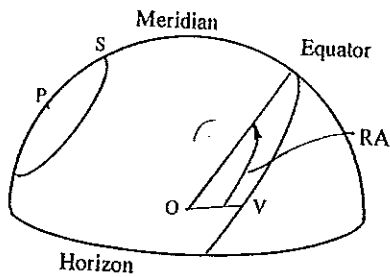


Fig. 1.12. Sidereal time is equal to the right ascension of a star on the meridian.

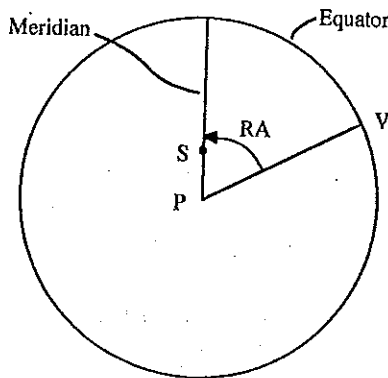


Fig. 1.13. Polar view of the previous figure.

fundamental catalogs (see Chapters 3 and 5). These fundamental positions form the reference system for the determination of the coordinates of fainter stars from the measurement of photographs. The method for doing this will be discussed in Chapter 5.

Finding the celestial pole

As the stars move across the sky each one will cross the meridian at some time in each interval of twenty four sidereal hours. Twelve hours after the initial transit the star will transit again, but this time its altitude at the time of transit will be less. This is seen in Fig. 1.14, and these crossings are referred to respectively as upper transit and lower transit. At the equator all lower transits will be invisible because they will all occur below the horizon. At any other latitude (except that of the North Pole or the South Pole) it is always possible to observe some stars at both upper and lower transits. Such stars are referred to as circumpolar stars.

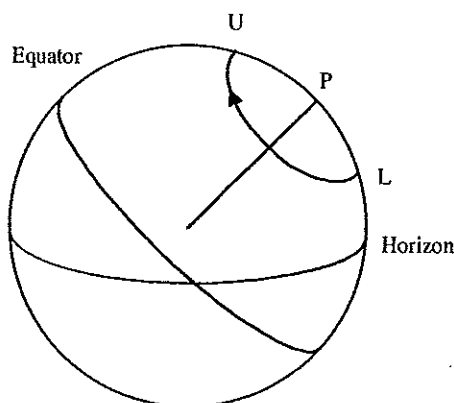


Fig. 1.14. Upper and lower transits of a circumpolar star.

Now if we wish to know the altitude of the pole, we have simply to observe the altitude of a star at both upper transit and at lower transit. Then

$$\text{Alt of pole} = A(L) + \frac{1}{2}(A(U) - A(L))$$

In practice the problem is not quite so simple because of the effects of refraction by the atmosphere of the earth. The rays of light from a star are refracted in such a way that the stars appear to be higher in the sky than they actually are. The correction for this effect will be discussed in detail in Chapter 4.

The azimuth of the celestial pole is always 0° regardless of the latitude of the observer. Therefore, as mentioned on p. 10 the meridian telescope's axis of rotation must lie along an east-west line. This alignment may be checked by observing a star six hours before and six hours after its upper transit. As indicated in Fig. 1.15 the altitude of the star should be the same at each of these two times. Furthermore, the angle between

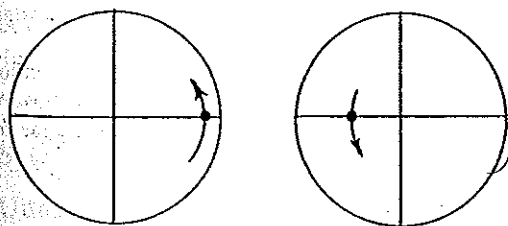


Fig. 1.15. Result of an error in the alignment of the axis of the meridian telescope.

the center of the field and the direction to the star should be the same in both cases. If these angles are not the same, then the mounting must be rotated in azimuth.

