# Gas Dynamics

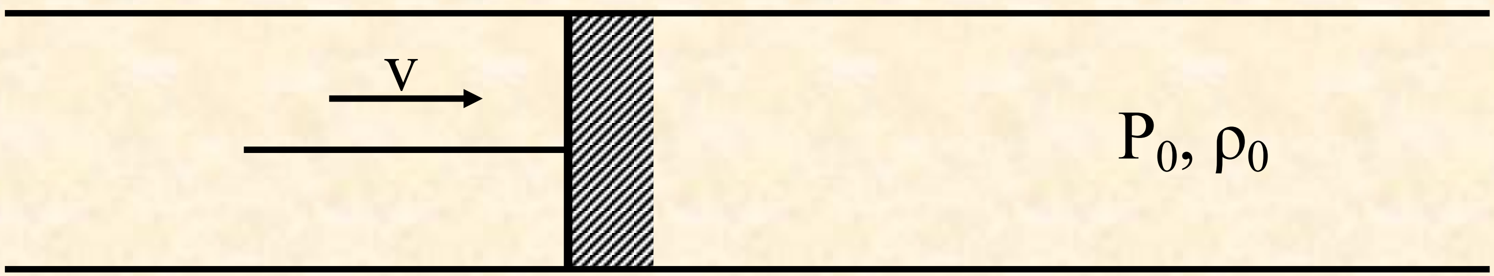
- Gaseous nebulae are not static entities:
- Expansion of ionized gas into the ISM (Planetary Nebula)
- Radiation driving of ionized clouds (stellar winds and AGN)
- Gravitational motions around a supermassive black hole (AGN)
- Expansion of ionized "sphere" of gas after a hot star turns on (H II region)

  - creates an ionization/shock front
- Explosions into the surrounding ISM (novae and supernovae)

  - create shock fronts

# Shock Fronts

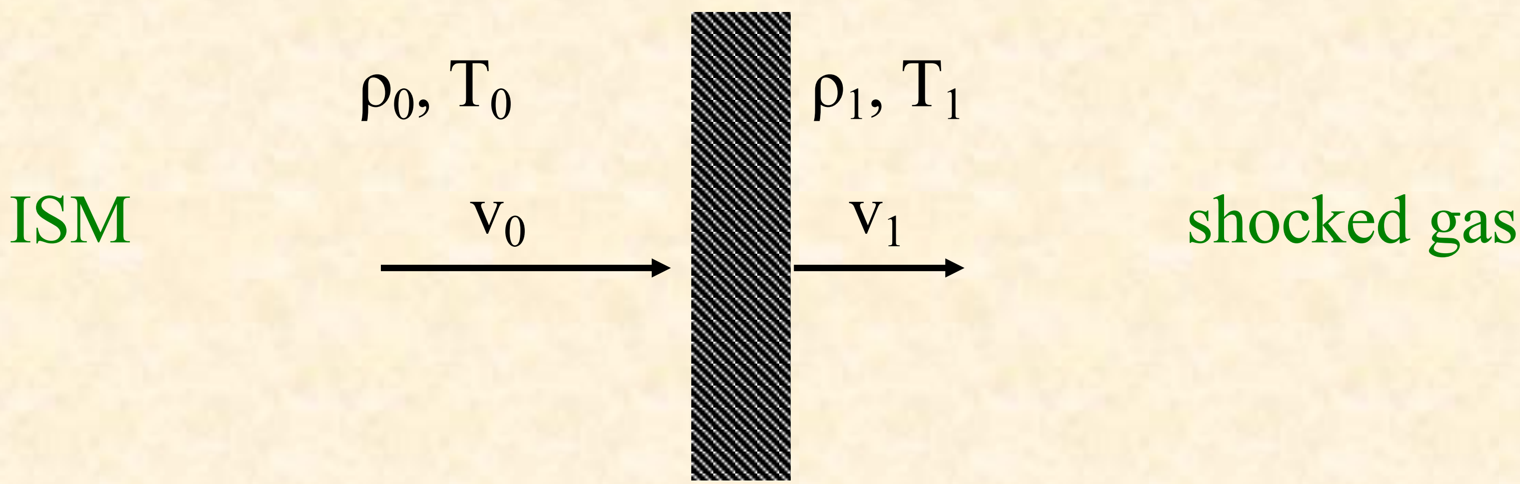
## Ex) Piston in a tube of gas (adiabatic case)




- Moving piston starts a compression wave (higher P, ρ).
- Speed of sound is higher in compressed region.
- Gas in the compressed region travels much faster, acting to steepen the pulse.
- A nearly discontinuous shock front is formed
- Note that velocity of piston must be supersonic (compared to unshocked gas), or the disturbance will spread out (no more shock front).

