ASTRONOMY 8300 – FALL 2024
Homework Set 3 Answers

1.a. O star: $L = 4\pi R^2 \sigma T^4 = 3.11\times10^{39}$ ergs s^{-1} $= 8.1\times10^{5}\,L_\odot$

B star: $L = 4\pi R^2 \sigma T^4 = 1.57\times10^{38}$ ergs s^{-1} $= 4.1\times10^{4}\,L_\odot$

1.b. Used IDL "planck" procedure to get B_λ and normalized blackbody curves to the O and B star luminosities to get L_λ :

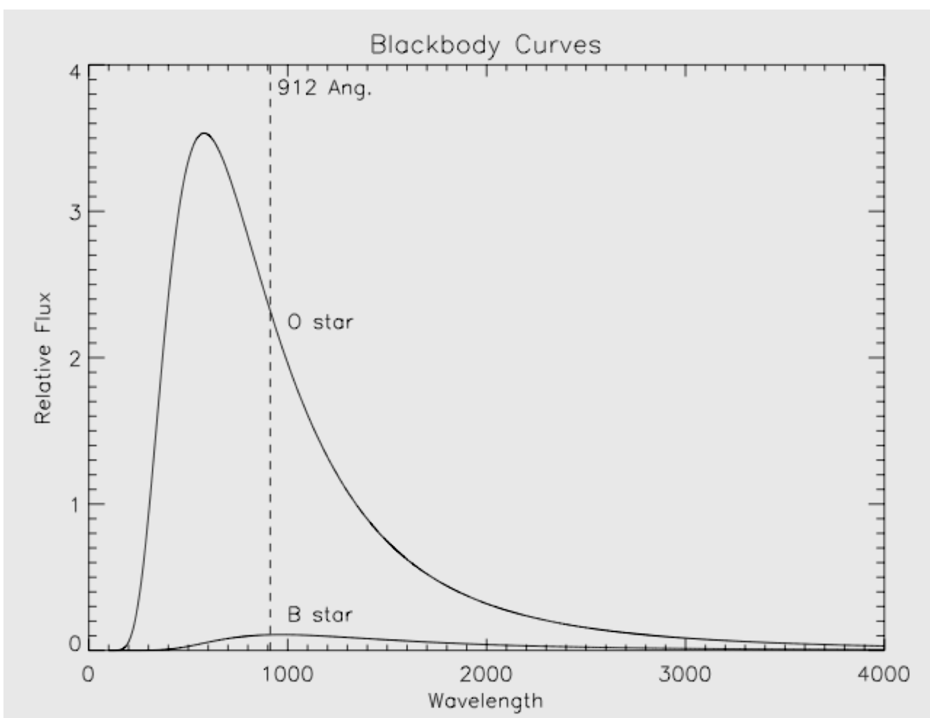
O star: $L_{ion} = \int_0^{912\text{Å}} L_\lambda d\lambda = 1.78\times10^{39}$ ergs s^{-1} (57% of total L)

B star: $L_{ion} = \int_0^{912\text{Å}} L_\lambda d\lambda = 3.45\times10^{37}$ ergs s^{-1} (21% of total L)

To get ionizing photons/sec, divide L_λ by photon energy and integrate:

O star: $Q_{ion} = \int_0^{912\text{Å}} \frac{L_\lambda}{hc/\lambda}\, d\lambda = 5.5\times10^{49}$ photons s^{-1}

B star: $Q_{ion} = \int_0^{912\text{Å}} \frac{L_\lambda}{hc/\lambda}\, d\lambda = 1.2\times10^{48}$ photons s^{-1}



1.c. $Q_{ion} = 4/3\pi r^3 n_H^2 \alpha_B \rightarrow r = \left(\frac{3Q_{ion}}{4\pi n_H^2 \alpha_B} \right)^{1/3}$

