ASTRONOMY 8300 – FALL 2024
Midterm- Answers

1.a. Equation of radiative transfer:

$$dF_\nu = -\kappa_\nu F_\nu ds, \text{ where } \kappa_\nu = \text{opacity}$$

$$d\tau_\nu \equiv -\kappa_\nu ds \quad (\tau_\nu = \text{optical depth})$$

$$\tau_\nu = \int_0^{\tau_\nu} d\tau'_\nu = \int_{F_c}^{F_\nu} -\frac{dF'_\nu}{F_\nu'} = \ln\left(\frac{F_c}{F_\nu}\right)$$

$$\tau_\nu = \ln\left(\frac{F_c}{F_\nu}\right) = -\ln\left(\frac{F_\nu}{F_c}\right) = -\ln\ (0.02) = 3.9$$

1.b. For resolution of FWHM = 0.2A = 39 km/sec at C IV, the line is resolved, so integrate optical depth over line and convert to column density

For resolution of FWHM = 5.0A = 970 km/sec at C IV, the line is unresolved. Can determine a lower limit to column density assuming line is unsaturated. The best technique is to measure other lines and use the curve-of-growth.

1.c. The abundances are determined from their column densities relative to that of hydrogen, using the above techniques. Most of the heavy elements in the ISM are depleted into dust grains, with the most refractory (highest condensation temperature) elements being the most depleted.

2.a. All reddening curves rise to shorter wavelengths, going from the optical to the UV. Compared to the standard Galactic curve, the LMC has a smaller 2200 A bump and rises more sharply to the UV (at $\lambda < 2500$ A). The SMC has no bump and a very sharp rise to the UV. The sharper rise to the UV indicates smaller average dust grain sizes. The lack of a 2200 A bump is likely due to the lack of C-based grains such as graphite or PAHs

2.b.

$$A_V = 3.1\ E(B\text{-}V), \text{ so } A_V = 0.78$$

$$E_{B\text{-}V} = A_B - A_V, \text{ so } A_B = E_{B\text{-}V} + A_V = 1.03$$

$$\frac{F_{V0}}{F_V} = 10^{0.4A_V} = 2.05 \text{ times brighter}$$

$$\frac{F_{B0}}{F_B} = 10^{0.4A_B} = 2.58 \text{ times brighter}$$

3. a.

$$n_{H^0}\int_{\nu_0}^{\infty}\frac{4\pi J_\nu}{h\nu}\,a_\nu(H^0)\,d\nu = n_e n_p \alpha(H^0,T) \quad (h\nu_0 = 13.6\text{ eV})$$

where: J_ν = mean intensity (ergs $s^{-1}cm^{-2}Hz^{-1}sr^{-1}$)

a_ν = ionization cross section

α = recombination coefficient for H (cm^3s^{-1})

The equation represents:

\# ionizations/sec/vol = # recombinations/sec/vol

3. b.

Case B: Use "on the spot" approximation:

$$n_{H^0}\int_{\nu_0}^{\infty}\frac{4\pi J_{\nu\text{ diffuse}}}{h\nu}\,a_\nu\,d\nu = n_e n_p \alpha_1(H^0,T) \quad \text{(recombinations to ground are absorbed locally)}$$

Subtract above from general equation, put in flux from a single star for $4\pi J_\nu$,

and deplete ionizing photons by absorption ($e^{-\tau_\nu}$):

$$n_{H^0}\int_{\nu_0}^{\infty}\frac{L_\nu}{4\pi r^2 h\nu}\,a_\nu e^{-\tau_\nu}\,d\nu = n_e n_p \alpha_B(H^0,T)$$

where $\alpha_B = \sum_{n=2}^{\infty}\alpha_n(H^0,T)$

3. c.

On a global scale: # ionizing photons/sec = #recombinations/sec

Let $Q(H^0)$ = #ionizing photons emitted by star per second

$$Q(H^0) = \int_{\nu_0}^{\infty}\frac{L_\nu}{h\nu}\,d\nu = \tfrac{4}{3}\pi r_1^3 n_H^2 \alpha_B$$

$$r_1 = \left(\frac{Q(H^0)}{\tfrac{4}{3}\pi\, n_H^2\alpha_B}\right)^{1/3}$$

4.a.

Heating – photoionization of atoms in gas, electrons establish a Mawellian distribution

Cooling –recombination emission lines

recombination continuum

bremsstrahlung (free-free) radiation

4.b. Collisionally excited emission lines become important. The temperature drops since more avenues of cooling become available.

4.c. The temperature increases, because electron levels responsible for forbidden lines are collisionally de-excited, resulting in less efficient cooling of the gas.

5. a. Lyα λ1216: emission – yes (recombination), absorption – yes (resonance line)
   b. Hβ λ4861: emission – yes (recombination), absorption – no (not a resonance line)
   c. [O III] λ5007: emission – yes (collisional excitation), absorption – no (forbidden line)
   d. Ga λ2874: emission – no (low abundance), no (low abundance)
   e. [O I] λ1302: emission – no (low ionization line), no (low ionization line)

6.a. [O III] λ5007/ λ4363: [O III] λ4363 arises from a much higher level, whose population increases more rapidly with temperature than the level that gives rise to [O III] λ5007, due to collisional excitation. So [O III] λ5007/ λ4363 decreases with increasing temperature of the ionized gas.

6.b. [O II] λ3729/ λ3726: Both lines are transitions to the ground state of O^+, and arise from upper levels that are close together (so temperature effects are not strong). The level responsible for λ3729 has a higher statistical weight, and will be populated through collisions more at low densities. However, its critical density is lower (due to the lower transition probability), so the [O II] λ3729/ λ3726 decreases with increasing density, and bottoms out when a Boltzman distribution is reached at high densities.

6.a. The recombination lines are due to cascades, and are therefore insensitive to temperature. The recombination continuum decreases with temperature, since the cross section for campture decreases with increasing electron velocity. The continuum/line ratio therefore decreases with increasing temperature.

7. Short written explanation of the following terms (2 pts each):
   a. Column density: measure of the amount of gas in the line-of-sight to a background emission source, measured in atoms (or other particles) cm^{-2}
   b. Equivalent width: measure of the absorption strength of a line relative to the continuum
   $$W_\lambda = \int (1 - F_\lambda / F_c)\, d\lambda$$
   c. Curve-of-growth: how equivalent width changes with column density for a specific velocity-spread parameter (b), assuming a distribution of velocities of the atoms
   d. Photodissociation region: zone between ionized and molecular gas in H II regions where the molecules have been dissociated by radiation
   e. PAHs: polycyclic aromatic hydrocarbons; linked benzene rings (C and H)
   f. Parahelium: S (total spin) = 0, Orthohelium: S = 1
   g. On-the-spot approximation: every recombination to n=1 (in H) leads to an ionizing photon that is absorbed locally; leads to Case B recombination
   h. Grotrian diagram: energy levels and transitions for a particular ion

i. Critical density: electron density for an atomic level at which #radiative decays/vol/sec = # collisional transitions/vol/sec $n_c(i) = \sum_{j<i} A_{ij} \Big/ \sum_{j\neq i} q_{ij}$

j. Bowen resonance-fluorescence: He II Lα radiatively pumps upper O III level, which leads to a series of emission lines in the 2800 – 3800 Å region

k. Bremsstrahlung: free-free continuum radiation from electrons passing near positive ions

l. Auger effect: X-ray ionization of inner-shell electron in heavy elements leads to an excited state, which decays by ejecting one or more electrons plus photons

m. Dielectronic recombination – recombining electron gives up energy to bound electron, leading to doubly excited state, which then decays by radiation

n. Charge exchange: two ions with similar ionization potentials trade an electron; Ex) neutral and singly ionized O and H trading electrons

o. Paschen continuum: recombination directly to n=3 in H, continuum emission at $\lambda < 8204$ Å

p. Two-photon continuum: forbidden $2^2S \rightarrow 1^2S$ yields two photons with combined energy of Ly-alpha photon, distribution gives continuum emission

q. Photoionization model input parameters: geometry, n_H, ionizing SED, ionizing flux, abundances

r. Resonance transition: transition to or from the ground state of an ion

s. Mid-IR dust features: 9.7 and 18 μm absorption from silicate grains, PAH emission, thermal dust emission

t. HI 21-cm emission: due to spin-flip of electron-proton pair (parallel to anti-parallel)

u. Quantum-mechanical rule for permitted transitions: $\Delta L = \pm 1$

v. Shock front: compression wave that exceeds the sound speed in the gas ahead of it

w. Ionization parameter:

$$U = \frac{\int_{\nu_0}^{\infty} \frac{L_\nu}{h\nu} d\nu}{4\pi r^2 c n_e} = \frac{\#\text{ ionizing photons/vol}}{\#\text{ electrons/vol}} \text{ at the incident face of cloud}$$

- U determines the ionization fractions of each element for a given SED:

x. Nebula energy conservation: $G = L_R + L_{FF} + L_C$, energy gained by photoionization equals energy loss to recombination, free-free radiation, and collisional excitation

y. Five phases of the diffuse ISM: molecular medium (MM), cold neutral medium (CNM), warm neutral medium (WNM), warm ionized medium (WIM), hot ionized medium (HIM)