As mentioned on p. 6, we also need to know the location of the vernal equinox, our unmarked and invisible reference point in the sky. This point can be located with remarkable precision, and the method for doing so will be described in detail in Chapter 4.

QUESTIONS FOR REVIEW

1. Why is it that two angles are sufficient to define the position of a point on a sphere?
2. Name the two coordinates in each of the four systems described in this chapter.
3. How is the meridian telescope used to determine stellar coordinates in the equatorial system?
4. Why is it that coordinates in the galactic system cannot be measured but must always be calculated?
5. What is a circumpolar star? What effect does the observer's latitude have on the number of circumpolar stars that can be seen?
6. A circumpolar star is observed to have an altitude of $49^{\circ} 38'$ at upper transit and an altitude of $18^{\circ} 21'$ at lower transit. Assume that these are true altitudes after correction for refraction and instrumental errors. What is the latitude of the observer?
7. What is the declination of the star observed in the previous question?

Further reading

- Green, R. M. (1985). *Spherical Astronomy*. Cambridge: Cambridge University Press.
- McNally, D. (1974). *Positional Astronomy*. Frederick Muller Limited. McNally covers much of the same material as the other authors listed here. This is a useful book for readers who wish to have another perspective.
- Pannekoek, A. (1961). *A History of Astronomy*. Interscience Publishers. Material on several early systems of coordinates may be found in the first few chapters of this interesting and authoritative work.
- Smart, W. M. (1977). *Textbook on Spherical Astronomy*, 6th edn. Cambridge: Cambridge University Press.
- Woolard, E. W. and Clemence, G. M. (1966). *Spherical Astronomy*. Academic Press. The level of detail in this book is such that it can be very useful to those who are working directly in positional astronomy. Chapter 20 is an excellent source for information on the terminology of coordinate systems.

Time

Time as we use it in our ordinary lives is based on the rotation of the earth with respect to the sun. As the earth rotates the sun appears to move continually around the heavens, and we define noon as the moment each day when the sun passes an observer's meridian. We could easily use the interval from one noon to the next to define the day, but in order to make all of the daylight hours part of the same calendar day, we start and end our days at midnight. The division of a day into twenty-four hours is strictly arbitrary and goes back to very early times. The Greeks divided the periods of daylight and darkness each into twelve equal parts to make twenty-four divisions in each day. The hours defined in this way were not always the same since the periods of light and dark vary in length with the seasons. The custom of dividing the hour into sixty minutes and the minute into sixty seconds is one of the last vestiges of the sexagesimal system of counting which was in use in the ancient world.

Today we keep track of time with a variety of clocks which range in complexity from sundials of the simplest sort to quartz clocks of the greatest precision. Our choice of one clock over another is made on the basis of our needs in particular situations. We can be on time for most appointments if we know the time to the nearest minute, but astronomers combining data recorded over a long period of time often need to know the time to the nearest thousandth of a second. In the end, however, we always come back to the recognition that the daily motion of the sun is the original basis of our whole system of timekeeping.

Solar Time

Unfortunately, the sun is a bit irregular as a time-keeper. If one were to make daily comparisons between an accurate watch and a sundial, one would soon find that the two were not in close agreement

very often. Noon on the sundial would be ahead of noon on the watch during the months of May and November and behind during the months of February and July. The watch, of course, runs at a constant rate, so the sun is apparently moving across the sky at a variable rate.

There are two principal reasons for this difference. The first is that the earth's motion around the sun is not uniform. The earth's orbit is slightly elliptical, so the earth moves faster when it is closest to the sun in January. Fig. 2.1 may help the reader to visualize the reason why the interval from noon to noon is actually longer in the winter than in the summer. At this point we are assuming that the period of the earth's axial rotation is constant, so when the earth's orbital speed is faster, as it is when the earth is closest to the sun, the earth must rotate further before noon will recur.