O star: $r = 7.7 \times 10^{19}\text{cm} = 25.0\text{pc}$

B star: $r = 2.1 \times 10^{19}\text{cm} = 6.9\text{ pc}$

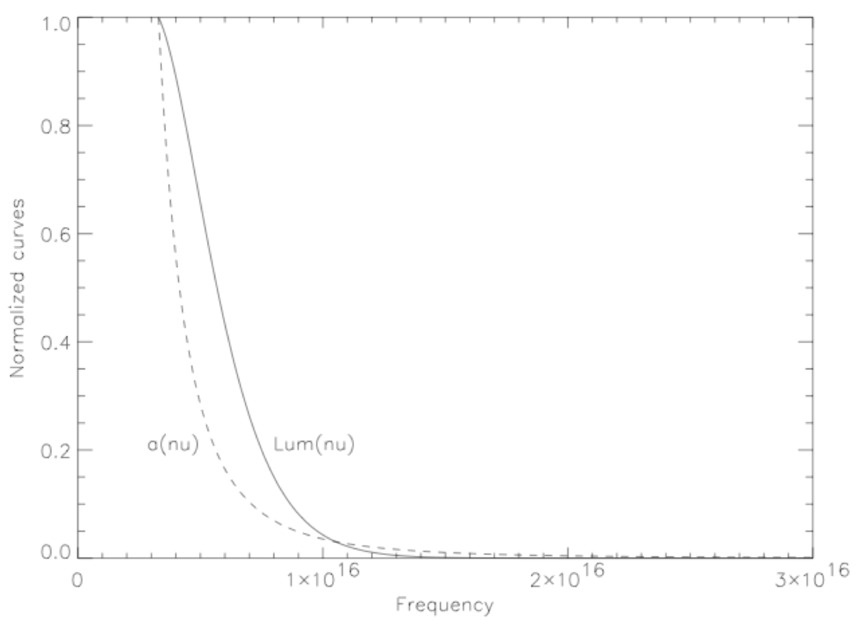
$$\frac{\text{\# B stars}}{\text{O star}} = \left(\frac{25.0}{6.9} \right)^3 \approx 48$$

2.a. $e^{-\tau_\nu} \approx 1, \text{ so: } t_{ion} = \left[\int_{\nu_0}^{\infty} \frac{L_\nu}{4\pi r^2 h\nu} a_\nu \, d\nu \right]^{-1} = \left[\frac{1}{4\pi r^2} \int_{\nu_0}^{\infty} \frac{L_\nu}{h\nu} a_\nu \, d\nu \right]^{-1}$

Use the Planck curve for L_ν (normalized to the previous luminosity)

Use the approximation: $a_\nu = 6.3 \times 10^{-18} \left(\frac{\nu}{\nu_0} \right)^{-3}$ where $\nu_0 = 3.29 \times 10^{15}$ Hz

Multiply, integrate numerically, and divide by $4\pi r^2 \rightarrow \quad t_{ion} = 1.9 \times 10^7$ sec



2.b. $t_{rec} = \frac{1}{n_e \alpha_B} = \frac{1}{(10\text{ cm}^{-3})(2.59 \times 10^{-13}\text{ cm}^3\text{s}^{-1})} = 3.86 \times 10^{11}$ sec

2.c. # photoionizations/vol/sec = # recombinations/vol/sec

$$\frac{n_{H^0}}{t_{ion}} = \frac{n_p}{t_{rec}} \rightarrow \frac{n_p}{n_{H^0}} = \frac{t_{rec}}{t_{ion}} = \frac{3.9 \times 10^{11}\text{ sec}}{1.9 \times 10^7\text{ sec}} = 2.0 \times 10^4$$

The time scales are per atom, so a large fraction of ionized atoms (protons) requires a much longer recombination time scale compared to the ionization time scale.

3.a. The equations we need are:

$$\tau_\nu = \int_0^r n_{H^0}(r')\ a_\nu\ dr'$$

$$n_H = n_p + n_{H^0}$$

$$n_{H^0} \int_{\nu_0}^{\infty} \frac{L_\nu}{4\pi r^2 h\nu}\ a_\nu e^{-\tau_\nu}\ d\nu = n_e n_p\ \alpha_B(H^0,T)$$

The ionization equilibrium equation becomes:

$$(n_H - n_p)\int_{\nu_0}^{\infty} \frac{L_\nu}{4\pi r^2 h\nu}\ a_\nu e^{-\tau_\nu}\ d\nu = n_p^2\ \alpha_B(H^0,T) \quad (\text{since } n_e = n_p)$$

$$\text{Let } b = \int_{\nu_0}^{\infty} \frac{L_\nu}{4\pi r^2 h\nu}\ a_\nu e^{-\tau_\nu}\ d\nu \Big/ \alpha_B(H^0,T)$$

$$\text{Let } c = b n_H$$

$$(n_H - n_p)b = n_p^2$$

$$n_p^2 + b n_p - c = 0$$

This is a quadratic equation with solution:

$$n_p = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-b \pm \sqrt{b^2 - 4c}}{2}$$

Starting at an arbitrarily small distance from the star, we take $\tau_\nu = 0$. Then we:

1) Calculate b and c from the ionizing luminosity, optical depth, distance, and atomic parameters.
2) Solve the quadratic equation to get n_p.
3) Calculate n_{H^0} $(= n_H - n_p)$ and thus n_p / n_H at position r.
4) Go to the next step outward and determine $\tau_\nu = \tau_\nu(\text{previous}) + n_{H^0} a_\nu \Delta r$
5) Repeat procedure for each step in radius.

3.b. Using double precision in IDL: