Francesco Bonaventura Cavalieri in Galileo's School of Thought.

K. Hujer

University of Chattanooga - Chattanooga, Tenn.

In his work *Augustine to Galileo*, A. C. Crombie makes a significant statement which throws revealing light on Cavalieri's role in the interpretation of natural phenomena as related to Galileian science:

« The intellectual revolution which had cost "Tuscan artist" such an anguish of effort, and yet left him still just short of reducing physics completely to mathematics, made it possible for his followers to take the geometrisation of the real world as evident. Cavalieri got rid of gravity as an innate physical property and said that like any other force it was due to external action » [1].

This view of Cavalieri and his relation to Galileo invites corroboration since it implies the role and trend Cavalieri, the geometer of Bologna, followed in the evolution of Galileian science.

Francesco Bonaventura Cavalieri (1598-1647), «gesuato» professor of mathematics at Bologna from 1629 until he died, is primarily celebrated for his opus major *Geometria indivisibilibus continuorum nova quadam ratione promota* (Bononiae, 1635). It contains the first exposition of his *method of indivisibles* sometimes described as the forerunner of the concept of calculus. Besides his great teacher, Galileo, whose lifetime almost engulfs the relatively brief life span of his disciple, we must be aware of another contemporary of Father Cavalieri, Johannes Kepler, that devotee of the Pythagorean-Platonic vision of a mathematical universe. It is suspected that the Bologna geometer taught the laws of Kepler. Cavalieri's life was also concurrent with several brilliant mathematicians—Descartes, Fermat, Roberval, and Wallis, and foremost with that mathematically most gifted genius, Pascal.

The seventeenth century was pregnant with new ideas, all preparing the ground for the discovery of calculus, that most powerful tool of ma-

---

[1] A. C. Crombie: *Augustine to Galileo,* London, 1952, p. 303.

GEOMETRIA
INDIVISIBILIBVS
CONTINVORVM
Noua quadam ratione promota.
AVTHORE
F. BONAVENTVRA CAVALERIO MEDIOLAN.
Ord. Iesuatorum S. Hieronymi, D. M. Mascarellæ Pr.
Ac in Almo Bonon. Gymn. Prim. Mathematicarum Professore.
AD ILLVSTRISS. ET REVERENDISS. D.
D. IOANNEM CIAMPOLVM.

BONONIÆ, Typis Clementis Ferronij. M. DC. XXXV. Superiorum permissu.

Fig. 1. – Title page of the first edition of Cavalieri's *Geometria indivisibilibus*, Bononiae, 1635 (Courtesy of Prague State University Library).

thematical physics. To some degree the Bologna geometer shared his famous teacher's fate of becoming the source of widely divergent views even of actual controversy. This was due to the circumstances around which Cavalieri's work on geometry took shape and the fact that it has been repeatedly maintained that because of the *method of indivisibles* he is the anticipator of calculus. The merit which Cavalieri deserves in the evolution that brought about the concept of calculus was ably exposed by Carl B. Boyer in his paper *Cavalieri, limits and discarded infinitesimals* [2]. Here we will consider the concept of the infinitely small which Cavalieri presented because it involves the perennially famous struggle between two pictures of the universe — the idealistic and the materialistic — each at one time or other leading to an absurd extreme.

The concept of indivisibles and its counterpart *method of exhaustion* was by no means new with Cavalieri. In the *Ephodos*, a treatise by Archimedes [3], discovered by the Danish scholar, Heiberg, in 1906 in a Constantinople palimpsest, it is indicated that even Archimedes, the supposed originator of the idea of indivisibles, has a forerunner, Democritos, founder of atomism, who through his concept of physical indivisibles, atoms, preoccupied his mind with the mathematical aspect of infinitesimals. With modifications, the *method of indivisibles* was used both by Eudoxos of Cnidos and Archimedes and rediscovered by Cavalieri almost twenty-one centuries later. It emerged as a Platonic-Pythagorean idea. In the *method of exhaustion* Archimedes compared rectilinear and curvilinear figures in view of determining their areas. When the tireless computer, Kepler, introduced the infinitely small into geometry while seeking elliptical areas, Cavalieri resumed upon this idea and converted Archimedes' method into his *method of indivisibles*. The question then arose: Is the infinitely small quantity, the indivisible, without dimension? Is it real or fiction? Is it a scaffold or a building itself? Cavalieri was generally cautious never to involve himself in the metaphysics of this problem, nevertheless he left the door sufficiently open to indicate his hidden predilection for the Pythagorean world view of mathematical atomism.

It was the vagueness with which Cavalieri presented his indivisibles that was the source of controversy. For example, Paul Guldin, a contemporary of Cavalieri, made a scathing attack on Cavalieri's unsatisfactorily stated principle. The irony is that Guldin, whose name unjustly is associated with the Guldin theorem which concerns the volume of a solid of revolution, actually of ancient origin and related to Pappus, never could prove his own theorem except by metaphysical speculation. Yet, Cava-

---

[2] C. B. Boyer: *Cavalieri, limits and discarded infinitesimals*, in *Scripta mathematica*, 8 (1941), 2.

[3] G. Sarton: *A history of science*, Cambridge Mass., 1959, II, pp. 78-79.

lieri proved Guldin's theorem by his *method of indivisibles.* In 1647 he clarified his principle and published his results in the *Exercitationes geometricae sex*, four years after Guldin passed away. The dilemma of this *method of indivisibles* was in the puzzle of the dimensionality of an aggregate made up of dimensionless entities of a lower order. In other words, how can a line be produced by the motion of dimensionless, indivisible points? In turn, how can a line, having absolutely no width, yield an area, or how can a moving plane of no thickness whatsoever, produce a solid?

Pascal, the mathematical prodigy, used Cavalieri's method in his interpretation of the cycloid, in which Galileo, Descartes and others were interested. But where others failed, Pascal succeeded while using the enigmatic *method of indivisibles.* Perhaps for this reason Pascal's work is sometimes regarded as the second chapter in the evolution of the concept of calculus. Yet, in our time, E. T. Bell makes an ardent and exaggerated attack, stating:

> « Cavalieri did not anticipate the calculus; he committed the unpardonable sin against it. But for his indivisibles and their absorption by scores of otherwise rational men who were to become college teachers, the common delusion that an infinitesimal is a "little zero" would have been extinct generations ago » [4].

Very much as Cavalieri's contemporary, Guldin, once did, E. T. Bell condemns these « indivisibly-divisible » nonentities as « non-sensical absurdities ».

Cavalieri was probably well aware of the dilemma associated with the concept of indivisibles. While his method lacks scientific foundation, nevertheless it yielded correct results. Thus, pragmatically expeditious, it raises the question: What makes it work? Is there a similarity with the case of the Balmer operational, empirical formula allowing us to compute the wavelengths of Balmer series of hydrogen before it was replaced by another formula based upon Bohr's hypothesis of the structure of hydrogen atom [5]? Is there a genuine reason for the controversial attitude toward one or the other interpretation of the concept of infinitesimal? No one considered disputing the virtue or falsity of Balmer's formula as against Bohr's since both formulations follows the Platonic line of thought in trying to « save the appearances », and consequently reduce the phenomena to rationality.

In our time we have numerous similar problems that became symptoms of a new science and in the epistemology of modern physics do not

---

[4] E. T. BELL: *The development of mathematics*, New York, 1945, pp. 137-138.

[5] M. BORN: *Atomic physics*, London, 1957, pp. 103-112.

raise the old style controversy. On the contrary, they are looked upon as mere stepping stones in the evolution of man's science. Thus, the once controversial theories on the nature of light, corpuscular and undulatory, had both equal experimental verification and light now is understood to diplay a dual nature. The result of the Michelson-Morley experiment performed in 1887 precipitated relativistic physics which negates the existence of the once seemingly very real, luminiferous ether [6]. Moreover, it created the necessity of an invention of Lorentz-Fitzgerald contraction formula which raises a similar question as Cavalieri's indivisible: is it real or is it a mere mathematical device or fiction in order to « save the appearances? » [7].

The mystery of the electron is involved in an increasingly acute conflict because of dichotomy in its dimensionality. Whether we consider its volume as having a definite dimension or as dimensionless point, both situations contradict its own existence [8]. Indeed, the most provoking are radical ideas in the field of modern cosmogony and the nature of the universe, involving such hypothesis, not without empirical reasons, as *creatio ex nihilo* [9]. Finally, what is the difference between the mental fiction of Cavalieri's indivisible and Jeans' statement that the universe is one great thought and « its creation must have been an act of thought » [10]. Certainly, the Bologna geometer committed no greater scientific crime than did Sir James Jeans, the venerated scientist, who for years occupied the chair of the secretary of the Royal Society of London, as once did Isaac Newton, who with Leibnitz shared in the discovery of calculus.

Thus the problem whether or not the indivisibles have any dimension involves the concept of space, specifically mathematical space, which is a mere abstract idea. The space whose properties are studied in geometry has no concrete extension that would be accessible by our visual and tactile means of perception. Hence its dimensionality is either a matter of our subjective convenience or of any individual arbitrariness upon our part to suit any immediate, practical purpose. Therefore there is no ground for the disputation concerning the dimensionality of the indivis-

---

[6] A. A. MICHELSON and E. W. MORLEY: *Method of making the wave-length of* Na-*light the actual and practical standard of length*, in *Silliman American Journal*, 34 (1887), pp. 333 sgg., pp. 427 sgg.; and also in *Philosophical* Magazine, 1887, 24, pp. 449 sgg. Also many physics textbooks.

[7] A. EINSTEIN: *Zur Elektrodynamik bewegter Korper*, in *Annalen der Physik*, 17 (1905), pp. 821-921. F. K. RICHTMYER and E. H. KENNARD: *Introduction to modern physics*, New York, 1947, pp. 113-117.

[8] P. CHAMBADAL: *La physique moderne et son interpretation*, Paris, 1956, pp. 152-153, 154-155.

[9] G. J. WHITROW: *The structure and evolution of the universe*, New York, 1959, p. 137.

[10] J. JEANS: *Mysterious universe*, Cambridge, 1932, p. 181.

ibles. They are only a convenient invention in the realm of the essential ideality of space.

After these considerations Crombie's statement [11] concerning Cavalieri will appear feasible since the Bologna geometer belonged to those followers of Galileo who reflected a strong tradition of Platonism and Pythagorean vision of mathematical universe. Galileo's inspiring epistolary activity gathered about him zealous disciples, each of them reflecting one of many facets of the master's prodigious genius. Analyzing the general atmosphere of the Galilean school of thought, it is, presumably, A. Koyré who maintains that in the case of Galileo it is not so much the question of Copernicus or Ptolemy as it is the rising mathematical as against non-mathematical physics. What position did Cavalieri occupy in this historic trend? Some scholars suspect that Cavalieri's *Geometria indivisibilibus* is the work of Galileo. This opinion could have arisen due to the fact that a number of the treatises which Galileo mentions in his occasional correspondence have been irrecoverably lost through the superstitious blindness of some of his relatives. After Galileo passed away they permitted the family confessor to examine his papers and to destroy whatever appeared to him objectionable. Consequently, according to what was then current opinion, some most valuable part of the papers were submitted to this expurgation.

In this way the loss of Galileo's *Essay on continuous quantity* is particularly to be regretted as it would be most revealing to determine how far he had succeeded in his thoughts on this important subject that apparently closely linked him to young Father Cavalieri. Cavalieri, who already in 1629 wrote his *Geometria indivisibilibus*, refused to publish his book hoping first to see Galileo's *Essay on continuous quantity* published. Indeed, in 1630 in his *Lo specchio ustorio* Cavalieri writes:

« How much is added by the knowledge of the mathematical sciences, which the famous school of Pythagoreans and Platonists considered supremely necessary for the comprehension of physical things, I hope will shortly become clear with the publication of the new science of movement promised by this marvellous Assayer of Nature, Galileo Galilei » [12].

However, these were the years of Galileo's most tumultuous and dramatic scenes connected with the publication of the famous Dialogues leading to the trial in 1633. This was also three years after the impov-

---

[11] A. C. Crombie: [*cit. n.* 1], p. 303.

[12] F. B. Cavalieri: *Lo specchio ustorio overo trattato delle settioni coniche et alcuni loro mirabili effetti intorno al lume*..., Bologna, 1632. Quoted from: P. P. Wiener and A. Noland ed.: *Roots of scientific thought*, New York, 1958; from the chapter by A. Koyré: *Galileo and Plato*, p. 171.

LO
SPECCHIO
VSTORIO
OVERO
TRATTATO
Delle Settioni Coniche,
ET ALCVNI LORO MIRABILI EFFETTI
Intorno al Lume, Caldo, Freddo, Suono,
e Moto ancora.
DEDICATO
A GL'ILLVSTRISSIMI
SIGNORI SENATORI
DI BOLOGNA
Da F. Bonauentura Caualieri Milaneſe Gieſuato
di S. GIROLAMO
AVTORE
E Matematico Primario nell' Inclito Studio dell'iſteſſa Città.

In Bologna, preſſo Clemente Ferroni 1632.
Con licenza de' Superiori.

Fig. 2. – Title page of Cavalieri's *Lo specchio ustorio*, Bologna, 1632 (Courtesy of Prague National Museum Library).

erished Kepler had passed away while journeying to beg for his unpaid salary from Emperor Rudolph.

Nothing then apparently stood in the way of the publication of Cavalieri's ecclesiastically harmless work in 1635. Throughout Galileo's works we find numerous indications of his preoccupation with the subject of infinitesimals but his remarks are vague and show little, if any, practical application of the method. Yet, this is the major part to which Cavalieri's book is devoted. At the beginning of the VII book of *Geometria indivisibilibus* we note that Cavalieri took a much more profound view of the subject than is generally implied and had approached very closely to the theories of his successors leading toward the concept of the calculus. Anticipating the objections to his hypothesis, he maintains that « there is no necessity to suppose the continuous quantities made of these indivisible parts, but only that they will observe the same ratios as those parts do ».