The second effect arises because of the obliquity of the ecliptic and is illustrated in Fig. 2.2. Imagine that the real sun moves along the ecliptic at a uniform rate and imagine a fictitious or mean sun which moves along the celestial equator at a uniform rate. In Fig. 2.2 the sun is at S and the mean sun is at M. Because we have assumed uniform motion for both S and M, $VS = VM$. S' is the projection of the true sun down to the celestial equator. From the time when the real sun crosses the vernal equinox until the time when it reaches the summer solstice in June it is behind the fictitious sun, M. Then it is ahead for three months; then behind and ahead again during the next two quarters. The combined effect of these two factors is shown in Fig. 2.3, and may be computed for any day of any year.

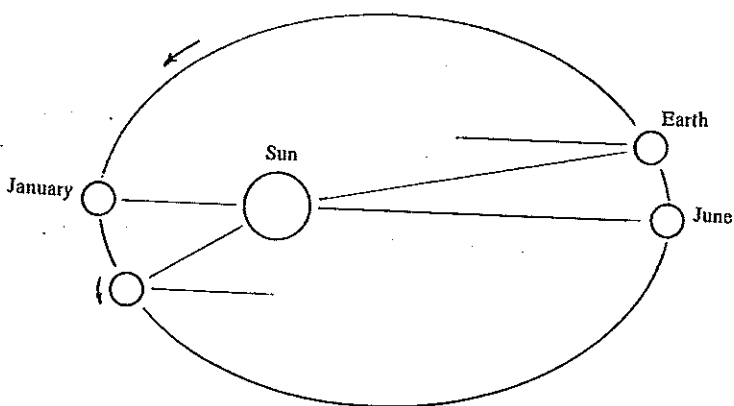


Fig. 2.1. Seasonal variations in the interval from noon to noon. The ellipticity of the earth's orbit is very much exaggerated in this sketch.

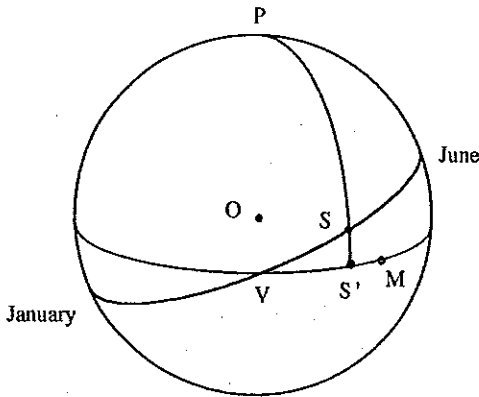


Fig. 2.2. The true sun projected to S' is lagging behind the mean sun at M. The angles VOS and VOM are equal.

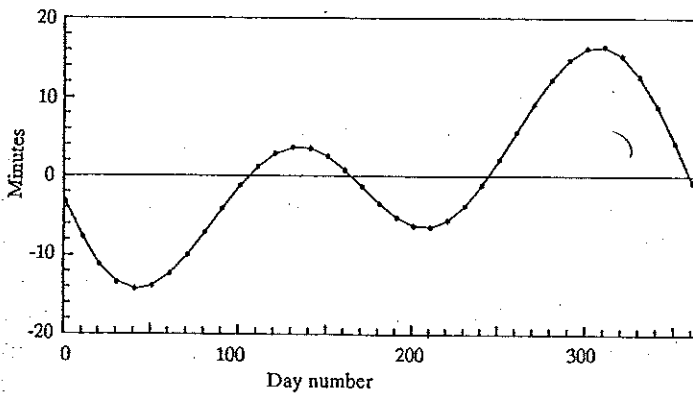


Fig. 2.3. Variation in the equation of time throughout the year.

The non-uniform time recorded by the sundial is referred to as solar time. The watch, however, is designed so that the day which it indicates has the average length of all of the days measured on the sundial in a year. Our watches and clocks are adjusted to measure mean solar time.

The difference between solar time (ST) and mean solar time (MST) is referred to as the equation of time (EqT), or

$$\text{Eq. of Time} = \text{App. Solar Time} - \text{Mean Solar Time}$$

$$\text{EqT} = \text{ST} - \text{MST}$$

This is sometimes written

$$\text{EqT} = \text{RAMS} - \text{RAS}$$

where RAMS is right ascension of the mean sun, and RAS is right ascension of the sun. A value for the equation of time on any particular day of the year may be determined from data published annually in the *Astronomical Almanac* (see bibliography and p. 41). At the equinoxes and at the solstices the equation of time is zero.

Another obvious property of solar time is that it is valid for an observer only at a specific longitude. For an observer at another longitude the solar time will be different (see Fig. 2.4). Up until the middle of the nineteenth century most cities and towns adopted their own local time which was essentially local mean solar time. Travel and communication were slow, and this was good enough for all purposes. With the coming of the railroads and the telegraph a more uniform system was needed and time zones were introduced. For convenience, then, we have arbitrarily marked off twenty-four zones of longitude on the earth, and we maintain the same time for all clocks within each of these zones. The time within each zone is known as standard time. On the eastern coast of North America we use Eastern Standard Time, while on the western coast we use Pacific Standard Time. Each zone is fifteen degrees wide, but for the convenience of the population the boundaries of the time zones are irregular.

In 1884 the meridian of Greenwich, England was chosen as the meridian of zero longitude, and longitude was defined to increase toward

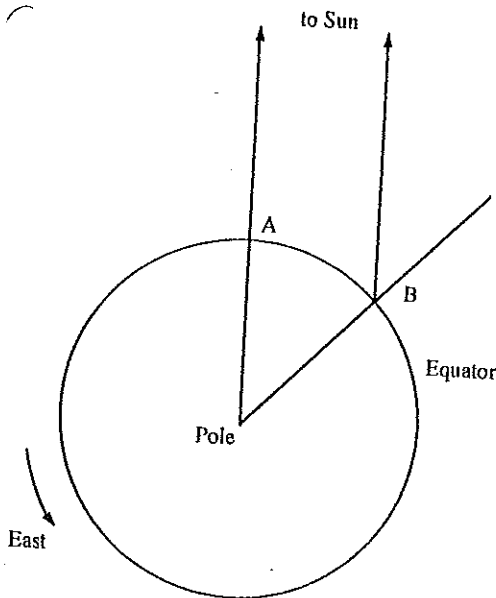


Fig. 2.4. Solar time depends upon the longitude of the observer.

the east and toward the west to a maximum value of 180 degrees. Navigators of ships at sea knew that the difference in local or mean solar time between two places was equivalent to the difference between the longitudes of the two places. Thus, they found it convenient to keep with them always an accurate timepiece set to the mean time in Greenwich. An observation which gave them their local time (i.e., apparent solar time corrected by the equation of time) then gave the longitude.

$$\text{longitude} = \text{Greenwich mean time} - \text{local mean time}$$

Observations of the altitude of the sun, for example, were routinely used to establish the moment of local noon. Today the term 'Greenwich Mean Time' is seldom used. Instead we commonly refer to 'Universal Time', but the meaning is the same.

Ephemeris Time and Dynamical Time

When a particular problem requires that intervals of time be measured with extreme precision, one may use Ephemeris Time. This system is necessary because the actual period of the earth's rotation is increasing in a slow but irregular manner. Therefore, in 1960 a new kind of time which increases in a steady, uniform way was introduced. It was given the name Ephemeris Time. In 1984 the concept of ephemeris time was refined even more, and the name was changed to Dynamical Time. Ephemeris Time and Dynamical Time are determined by adding a correction to the Universal Time in the sense $DT = UT + T$. The correction, T , cannot be computed in advance, so the user must always wait until the next value of the correction has been published. At the beginning of 1989 the correction was just over 56 seconds. Instructions for the computation of Dynamical Time are to be found in the *Astronomical Almanac*. There are actually two forms of dynamical time. One, Terrestrial Dynamical Time or TDT, is referred to the earth. The other, Barycentric Dynamical Time or BDT, is referred to the barycenter of the solar system.

