Cepheids at high angular resolution

recent results obtained by stellar interferometry*

Antoine Mérand[1 †]

[1]CHARA / Georgia State University

ABSTRACT

We present the latest results obtained by stellar interferometry (SI) in the field of pulsating Cephieds stars. The SI technique is particularly well suited to measure distances using the quasi-geometric pulsation parallax method known as the Baade-Wesselink (BW) method. The improvements in angular resolution and precision in the measurements have led to high precision calibration of the Cepheids period luminosity relation. On the other hand, the assumptions of the BW method can be investigated using SI: we present a calibration of the spectroscopic projection factor and the study of center to limb darkening and circumstellar envelopes. These later studies clearly demonstrate that SI has recently entered a regime where the BW implemetation is no longer limited by the precision of the angular diameter measurements but by the classical assumptions underlying the method itself. This calls for a better understanding of Cepheids, using numerical models and diverse observation techniques, including SI.

Keywords: Star: variable: pulsating: Cepheids. Technique: high angular resolution: Interferometry

1. INTRODUCTION

Cephieds are known for being 'standard candles': they allow astronomers to measure distances easily, from the solar neighborhood to the closest galaxies. For this reason, they occupy a niche in distance determination methods since they lie between astrometric methods (very high precision, solar neighborhood) and cosmological distances techniques (which require calibration).

Their intrinsic average luminosity and pulsation period follow a bijective relation (linear in log-log) known as the P-L relation. Once one knows the pulsation period, one can derive the intrinsic luminosity and because the apparent luminosity can be measured, the distance can be inferred. This method was first used to derive the distance to the galaxies M31 and M33. At this time, there was a problem with the result (a factor of 2) but nobody knew the P-L calibration was wrong until the error was corrected in 1952 by W. Baade.

Nowadays, the use of Cepheids as standard candles is still limited by the precision on the calibration of the P-L relation. Even if the relation is understood by theoreticians using numerical models, these models still fail to unambiguously calibrate the P-L relation. We have then to rely on different observational techniques to calibrate the P-L relation, all of them requiring to know the actual distance to the stars, especially for the zero point of the linear relation between Log P and log L. It is still possible to get the slope from a set of Cepheids of different periods, all at the same distance. For the zero point, though, the main problem remains the measurement of Cepheids distances with a great accuracy.

Moreover, apart from the P-L relation calibration, Cepheids are interesting by themselves: their are one of the simplest objects for modeling the coupling between radiative transfer and hydrodynamics. High angular resolution techniques, such as stellar interferometry, offer unique insights in this respect.

†formerly at LESIA - Observatoire de Paris

Further author information: Send correspondence to A.M. : E-mail: antoine@chara-array.org

2. THE BAADE-WESSELINK METHOD

2.1. A bit of (forgotten) history

Almost a hundred years ago (in 1918), when there was still a debate as to whether Cepheids radial velocity variations (detected by spectroscopy) were due to binarity or photospheric pulsation. F.A. Lindemann[‡] noted[1] that if the Cepheids were to be pulsators, then one can measure their parallaxes (hence their distances). He noted that the linear radius at time t can be obtained by integration of the pulsation velocity:

$$R(t) = R_0 + \int_0^t v_p dt - \frac{t}{P}\int_0^P v_p dt \tag{1}$$

where R_0 is the radius at zero time, P the period, v_p the pulsation velocity. The last term of the equation is a correction for the systematic velocity (the average velocity as measured). On the other hand, if one knows the effective temperature T as a function of time (which was the case at that time, from spectroscopy), one can determine their surface brightness $S_b(T)$ in a given photometric system. Then, the apparent flux F of the star is:

$$F = \frac{\pi R(t)^2}{d^2} S_b(T) \tag{2}$$

where d is the geometrical distance. Lindemann wrote, in his very short paper, that he did not have the time to actually study a real case but he recognized that this should close the debate if one knew the distance to a Cepheid from a previous parallax measurement.

Oddly enough, the method is known today as the Baade-Wesselink method.[2,3] These later authors introduced the idea of using photometric colors in order to estimate the surface brightness: Lindemann suggested a black-body integration, which proved not accurate enough, since different parts of the spectra leads to different color temperatures. Nonetheless, to our knowledge, Lindemann was probably the first to suggest the method, and it was used exactly in the context he thought about it: proving that Cepheids are pulsating stars and, in the mean time, measuring their distances. This is what Wesselink did in 1946.

2.2. Early applications of the BW method, the role of stellar interferometry

If one has access to stellar angular diameter measurements ($\theta = D/d$, where d is the distance), the fundamental BW equation can be written as (assuming that the pulsation velocity as a null average over one period):

$$\theta(t) - \theta(0) = -\frac{2}{d}\int_0^t v_p \, dt \tag{3}$$

The pulsation velocity ($v_p = \partial R/\partial t$) is measured from high resolution spectroscopy. The velocity varies by tens of kilometers per second over a pulsation phase, thus modern spectrographs can measures the variation with great precision. On the other hand, the angular diameters are rather small, and get smaller for Cepheids further from the Sun. The largest Cepheids angularly (α UMi or ℓ Car) have apparent diameters of 3 milli arcseconds (mas), whereas the next ten followings (δ Cep, ζ Gem, etc.) have typical angular diameters of the order of 1 mas.

The angular resolution required to directly measure angular diameters and variations (of the order of 15% peak-to-peak) was accessible only in the early 2000's. Nonetheless, if one looks carefully, stellar interferometry was involved with the BW method before this time.

In the 1990's, the BW method was successful in obtaining an indirect measurement of Cephied angular diameters (e.g. Gieren et al. 1998[4]). This approach is based on the Wesselink idea: using photometric colors to measure surface brightness. Then, one only needs two magnitudes to recover the angular diameter: one gives the flux measurement, while the surface brightness in that band is derived from the difference (called the color). This method was really successful following the advent of near-infrared photometry (K band, in particular): it appears that the angular diameter can be recovered with a great precision from simultaneous V

[‡]later known as Lord Cherwell

and K photometric measurements (e.g. Fouqué & Gieren 1997[5]). This flavor of the BW method, using visible and near infrared photometry, is known as the Barnes-Evans method.

Where stellar interferometry (SI) played an unexpected role was in the calibration of the relation $\theta = f(V, V-K)$. Early interferometers, such as Mark III or IOTA, devoted time to the measurement of angular diameters and calibration of the relation $\theta = f(V, V-K)$ for non-pulsating stars of different spectral types (then accessing a large range of $V-K$ values, including the F to G spectral types of Cepheids).

Naturally, the Barnes-Evans method relies on different assumptions, in particular that calibration of the brightness surface relations obtained for non-pulsating yellow giants and supergiants is still applicable to Cepheids. Moreover, the photometric method is really powerful because it can access faint (thus distant) stars. On the other hand, it suffers from possible biases: the interstellar absorption for example.

SI offers a far more direct angular diameter measurement, which makes the interferometric BW method quasi-geometric (it is also known as the parallax pulsation method). For all these reasons, interferometrists have been pursuing Cepheid observations any time they could.

2.3. The interferometric BW method, formalism

In this section, we will introduce a simple formalism to quantify the expected precision of angular diameter measured by interferometry.

We will consider that a Cepheid appears, at a given phase, as a uniform disk (a limb darkening disk hypothesis would not change the further conclusions). Then, its interferometric squared visibility can be written as a function of baseline B, wavelength λ and angular diameter θ as follows:

$$V^2(B\theta/\lambda) = \left(2\frac{J_1(\pi B\theta/\lambda)}{\pi B\theta/\lambda}\right)^2 \quad (4)$$

where J_1 is the Bessel function of the first kind. In the simplest instrumental configuration, one will use a single baseline, single wavelength instrument. The quantity of interest is then the relative variation of the squared visibility as a function of the angular diameter. We should introduce what we call the amplification factor A_f:

$$A_f(B/\lambda, \theta) = \frac{\partial V^2(B/\lambda, \theta)}{\partial \theta}\frac{\theta}{V^2(B/\lambda, \theta)} \quad (5)$$

where an increase of $\delta\theta$ leads to an increase in the visibility of $\delta V^2/V^2 = A_f \times \delta\theta/\theta$. Thus, if one assumes that the stellar interferometer has a constant relative precision $\sigma V^2/V^2$ with respect to the squared visibility level, in order to optimize the precision on the angular diameter, one should maximize A_f in order to maximize $\delta V^2/V^2$ for a given diameter variation $\delta\theta/\theta$.

It happens that, for a given angular diameter, A_f increases from the zero baseline to the first null of the visibility curve (Eq. 4, $B\theta/\lambda \approx 1$), then decreases in the second lobe. Thus, one should measure the visibility of a Cepheid as far a possible in the first lobe, down to the first null. This probably breaks the simple assumptions we made earlier, especially the fact that the squared visibility error is constant with respect to the visibility level, even at very low visibilities. Anyway, the expected conclusions are that, for a given angular size, the angular resolution B/λ should be as large a possible but still short of the second lobe in order to get the best precision on the angular diameter measurement. For a given angular diameter and a given interferometer configuration (B/λ), the best results are obtained with the best intrinsic instrumental precision.

3. THE INTERFEROMETRIC BW METHOD, FIRST RESULTS

3.1. Direct application of the IBW method

In order to be able to apply the IWB method, not only one should resolve the star (measure its diameter), but at the given baseline/wavelength, the visibility variations, due to the change in angular diameters, should be measurable. One can look back to the early attempts considering the angular resolution compared to the angular

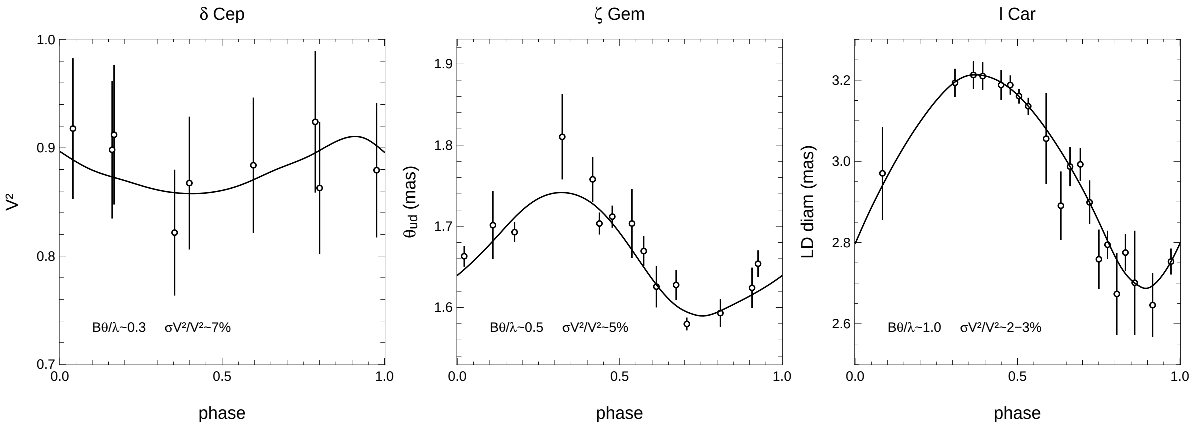

Figure 1. Different interferometric attempts to measure Cepheid angular diameter variations. From left to right: Mourard et al. (1997[6]), Lane et al. (2000[7]) and Kervella et al. (2004[8]). The left panel is V^2 as a function of phase, while the panels to the right are angular diameters with respect to phase. The thin, continuous line is the integration of the pulsation velocity (distance has been adjusted). From left to right, one can see the effect of increasing resolution ($B\theta/\lambda$) and improving precision ($\sigma V^2/V^2$). In the left panel, the pulsation was not claimed to be detected; the middle panel was the first detection, with a 10% precision on the distance; the right panel displays one of the best: 4% in the distance.

diameter ($B\theta/\lambda$) and instrumental precision (Fig. 1): the recent instruments tend to have larger baselines and higher internal precision, allowing interferometrists to measure distances with a better formal precision.

Using Cepheids of different periods, it is possible to calibrate the P-L relation, especially the zero-point. This was done by Kervella et al. (2004[9]): they obtained one of the best formal precisions on the parameters of the P-L relation so far.

Considering this simple point of view, the improvement of the P-L relation calibration requires large baselines in order to increase the number of Cepheids observable. In order to visualize this, for a given instrument (baseline, wavelength, precision) it is possible to plot Cepheids in angular diameter/angular amplitude coordinates and a delimitation indicating whether or not the pulsation could be measured at a given precision. The better instrument will be able to observe a large number of stars with a good precision.

3.2. Straightforward extensions

Considering existing facilities, only two of them can observe Cepheids in order to improve distance measurements: VLTI and CHARA Array. I will consider these two facilities with particular interferometric instruments: AMBER for VLTI and FLUOR for the CHARA Array.

So far, only a very few Cepheids have a distance estimated by the IBW method with a precision better than 10%, because of poor interferometric datasets. Considering Fig. 2, it appears that current facilities (VLTI/AMBER and CHARA/FLUOR) should be able to increase this number by a great amount (up to slightly more than 20 targets, considering both hemispheres, can have their distances determined with a precision better than 10%).

3.3. Effect of the center-to-limb darkening correction in the IBW method

The IBW method is thought to be better to the Barnes-Evans method, because it has a more direct measurement of the angular diameter. However, stellar interferometry does not measure directly the angular diameter, but visibilities. Every interferometric measurement should assume then a visibility model in order to be turned into an angular diameter measurement.

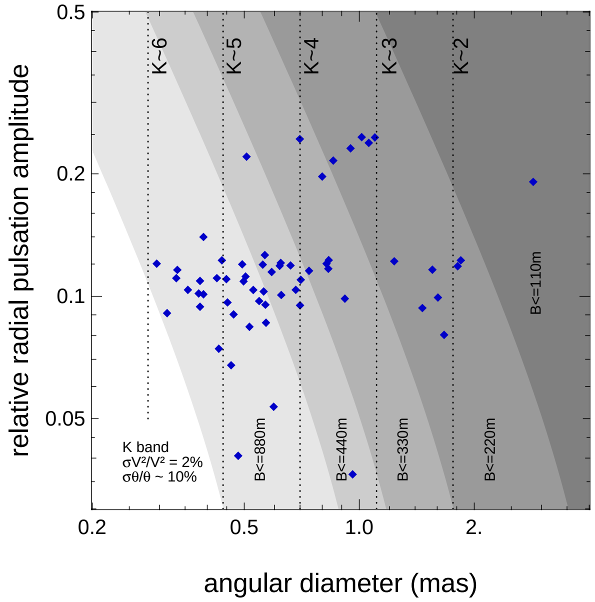

Figure 2. Expected performance of CHARA/FLUOR when observing Cepheids. Cepheids (northern and southern hemispheres, from Moskalik & Gorynya 2005[10]) are dots in the radial variation amplitude / angular diameter coordinates, different gray areas correspond to observability (distance determined with a 10% error) for a given maximum baseline and for the given instrumental configuration: precision of 2% and K band combiner. The upper scale is the approximative K band magnitude. CHARA/FLUOR has an actual 330m maximum baseline, thus it can observe all Cepheids down to $K \approx 4$ (approximately 10 stars in the northern hemisphere). VLTI/AMBER, in J band, with a precision of 2% and a maximum baseline of 220m is roughly equivalent to CHARA/FLUOR with a 440m baseline and is thus complete down to $K \approx 4.5$. A full coverage down to $K \approx 6$ requires a 900m baseline for FLUOR and a 450m baseline for AMBER in J band.

Usually, a stellar model consists of a center-to-limb darkened (CLD) disk. The CLD depends on the star itself: mainly the surface gravity, chemical composition and effective temperature, though only this last one has a strong influence. Often, this correction appears as a multiplicative factor $k = \theta_{LD}/\theta_{UD}$. The use of such a factor is made possible because the visibility profile always locally looks like the profile one gets from a uniform disk. However, this is only true for the highest visibilities: going near the first null and in the second lobe makes things different at the sub-1% level. Anyway, if one assumes that the k approximation is valid (which is true to the first order), the IBW equation turns into (see Eq. 3):

$$\theta_{UD}(t) - \theta_{UD}(0) = \frac{2k}{d} \int_0^t v_p \, dt \tag{6}$$

where k introduces a multiplicative bias to the distance. Any uncertainty on this factor k leads to the exact same uncertainty in the derived distance.

As mentioned before, k is mostly a function of effective temperature. It has been known for decades that Cepheids experience spectral types changes throughout a pulsation period. Thus, it is legitimate to consider that, at some point, a precise estimation of the distance should take this effect into account. It is important to note that CLD corrections rely on photospheric modeling. These models assume quiet, non-pulsating photospheres, whereas Cepheids have strongly pulsating photospheres, with compression and even shock waves. Some recent attempts have been made to compute radiative transfer in pulsating Cepheid model. Doing so, Marengo et al. (2003[11]) have been able to estimate the change of k with respect to the pulsation phase. In the near infrared H band, k is fairly stable: it varies by less than 0.2%, only during the rebound (Fig. 3, left panel). In the visible, the factor has more dramatic changes, 2.5% (Fig. 3, right panel). Thus, even if the visible wavelengths offer

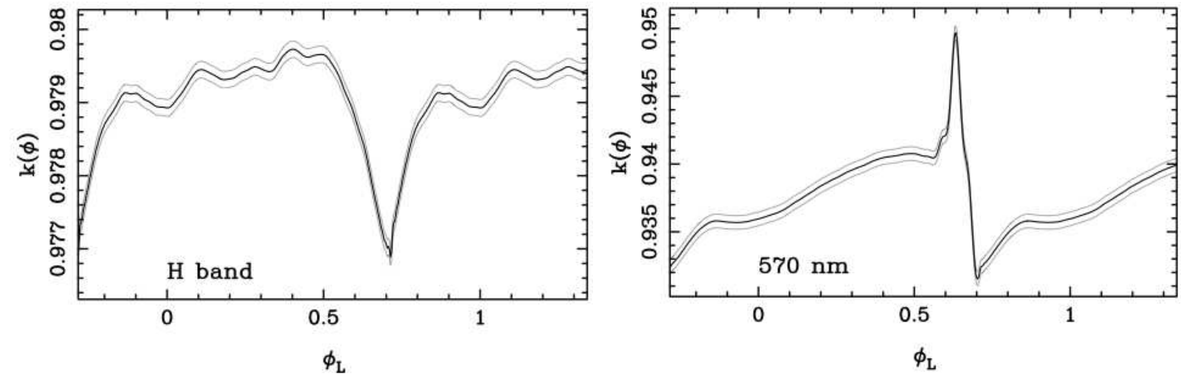

Figure 3. $k = \theta_{LD}/\theta_{UD}$ models as a function of phase for the ten days period Cepheid ζ Gem (Marengo et al. 2003[11]). Left is for near-infrared H band, right is for visible wavelengths. Note that the vertical scales are different: k_{570nm} varies ten times more than k_H over one pulsation cycle. The dramatic changes around $\phi \approx 0.7$ correpsonds to the photospheric rebound.

better spatial resolution, interferometric observations at these wavelengths may be biased by changes in k, if this effect is not taken into account.