# Shock Fronts

- Ex) Explosion of a Supernova into the ISM
- Supernova remnant moves at supersonic speed in the ISM at $v_0$
- Builds up a pulse of increased pressure
- Pulse steepens because sound velocity is higher in the compressed region. In this case, gas can escape out the back end.
- Consider a shock front propagating through the ISM:
  - Reference frame: traveling with the shock




$\rho$ – mass density, T – temperature, v - velocity

# Conservation laws

1) $\rho_0 v_0 = \rho_1 v_1$ (cons. of mass)

2) $P_0 + \rho_0 v_0^2 = P_1 + \rho_1 v_1^2$ (cons. of momentum)

## I. Adiabatic Case (e.g., initial SNR explosion)

For adiabatic expansion (no radiation loss from compression):

$P = K\rho^\gamma$ ($\gamma = 5/3$ for monatomic gas)

"It can be shown that (see Osterbrock, p. 162)":

3) $$\frac{1}{2}v_0^2 + \frac{\gamma}{\gamma-1}\frac{P_0}{\rho_0} = \frac{1}{2}v_1^2 + \frac{\gamma}{\gamma-1}\frac{P_1}{\rho_1}$$ (cons. of energy)

$$\frac{1}{2}v_0^2 + \frac{5}{2}\frac{P_0}{\rho_0} = \frac{1}{2}v_1^2 + \frac{5}{2}\frac{P_1}{\rho_1}$$ (for monatomic gas)

(1st term - flow kinetic energy per mass, 2nd term - thermal kinetic energy plus compression energy)

## II. Isothermal Case (e.g., stellar wind bubble inside H II region)

- gas just ahead and just behind shock has same temperature ($T_0 = T_1$)
- applies to shocks *within* H I or H II regions since the heating and cooling time scales are much smaller than the expansion time scales (works also for planetary nebula)

$$P = \frac{\rho k T}{\mu m_H} \quad (\mu \text{ - atomic weight, } T = \text{const.})$$

$$P \propto \rho \qquad (\gamma = 1)$$

So for the conservation of energy:

$$3) \quad \frac{P_0}{\rho_0} = \frac{P_1}{\rho_1} = \frac{kT}{\mu m_H}$$

The sound speed in the undisturbed gas is given by:

$$c_0 = \sqrt{\frac{\gamma P_0}{\rho_0}} = \sqrt{\frac{\gamma k T_0}{\mu_0 m_H}} \quad \left(\sim 1 \text{ km s}^{-1} \text{ for } T = 100 \text{ K}\right)$$

and the Mach number of the shock front is:

$$M = \frac{|v_0|}{c_0}$$ Supersonic if $M > 1$

Combining the conservation equations, we get:

1) $$\frac{P_1}{P_0} = \frac{2\gamma}{\gamma+1} M^2 - \frac{\gamma-1}{\gamma+1}$$

2) $$\frac{\rho_1}{\rho_0} = \frac{(\gamma+1)M^2}{(\gamma-1)M^2 + 2}$$

Ex) For a strong shock (large M), $P_1/P_0 \to \infty$

Adiabatic monatomic case: $\rho_1/\rho_0 \to 4$, $v_1 \to \frac{1}{4} v_0$

Isothermal case: $\rho_1/\rho_0 \to \infty$, $v_1 \to 0$

# Ionization Fronts

- Ex) An O or B star turns on
- An H II region expands at the rate that ISM is ionized
- The momentum conservation law is the same. The degree of ionization changes sharply across the boundary:

1) $\rho_0 v_0 = \rho_1 v_1 = m_i \Phi_i$ (cons. of mass)

where $m_i$ = mass of ionized gas per created ion / electron pair

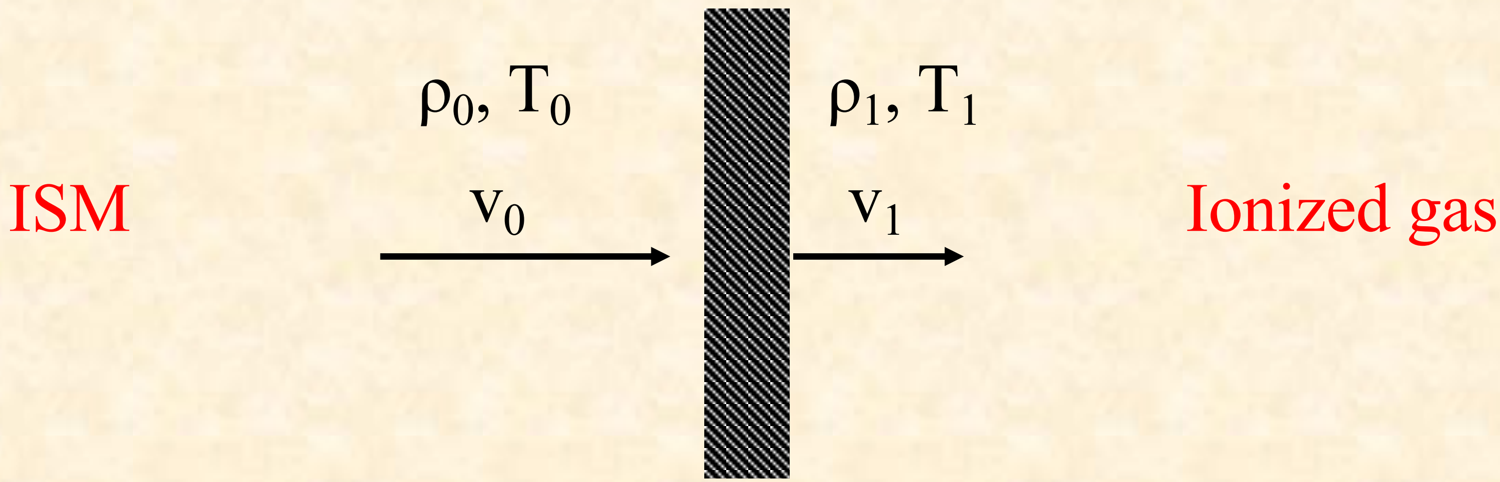
($m_i = m_H$ for a pure H nebula)

$$\Phi_i = \int_{\nu_0}^{\infty} \frac{F_\nu}{h\nu} d\nu \ = \text{ flux of ionizing photons}$$

# Ionization Fronts (H II Region)

For an ionized nebula, we have two very different temperatures. The conservation of energy becomes:

3) $\frac{P_0}{\rho_0} = \frac{kT_0}{\mu_0 m_H}$ and $\frac{P_1}{\rho_1} = \frac{kT_1}{\mu_1 m_H}$ ($T_1$ fixed by photoionization, $T_0$ is temperature of ISM)




Solving for the density ratio:

$$\frac{\rho_1}{\rho_0} = \frac{c_0^2 + v_0^2 \pm \left[\left(c_0^2 + v_0^2\right)^2 - 4c_1^2 v_0^2\right]^{1/2}}{2c_1^2}$$

(Osterbrock, p. 166)

There are two allowed ranges for $v_0$ (speed of ionization front),

since $\frac{\rho_1}{\rho_0}$ must be real:

1) $v_0 \geq c_1 + \sqrt{c_1^2 - c_0^2} \;\equiv\; v_R \approx 2c_1$ (for $c_1 >> c_0$ in H II region)

$v_R$ is the velocity of an "R - critical front" (R - rare or low density),

since as $\rho_0 \to 0$, $v_0 \to \infty$ and therefore exceeds $v_R$

For $v_0 > v_R$, an R - type front moves supersonically into the ISM

2) $v_0 \leq c_1 - \sqrt{c_1^2 - c_0^2} \;\equiv\; v_D \approx \frac{c_0^2}{2c_1}$ ($c_1 >> c_0$)

$v_D$ is the velocity of a "D - critical front" (D - dense),

When $v_0 < v_D$, a D-type front moves subsonically into the ISM

# Consider an H II region expanding into the ISM

In this case, $c_0$ is small, and $v_0 \gg c_1$, so again there are two cases:

1) $\dfrac{\rho_1}{\rho_0} = \dfrac{v_0^2}{c_1^2}\left(1 - \dfrac{c_1^2}{v_0^2}\right) \gg 1$ (strong R - type front)