Sidereal Time

Because of the earth's orbital motion the solar day does not really represent the true rotation period of the earth. The diagram in Fig. 2.5 may help to illustrate this point. If the earth moves from position 1 to position 2 in the course of a day, then after one complete rotation an observer who was originally facing the sun will now be facing toward V.

Noon will not occur again until the earth has rotated through the additional angle a . Thus the solar day is really longer than the true rotation period of the earth. The stars are the logical frame of reference against which to measure the true rotation period, and as mentioned in Chapter 1, time based on the earth's rotation with respect to the stars is known as 'sidereal time'. It is perhaps obvious that sidereal time is of great importance to astronomers.

In discussing the methods for determining the sidereal time we may note simply that if the earth moves around the sun in 365 days, then in one day the earth will have moved through an angle equal to $360/365$ or 0.98 degree. Recall now these correspondences:

$$\begin{aligned} 24 \text{ hours} &= 360 \text{ degrees} \\ 1 \text{ hour} &= 15 \text{ degrees} \\ 4 \text{ minutes} &= 1 \text{ degree} \end{aligned}$$

From the above it should be clear that the solar day is approximately four minutes longer than the sidereal day. Let us consider some more precise values and see how the correct sidereal time may be calculated. There are several convenient methods by which this may be done.

Finding the Sidereal Time

We mentioned earlier that many of the older observatories were originally equipped with a small transit telescope. Most of these telescopes have since been removed from active use, and their housings have been converted to other purposes. These telescopes were originally used

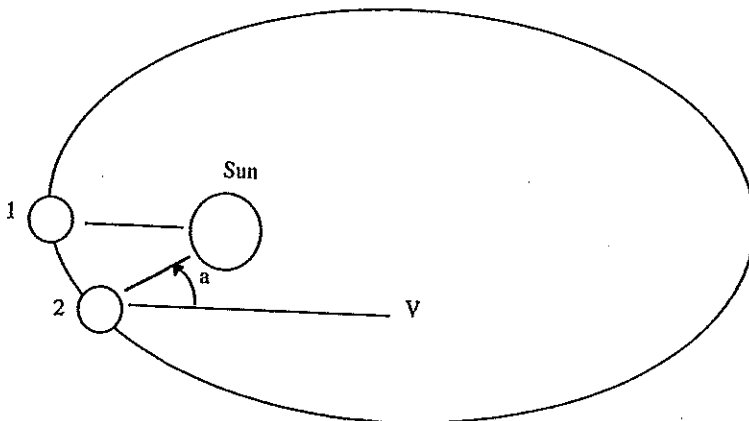


Fig. 2.5. The sidereal day is shorter than the solar day.

to determine the sidereal time directly from observations of stars of known right ascension. The process is just the reverse of that for finding the right ascension. A star is observed during transit, and the local sidereal time at the moment of transit is the same as the previously known right ascension of the star. Local time or standard time may then be calculated.

We could still find the sidereal time from the same sort of telescopic observations, but in the modern world it is much simpler and less time-consuming simply to compute the sidereal time from the known value of standard time. We begin with the standard time which can be quite easily obtained from time signals regularly transmitted *via* radio or telephone by government agencies in a number of countries. A list of the call letters and frequencies and the telephone number on which the signals are transmitted is included in Appendix 2. The time signals consist of a series of tones to mark the seconds. Hours and minutes are identified by frequent vocal statements or by codes. By suitably recording the time signals along with other data, clocks can be set to the nearest thousandth of a second. In many practical situations such precision is not really necessary and the time obtained by dialing an appropriate local telephone number is adequate.

Now as we proceed to methods for the conversion from mean solar time to local sidereal time, let us first note that the *Astronomical Almanac* (see p. 41) contains a table which lists the sidereal time for zero hours UT for every day of the year. These tables are not the same from one year to the next, so one must always use the Almanac for the appropriate year. The sidereal time may be computed by proceeding through the following steps:

Step 1. Convert Standard Time to UT. This is done by adding one hour for each time zone between the observer and Greenwich if the observer is west of Greenwich. For example, an observer in Boston or Quebec would add five hours to convert from Eastern Standard Time to UT. An observer in Chicago would add six hours; in Winnipeg, seven hours, and so forth. Observers east of Greenwich would of course subtract an hour for each time zone.

Step 2. Define the 'solar interval' as the time interval from 0 h UT to the time in question. Convert this solar interval into a sidereal interval. This is done by multiplying by the factor, 1.00273791, the ratio of the number of solar minutes per sidereal day to the number of sidereal minutes per day (1443.9425/1440).

Step 3. Add the sidereal interval to the sidereal time at 0 h UT taken

from the Astronomical Almanac. This gives us the Greenwich Sidereal Time.

Step 4. Subtract the longitude of the observer in order to know the Local Sidereal Time.

Example: Find the local sidereal time at 1900 hours on September 29, 1981 for an observer in Wellesley, Mass., USA.

1. $1900+0500=2400$ UT=0000 UT on September 30
2. The interval from 0 h UT is 0.
3. The Astronomical Almanac gives 0 h 30 m 40.0511 s as the GST at 0 h UT on September 30, 1981. Thus the Greenwich Sidereal Time is 0 h 30 m 40.0511 s.
4. Subtract the longitude of Wellesley (4 h 45 m 13.3 s) 0 h 30 m 40.0511 s - 4 h 45 m 13.3 s = 19 h 45 m 26.7 s, the local sidereal time.

This example was obviously simplified because the universal time turned out to be 0 h. Let us consider another example.

Example: Find the local sidereal time at 1100 hours on September 29, 1981 for the observer in the first example.

1. $1100+0500=1600$ UT
2. 16 hours = 960 min.
 $960 \text{ min} \times 1.00273791 = 962.62839 \text{ min}$ or 16 h 2 m 37.7 s. This is the sidereal interval.
3. GST = Sid Int + ST 0 h UT
 $16 \text{ } 02 \text{ } 37.7 + 0 \text{ } 26 \text{ } 43.5042$ (ST 0h UT Sept. 29) = 16 29 21.2
4. LST = GST - longitude
 $16 \text{ } 29 \text{ } 21.2 - 4 \text{ } 45 \text{ } 13.3 = 11 \text{ } 44 \text{ } 7.9$, the LST.

It is often convenient to convert hours, minutes and seconds into hours and a decimal part of an hour in order to facilitate addition and subtraction of numbers such as these. Most small scientific calculators provide keys for the conversion of degrees, minutes and seconds to degrees, and the same keys work, of course, for the conversion of hours, minutes and seconds into hours.

Those individuals who routinely use a programmable calculator or a computer will find that a second method of computing the sidereal time will be more convenient. In this method there is one constant which must be changed in the program at the beginning of each year. The formula is as follows:

$$\text{GST} = G + 0.0657098232 \times N + 1.0027379093 \times \text{UT}$$

where G is the GST at 0 h UT on the zeroth day of the year. (The zeroth day is obviously the same as the last day of the previous year); N is the number of the day from the beginning of the year (see below); and UT is the universal time in hours. If the computation using the formula gives a number greater than twenty-four hours, then twenty-four hours should be subtracted and the date should be made one day later. The user may find the new value of the constant, G , in the *Astronomical Almanac* at the beginning of each year and proceed with confidence.

The day number may be found from another formula which lends itself nicely to computer programs:

$$N = \langle 275 \times M / 9 \rangle - 2 \times \langle (M + 9) / 12 \rangle + I - 30$$

where M is the number of the month and I is the day of the month. The brackets $\langle \rangle$ indicate that one should use the integer value of the enclosed quantity. In a leap year the coefficient of the second term should be changed from 2 to 1. More information on this method of computing the local sidereal time may be found in *Almanac for Computers* published annually by the U.S. Naval Observatory.

The convenience of calculating the sidereal time means that it is no longer really necessary to maintain an accurate sidereal clock in an observatory. The standard time of an observation may be recorded directly from a clock or a radio, and the sidereal time may be calculated at a later time. However, the sidereal clock is still valuable in providing the observer with a quick means of knowing where to look for an object of a particular right ascension.

Julian Date

There are in astronomy a number of situations in which it is convenient to use a system of consecutively numbered days. Such a sequence is useful whenever the number of days between two events is required. The number of the earlier day may simply be subtracted from the number of the later day. Applications of this type arise, for example, in the study of variable stars, and these will be discussed in Chapter 14. The most widely used scheme for numbering days was introduced in 1582 and is known as the system of Julian Day Numbers. The numbers begin with January 1, 4713 BC, and each day is considered to begin at noon rather than at midnight. Julian Day numbers may be found in the *Astronomical Almanac* in the tables in which data for the sun are tabulated. See, for example, Fig. 3.9.

In computational work it is often more convenient to calculate the Julian Day number than it is to look it up and enter it from a table. The following formula is quite simple and may be used to find the proper number for any day since the year 1900.

$$JD = 2\,415\,020 + 365 \times (\text{year} - 1900) + N + L - 0.5$$

Here N is, again, the number of the day counted from the beginning of the year, and L is the number of leap years which have occurred between 1901 and the beginning of the year in question. 2 415 020 is the Julian Day number of January 1, 1900, and the 0.5 accounts for the fact that the Julian Day begins at noon. This calculation gives the Julian Day number at 0 h UT, so one must also add the decimal fraction of a day at the required universal time.

Example: Find the Julian Day Number for April 18, 1983.

First we must calculate N , the number of days which have elapsed from the beginning of the year until April 18.

$$N = \langle 275 \times 4/9 \rangle + 2 \times \langle (4+9)/12 \rangle + 18 - 30$$

$$N = 108$$

The number of leap years from 1901 to 1983 is simply $\langle 82/4 \rangle$ and is 20. Then we may find the Julian Day Number from the formula given above.

$$JD = 2\,415\,020 + 365 \times (1983 - 1900) + 108 + 20 - 0.5$$

$$JD = 2\,445\,442.5$$

Example: Find the Julian Day Number at 13 h 45 m on April 18, 1984. Note that 1984 was a leap year. Following the procedure used in the previous example:

$$N = \langle 275 \times 4/9 \rangle - 1 \times \langle (4+9)/12 \rangle + 18 - 30$$

$$N = 109$$

$$JD = 2\,415\,020 + 365 \times 84 + 109 + 20 - 0.5 + 0.5729$$

$$JD = 2\,445\,809.0729$$

The final term (0.5729) is the fraction of a day equivalent to 13 hours and 45 minutes. Note also that the number of leap years was 20 in both examples even though 1984 was a leap year. The extra day in 1984 was taken care of when the day number, N , was calculated.

This useful method of numbering days was introduced, as we have said, in 1582, and it was devised by Joseph Justus Scaliger, one of the most brilliant men of his time but one who is little known today. He chose the name 'Julian' to honor his father, Julius Caesar Scaliger. There is no connection at all between the Julian Day numbers and the Julian

Calendar which was introduced during the reign of Julius Caesar. Joseph Scaliger defined his Julian Period as 7980 years, the product of the numbers 28, 19 and 15. The first of these is the number of years required for the same day of the week to re-occur on the same day of the year. The second is the Metonic Cycle in which 235 lunar cycles are equal in length to 19 solar years. The third number is the length of the Roman Indiction, a period of years used by the emperor, Constantine, in the fourth century AD. It hardly needs to be said that there is no astronomical significance to Scaliger's Julian Period.

