Emitted Spectrum

- Summary of emission processes
- Emissivities for emission lines:
  - Collisionally excited lines
  - Recombination cascades
- Emissivities for continuum processes
  - recombination
  - brehmsstrahlung
  - H I two-photon
- Radiative transfer effects
- Bowen Resonance-Fluorescence

Summary of Emission Processes

- Emission Lines:

  1) Recombination:

     - essentially just H I, He I, He II due to abundances

  2) Collisionally excited

     - levels above ground where $h\nu \approx kT$
     - forbidden lines (e.g., [O III], [N II], in the optical, IR)
     - semi-forbidden (e.g., C III] in the UV)
     - permitted (e.g., C IV, Si IV, Mg II in the UV)

- Continuum:

  1) brehmsstrahlung

  2) continuum recombination (H I, He I, He II)

  3) H I two-photon

Emissivities for Emission Lines

- want j_{ij} (ergs $s^{-1}cm^{-3}$) for transition $i \rightarrow j$

(Note Osterbrock & Ferland use $4\pi j$ for emissivity)

Collisionally excited lines:

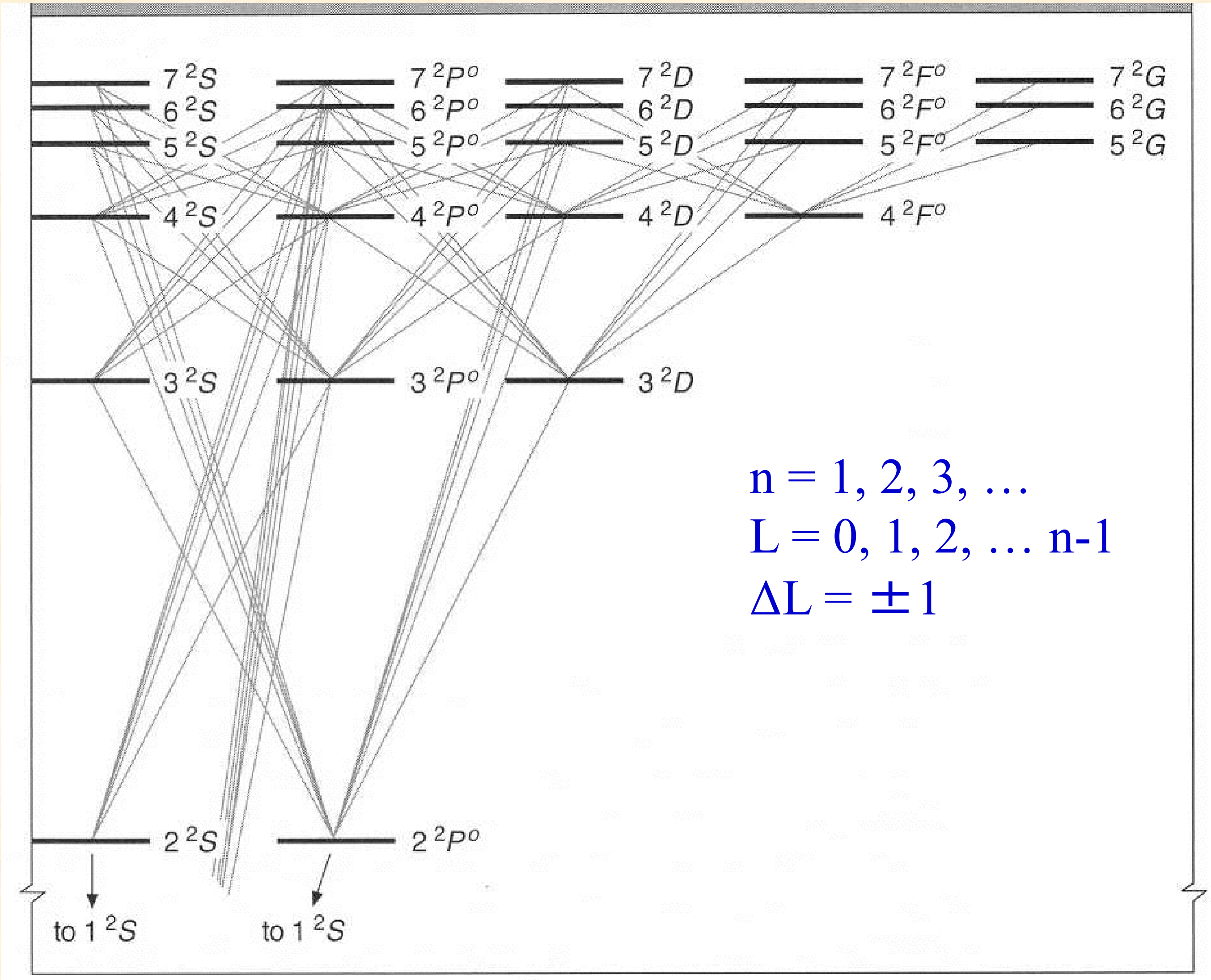
- calculate the level population from detailed balancing:

$j_{ij} = h\nu_{ij} n_i A_{ij}$ (see previous lecture to get n_i)

Recombination lines:

- use cascade matrices
- transitions from nL to n'L' (instead of ij notation)

Recombination for H I - Cascades



$n = 1, 2, 3, \ldots$

$L = 0, 1, 2, \ldots n-1$

$\Delta L = \pm 1$

(Osterbrock & Ferland, p. 19)

Cascades

The relative probability of a direct transition from level nL to level n'L' is:

$$P_{nL,n'L'} = \frac{A_{nL,n'L'}}{\sum_{n''=1}^{n-1} \sum_{L''} A_{nL,n''L''}}$$

Let $C_{nL,n'L'}$ = cascade matrix

= transition probability for all possible routes from nL to n'L'

For n' = n - 1: $C_{nL,n'L'} = P_{nL,n'L'}$

To obtain cascade matrix: $C_{nL,n'L'} = \sum_{n''>n'}^{n} \sum_{L''} C_{nL,n''L''} P_{n''L'',n'L'}$

- the cascade matrix is fixed for a given ion (e.g., H I)

To get the level populations:

- consider transitions to and from level nL:

$$n_e n_p \alpha_{nL}(T) + \sum_{n'>n}^{\infty} \sum_{L'} n_{n'L'} A_{n'L',nL} = n_{nL} \sum_{n''=1}^{n-1} \sum_{L''} A_{nL,n''L''}$$

recomb to nL + # cascades to nL = # cascades from nL

$(\Delta L = \pm 1)$

The equilibrium equation becomes:

$$n_e n_p \sum_{n'=n}^{\infty} \sum_{L'=0}^{n'-1} \alpha_{n'L'}(T)\, C_{n'L',nL} = n_{nL} \sum_{n''=1}^{n-1} \sum_{L''} A_{nL,n''L''}$$

- $n_{n'L'}$ has been eliminated; n_{nL} can be solved for iteratively once the ionization balance is determined

The emissivity is then :

$$j_{nn'} = h\nu \sum_{L=0}^{n-1} \sum_{L'=L\pm1} n_{nL} A_{nL,n'L'}$$

(ergs s^{-1} cm^{-3})

Case A (optically thin): all levels included

Case B (optically thick): all Lyman transitions with $n \geq 3$ are converted to higher order lines plus Lyman α or two - photon emission

- for $\tau\,(h\nu_0) = 1$, $\tau\,(Ly\alpha) = 10^4$, $\tau\,(Ly\beta) = 10^3$

$\rightarrow$ No Lyβ, Lyγ, etc. emission from typical nebulae

Density effects for recombination :

- collisions change nL to nL' (small energy difference)
- the collision cross section is larger for higher n
- for n ≥ 15 (H I), the population is the same as for thermodynamic equilibrium:

$$n_{nL} = \frac{(2L+1)}{n^2} n_n \quad (n_n = \text{ density of ion at quantum level } n)$$