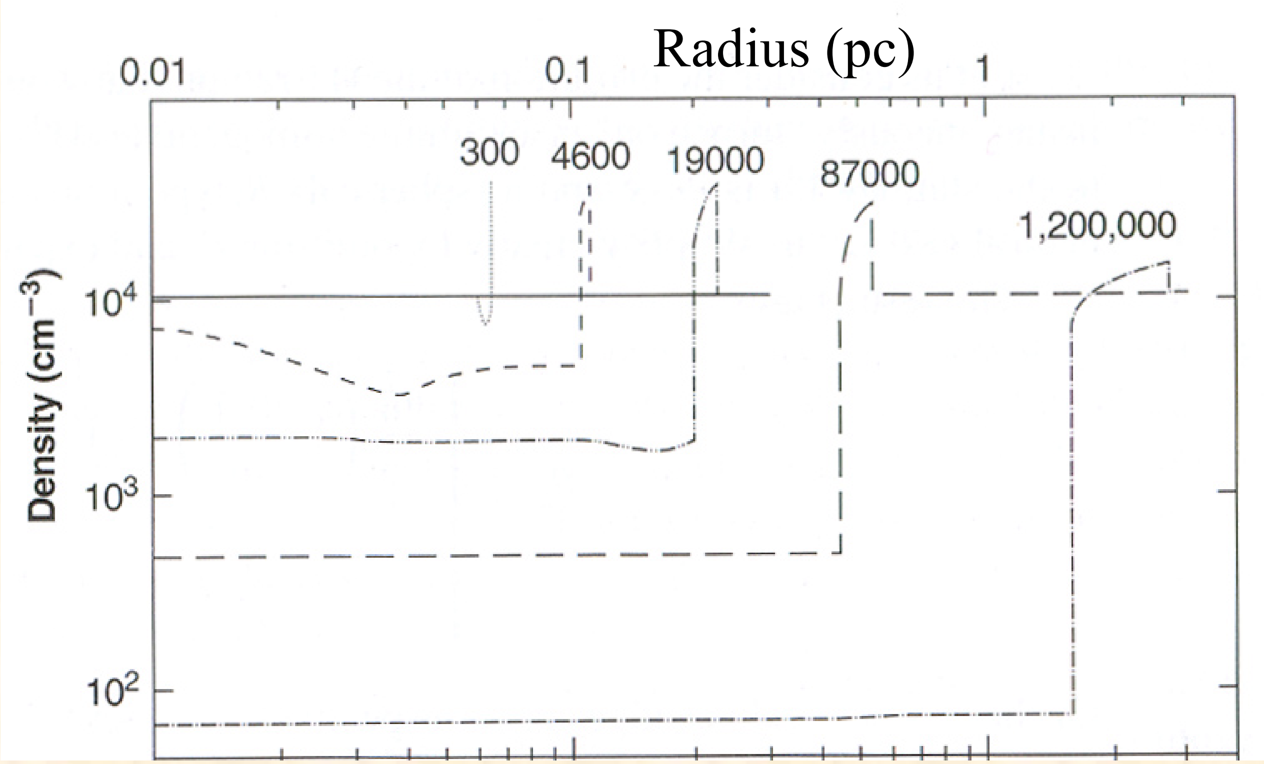
2) $\dfrac{\rho_1}{\rho_0} = 1 + \dfrac{c_1^2}{v_0^2} \approx 1$ (weak R - type front)

Case 1) is not physical, since high densities in the ionized gas disrupt the shock. Using the conservation of mass for case 2):

$$v_1 = v_0\left(1 - \frac{c_1^2}{v_0^2}\right) \approx v_0 \gg c_1$$

So the ionization front initially moves supersonically into the ISM, due to the large number of ionizing photons.

# Model of Expanding H II Region around an O Star




(Osterbrock & Ferland, p. 167)

- t = 0 - 300 years: weak R-type ionization front moves out at $v_0 \approx 300$ km/sec
- t = 300 - 4600: $\Phi_i$ decreases as the front expands, due to geometric dilution and photoionization; $v_0$ decreases until it reaches $v_R \approx 2c_1$
- $t \approx 4600$ years: a shock front breaks off and compresses the gas ahead of the ionization front, which becomes D-type; $v_0 \approx v_D$, $v_1 \approx c_1$
- t = 4600 - 1 million years: the ionization front continues as D-type, which slows down until it terminates at the Stromgren radius.