In order to avoid the inconvenience of using very large numbers some people have chosen to count the days only from 1900. This scheme is referred to as the system of Modified Julian Day numbers (MJD). The reader should be able to modify the formula given above to calculate the correct MJD rather than the JD.

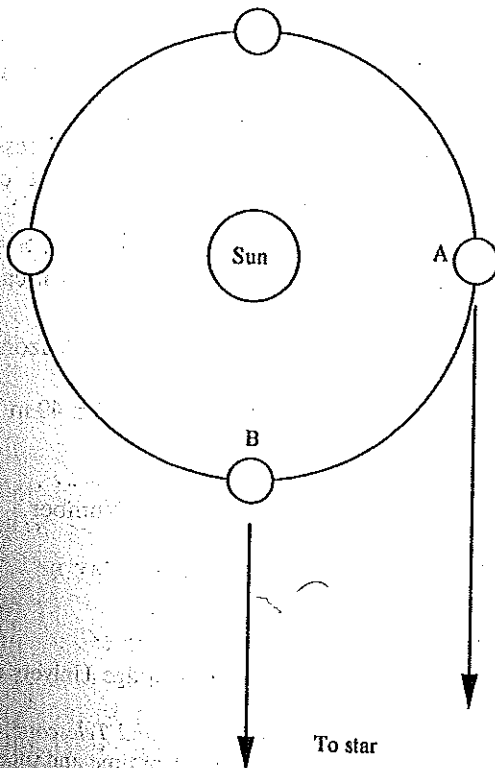


Fig. 2.6. The need for heliocentric time.

Heliocentric Time

When precisely-timed celestial events are observed at several times during the year, we must take into account the time required for light to travel across the earth's orbit. Depending upon the direction of an object in space light can reach the earth as much as eight minutes earlier when the earth is at one position in its orbit than when it is at another. This effect is illustrated in Fig. 2.6. A recurring event such as the eclipse of a binary star would seem to occur earlier when the earth is at position A than it would three months later when the earth is at position B. In order to correct for this, we calculate the heliocentric time, that is, the time at which the event would have been recorded by an observer on the sun.

The method of determining the heliocentric time will be described in detail in Chapter 14 as part of the discussion of variable stars.

QUESTIONS FOR REVIEW

1. What is the basis for our everyday system of timekeeping?
2. Why is the sidereal day not the same length as the solar day? Which is longer?
3. Define solar time and mean solar time. Why is a distinction necessary?
4. Why is the difference (solar time—mean solar time) not constant throughout the year?
5. How can a precise clock be used to determine longitude?
6. Describe the reasons why dynamical time is needed and why it cannot be determined at the time of an observation.
7. Calculate the local sidereal time for an observer in Tucson, Arizona, USA for May 1, 1990 at 21 h 31 m Mountain Standard Time (MST).
8. Calculate the Julian Day Number for May 12, 1990, at 22 h 40 m Eastern Standard Time.
9. Write a computer program to compute local sidereal time.
10. Write a computer program to compute the Julian Day Number.

Further reading

- Green, R. M. (1985). *Spherical Astronomy*. Cambridge: Cambridge University Press. Consult Chapter 10 for descriptions of many systems of time.
- MacRobert, A. (1989). Time and the amateur astronomer. *Sky and Telescope*, 77, 378. This is a concise and clear description of the important kinds of time and the need for each.

- Pannekoek, A. (1961). *A History of Astronomy*. Interscience Publishers. Good discussions of time-keeping and clocks are scattered throughout this book. Consult the index.
- Smart, W. M. (1977). *Textbook on Spherical Astronomy*. Cambridge: Cambridge University Press. Chapter VI is a very complete discussion of the many ways of measuring time.