Although Galileo is generally understood as the founder of empiricism, it has been amply evident that particularly Cavalieri's link to Galileo indicates sufficiently that the latter was the transitory adherent of both the experimental as well as idealistic schools of thought. Then it should not be surprising to find the seemingly wavering attitude of Galileo towards some outstanding ancient as well as contemporary philosophers concerning the leading schools of thought. With Cavalieri, the intellectual cosmos Galileo inherited was that of the middle ages. On the one hand he would subscribe to what the Platonic dialogue *Philebus* first stated, *i.e.*, that each science is only a science insofar as it contains mathematics. With the rise of the Renaissance this idea was strongly promulgated first by Leonardo da Vinci followed by a number of others including Galileo and Cavalieri. When he was 25 years old, Galileo writes in his earliest volume *De motu*: « My method is the one that mathematics taught me ». On the other hand, despite the exchange of correspondence with Kepler, it is rather startling that Galileo's imagination was never inflamed by Kepler's elliptical orbits, and above all, by Kepler's cherished third law which according to the Pythagorean doctrine of the harmony of the spheres he calls the law of harmonics. Kepler was particularly jubilant over this accomplishment. It is at this point it may be possible for us to reveal Cavalieri's intellectual center of gravity when as a geometer he was drawn to Kepler and his mathematical world view in the very region that Galileo passed by without notice. Kepler's law of harmonics was the result of tireless search and tedious computation which, according to Boyer, Galileo avoided, indicating this interesting distinction. In a similar case, although influenced, probably through Cavalieri, Galileo was relatively little attracted by Archimedes, famous for his search of a cosmic foothold in order to obtain his leverage.

Foremost, and whether or not we have direct documentation, the most important aspect of Cavalieri's role is in being a methodological bridge between Kepler and Galileo. Kepler's initial influence can be traced to his *Nova stereometria doliorum* published 1615, in which Kepler's consideration of infinitely small quantities gave impulse to Cavalieri's principal work on geometry. Here the immediate heritage that links Cavalieri with scholastic philosophy is in the concept of indivisibles that he found in the reasonably influential mathematical writings of Thomas Bradwardine, Archbishop of Canterbury, who also initiated the concept of kinematics as against dynamics. If Bradwardine's and even Thomas Aquinas' influence may have been responsible for Cavalieri's exposition of his indivisibles on a methaphysical plane, it is Bradwardine's kinematic concept that appears to be reflected in both Cavalieri's motion of dimensionless point producing a line and in Kepler's three laws of motion which, against the later Newtonian dynamical laws, are kinematic in their nature. This means that Kepler's laws are distinctly Pythagorean-Platonic in their intrinsic character. Consequently, matter which contains no inherent physical property is here merely to obey the impulse of an external impetus. Therefore this impetus, force, is extrinsic. Action of extrinsic impetus upon matter, due to the inherent geometricity of the universe, is thus manifested by the regularity of motion.

A less known though no less significant contribution of Cavalieri is his introduction and application of logarithms, the invention of which was published by Napier in 1614. In fact, at one occasion Galileo praises the Bologna mathematician for his work on *A hundred varied problems to illustrate the use of logarithms* [13], which as Cavalieri states, are a different kind than those of Kepler. It is additionally poignant that this book deals especially with astrological instructions apparently intended for Cavalieri's students at his Direttorio Uranometrico in Bologna. Although astrology and astronomy were almost systematically confounded in Cavalieri's time, this significant information merely confirms the mental set-up of Cavalieri's Pythagorean-Platonic world view.

In conclusion, we are justified to remind ourselves of Cavalieri's readiness « to discard gravity as an innate physical property of matter » [14] because it was in the true spirit of the Pythagorean vision of a mathematical universe whose harmony in its geometrical manifestation was a result of action of some extrinsic force of supreme Geometer. Finally, it implies

---

[13] F. B. CAVALIERI: *Centurie di varii problemi per dimostrare l'uso e la facilità de' logaritmi nella gnomonica, astronomia, geografia, altimetria, pianimetria, stereometria et aritmetica prattica ...*, Bologna, 1639. Quoted from: L. THORNDYKE: *History of magic and experimental science*, New York, 1958, VII, p. 119.

[14] A. C. CROMBIE: [*cit. n.* [1]], p. 303.

the fact that Father Cavalieri in the Galilean science represents the distinct symbol of Platonism. Through his metaphysical approach Cavalieri suggested the ultimate intractability of natural phenomena to rationality. It means that man with his inadequate senses of perception and state of reasoning will always be « saving the appearances » in an imperfect way and in ephemeral form. Thus, each interpretation of natural phenomena, no matter how clever, is only temporary, subject to being replaced by a more convenient explanation in the course of man's advancing, unending search for reality. This is the noblest implication of Father Cavalieri's role as an outstanding disciple in the intellectual community that gathered around Galileo Galilei.