We can define an effective recombination coefficient :

$$n_p n_e \alpha^{eff}_{nn'} = \sum_{L=0}^{n-1} \sum_{L'} n_{nL} A_{nL,n'L'}$$

so that $\alpha^{eff}_{nn'}$ is the # transitions per $n_p n_e$ (from n to n')

H I Case A Recombination

H I recombination lines (Case A, low-density limit)

	T			
	2,500 K	5,000 K	10,000 K	20,000 K
$4\pi j_{H\beta}/n_e n_p$ (erg cm^3 s^{-1})	2.70×10^{-25}	1.54×10^{-25}	8.30×10^{-26}	4.21×10^{-26}
$\alpha^{eff}_{H\beta}$ (cm^3 s^{-1})	6.61×10^{-14}	3.78×10^{-14}	2.04×10^{-14}	1.03×10^{-14}
Balmer-line intensities relative to Hβ				
$j_{H\alpha}/j_{H\beta}$	3.42	3.10	2.86	2.69
$j_{H\gamma}/j_{H\beta}$	0.439	0.458	0.470	0.485
$j_{H\delta}/j_{H\beta}$	0.237	0.250	0.262	0.271
$j_{H\varepsilon}/j_{H\beta}$	0.143	0.153	0.159	0.167
$j_{H8}/j_{H\beta}$	0.0957	0.102	0.107	0.112
$j_{H9}/j_{H\beta}$	0.0671	0.0717	0.0748	0.0785
$j_{H10}/j_{H\beta}$	0.0488	0.0522	0.0544	0.0571
$j_{H15}/j_{H\beta}$	0.0144	0.0155	0.0161	0.0169
$j_{H20}/j_{H\beta}$	0.0061	0.0065	0.0068	0.0071
Lyman-line intensities relative to Hβ				
$j_{L\alpha}/j_{H\beta}$	33.0	32.5	32.7	34.0

(Osterbrock & Ferland, p. 72)

H I Case B Recombination

H I recombination lines (Case B)

	T								
	5,000 K			10,000 K			20,000 K		
n_e (cm^{-3})	10^2	10^4	10^6	10^2	10^4	10^6	10^2	10^4	10^6
$4\pi j_{H\beta}/n_e n_p$ (10^{-25} erg cm^3 s^{-1})	2.20	2.22	2.29	1.23	1.24	1.25	0.658	0.659	0.661
$\alpha^{eff}_{H\beta}$ (10^{-14} cm^3 s^{-1})	5.37	5.43	5.59	3.02	3.03	3.07	1.61	1.61	1.62
Balmer-line intensities relative to Hβ									
$j_{H\alpha}/j_{H\beta}$	3.041	3.001	2.918	2.863	2.847	2.806	2.747	2.739	2.725
$j_{H\gamma}/j_{H\beta}$	0.458	0.460	0.465	0.468	0.469	0.471	0.475	0.476	0.476
$j_{H\delta}/j_{H\beta}$	0.251	0.253	0.258	0.259	0.260	0.262	0.264	0.264	0.266
$j_{H10}/j_{H\beta}$	0.0515	0.0520	0.0616	0.0530	0.0533	0.0591	0.0540	0.0541	0.0575
$j_{H15}/j_{H\beta}$	0.01534	0.01628	0.02602	0.01561	0.01620	0.02147	0.01576	0.01612	0.01834
$j_{H20}/j_{H\beta}$	0.00657	0.00819	0.01394	0.00662	0.00755	0.01058	0.00664	0.00717	0.00832

(Osterbrock & Ferland, p. 78)

-recombination lines are good reddening indicators, since their intrinsic ratios are approximately constant over a wide range in temperature

Continuum radiation

- Recombination (free-bound): Hydrogen

$$h\nu = \tfrac{1}{2} m v^2 + \chi_{n_1} \quad \text{where } \chi_{n_1} = \frac{h\nu_0}{n_1^{\ 2}}$$

- Lyman continuum (to $n_1 = 1$): $\lambda \leq 912$ Å (UV)
- Balmer continuum (to $n_1 = 2$): $\lambda \leq 3646$ Å (optical)
- Paschen continuum (to $n_1 = 3$): $\lambda \leq 8204$ Å (IR)

$$j_\nu = n_p n_e \sum_{n=n_1}^{\infty} \sum_{L=0}^{n-1} v\sigma_{nL}(H^0, v)\, f(v)\, h\nu \,\frac{dv}{d\nu}$$

- Brehmsstralung (free-free) (important in IR and radio)

Total b-f and f-f can be written as:

$$j_\nu(\text{H I}) = n_p n_e \gamma_\nu(H^0, T) \qquad \gamma = \text{continuous emission coef.}$$

$$j_\nu(\text{He I}) = n_{He^+} n_e \gamma_\nu(He^0, T) \qquad \text{(Osterbrock p. 83)}$$

$$j_\nu(\text{He II}) = n_{He^{+2}} n_e \gamma_\nu(He^+, T)$$

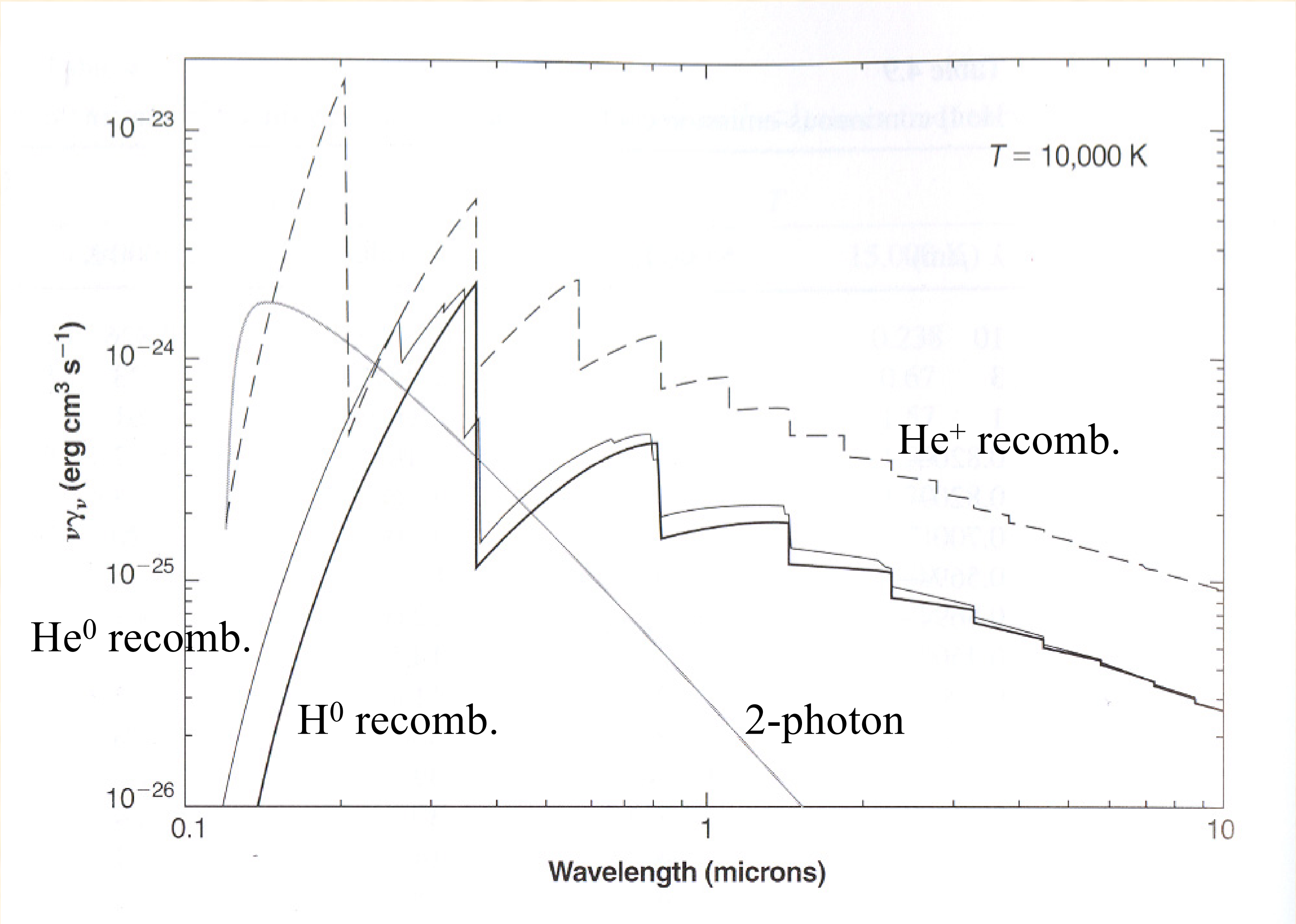
- H I two-photon emission

$$j_\nu(2q) = n_{2^2S} A_{2^2S,1^2S} 2hyP(y)$$

where P(y)dy is the probability per decay that a photon is emitted in the range of frequencies $y\nu_{12}$ to $(y + dy)\nu_{12}$

- hν' + hν'' = hν (Lyα)
- 2 photon continuum peaks at 1/2 energy of Lyα photon (corresponding to λ = 2431 Å)

Continuous Emission



(Osterbrock & Ferland, p. 86)

Radiative Transfer Effects

- Most emission lines escape a nebula after their creation, since the probability of re-absorption is low

  → for most lines, you can ignore radiative (photo-) excitations
- However, some lines are "optically thick" in most circumstances:
- 1) H I Lyman series

  2) He I triplet state

  3) Bowen resonance-fluorescence lines

- In general, the optical depth depends on column density, velocity profile, and geometry

1) H I Lyman Series

Case A – Lyman lines are optically thin

Case B – Lyman lines are optically thick

2) He I triplet state:

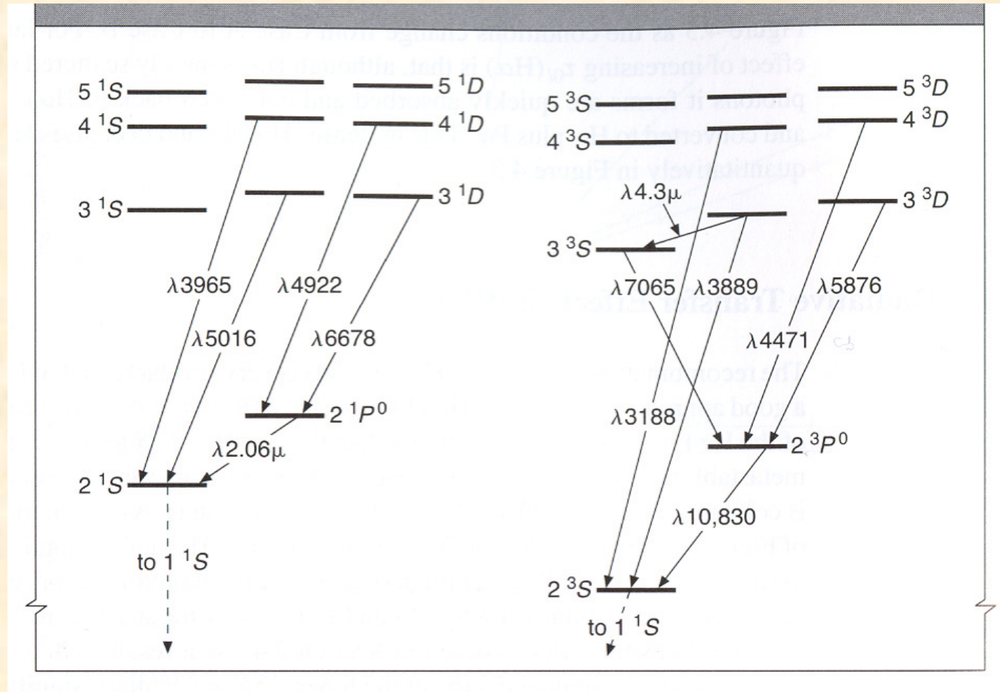
- 2^3S is metastable (transition to 1^1S is highly forbidden)
- escape can occur by:

1) Collision to 2^1S or 2^1P
2) Photoionization (particularly by H I Lyα)
3) Radiative excitation to 3^3P (absorption of λ3889), followed by radiative transitions:

→He I λ3889 weakened,

→He I λ7065 strengthened,

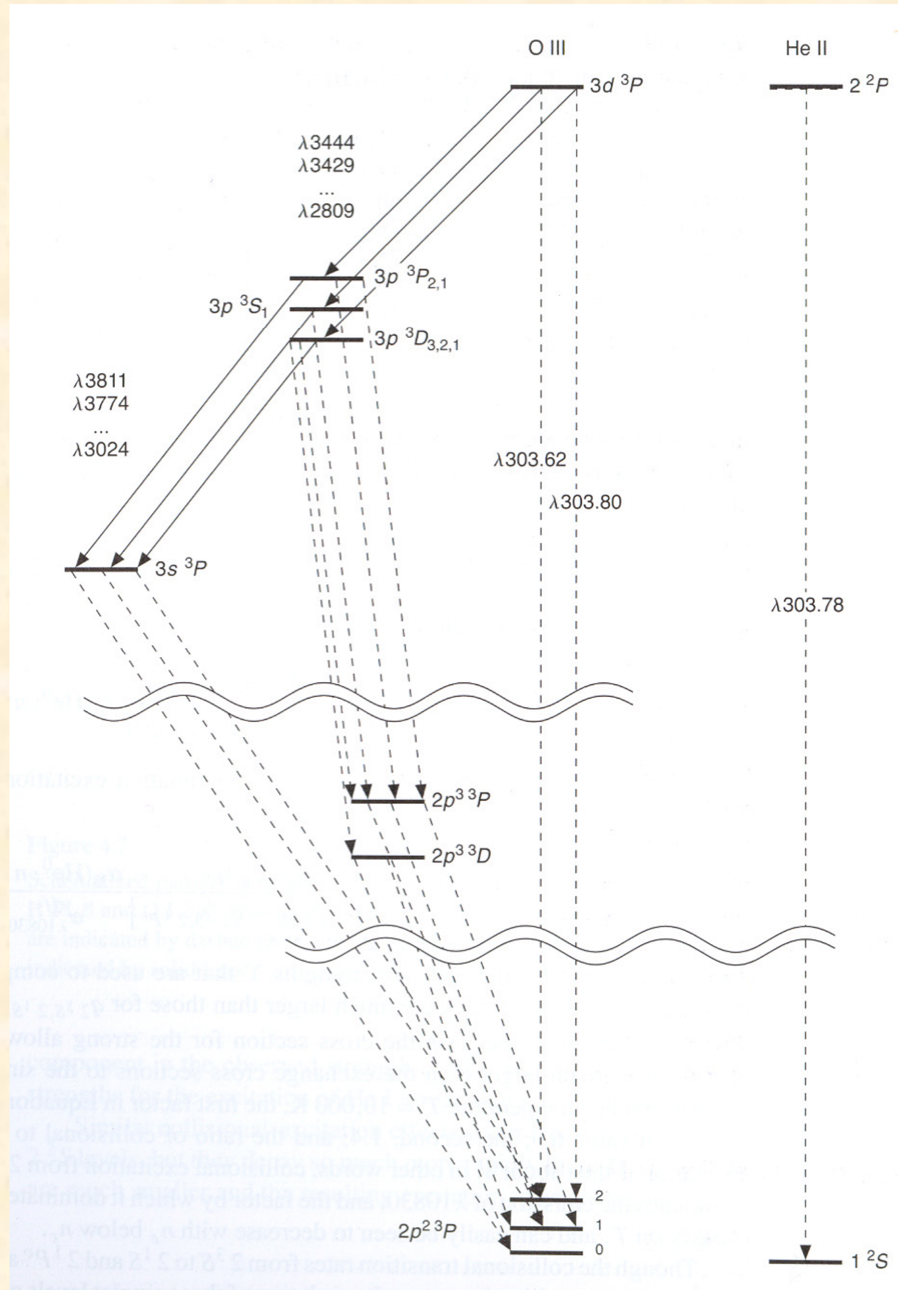
→He I λ4471 not affected



(Osterbrock & Ferland, p. 98)

3) Bowen Resonance-Fluorescence Mechanism:

- He II Lyα is very optically thick, and scatters throughout nebula
- close correspondence between He II Lyα (λ303.78) and O III $2p^2\ ^3P_2 - 3d\ ^3P_2$ (λ303.80)
- the He II Lyα photons can thus be captured by ground-state O III (resonance absorption) and converted to UV/optical photons (fluorescence)
- a competing process is ionization of H^0 or He^0 by λ303.78
- the relative importance depends on the optical depth (a function of geometry, velocity dispersion, and column)



(Osterbrock & Ferland, p. 101)

Bowen UV/optical Lines

O III Resonance-fluorescence lines

Transition	Wavelength (Å)	Relative probability	Relative intensity
$3p\ ^3P_2 - 3d\ ^3P_2^0$	3444.10	3.74×10^{-3}	0.277
$3p\ ^3P_1 - 3d\ ^3P_2^0$	3428.67	1.25×10^{-3}	0.093
$3p\ ^3S_1 - 3d\ ^3P_2^0$	3132.86	1.23×10^{-2}	1.000
$3p\ ^3D_3 - 3d\ ^3P_2^0$	2837.17	1.16×10^{-3}	0.104
$3p\ ^3D_2 - 3d\ ^3P_2^0$	2819.57	2.08×10^{-4}	0.019
$3p\ ^3D_1 - 3d\ ^3P_2^0$	2808.77	1.38×10^{-5}	0.0013
$3s\ ^3P_2^0 - 3p\ ^3S_1$	3340.74	1.79×10^{-3}	0.136
$3s\ ^3P_1^0 - 3p\ ^3S_1$	3312.30	1.07×10^{-3}	0.082
$3s\ ^3P_0^0 - 3p\ ^3S_1$	3299.36	3.57×10^{-4}	0.028
$3s\ ^3P_2^0 - 3p\ ^3P_2$	3047.13	2.14×10^{-3}	0.179
$3s\ ^3P_1^0 - 3p\ ^3P_2$	3023.45	7.12×10^{-4}	0.060
$3s\ ^3P_2^0 - 3p\ ^3P_1$	3059.30	3.95×10^{-4}	0.033
$3s\ ^3P_1^0 - 3p\ ^3P_1$	3035.43	2.37×10^{-4}	0.020
$3s\ ^3P_0^0 - 3p\ ^3P_1$	3024.57	3.17×10^{-4}	0.027
$3s\ ^3P_0^0 - 3p\ ^3D_1$	3757.21	3.57×10^{-6}	0.0002
$3s\ ^3P_1^0 - 3p\ ^3D_1$	3774.00	2.67×10^{-6}	0.0002
$3s\ ^3P_2^0 - 3p\ ^3D_1$	3810.96	1.80×10^{-7}	0.0001
$3s\ ^3P_1^0 - 3p\ ^3D_2$	3754.67	7.22×10^{-5}	0.0049
$3s\ ^3P_2^0 - 3p\ ^3D_2$	3791.26	2.41×10^{-5}	0.0016
$3s\ ^3P_2^0 - 3p\ ^3D_3$	3759.87	5.39×10^{-4}	0.037