At the level of precision we are using the IBW method in the infrared (a few percent in distances), one should not worry much. However, this assumption is based on models that still use approximations to treat the hydrodynamics and radiative transfer. Thus, it is mandatory, as interferometers grow in precision and spatial resolution to measure the CLD of Cepheids at different phases in order to validate the modeling results.

4. RECENT INTERFEROMETRIC RESULTS

4.1. Increasing the formal precision on distances

So far, interferometrists have been able to determine a Cepheid distance with a formal precision of about 4%: Kervella et al. (2004[8]) obtained this result on ℓ Car with VLTI/VINCI. In a natural extension, one could imagine observing other stars and getting similar results in order to calibrate the P-L relation. This was done by the same team on half a dozen Cepheids, but with non-uniform precision. This is because ℓ Car is 3 mas in angular diameter, whereas the others were only 1.5 mas or less. In order to obtain the same result, all things being equal, they should have used a baseline twice as long for these smaller stars, which was not available. However, their calibration of the zero point of the P-L relation remains one of the best, mainly thanks to the nice result on the long period ℓ Car ($P \approx 35$ days) and the inclusion of the short period δ Cep ($P \approx 5$ days) in the data set, where the geometrical parallax has been measured by the Hubble Space Telescope (HST) with a high precision (4%, Benedict et al. 2002[12]).

Considering the configuration baseline / wavelength / instrumental precision, the CHARA/FLUOR instrument (B=330m, K band and $\sigma V^2/V^2 \approx 1-2\%$) is probably one of the most appealing currently available stellar interferometers for IBW application. Using this instrument, we (Mérand et al. 2005[13]) were able to lower the formal precision on the distance down to 1.6% by observing δ Cep, which has an average angular diameter of 1.5 mas.

The assumptions for this work were the most standard ones: CLD correction (k) was constant over the the pulsation cycle and taken from static photospheric simulations. However, the integration of the pulsation velocity does not differ, in shape, from the interferometric angular diameter measurements. Then, this is a confirmation that at some level (a few percent), one does not need to use non-constant CLD correction, at least in K band.

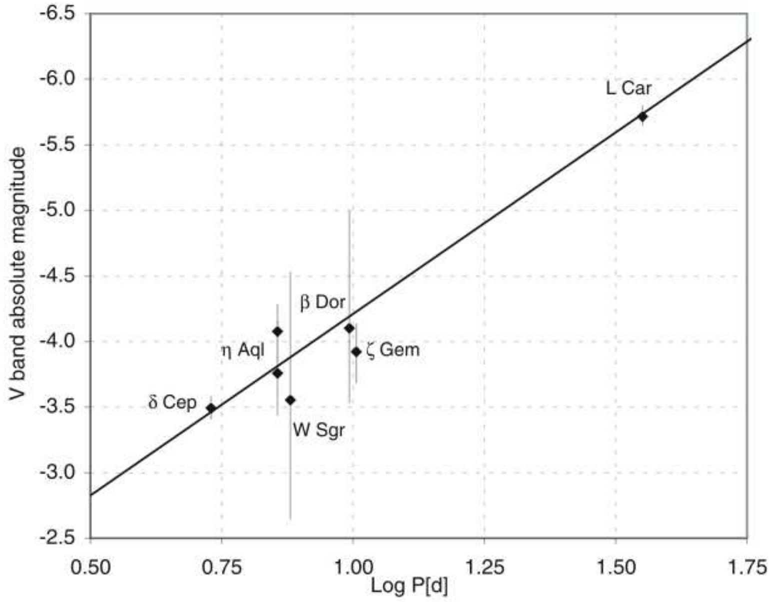

Figure 4. Cepheids P-L relation in V band. Data points and zero point of the linear relation comes from Kervella et al. (2004). The slope comes from Gieren et al. (1998[4]) and shows a nearly perfect agreement. All distances with the exception of δ Cep (shortest period, direct parallax by Benedict et al. 2002[12]) have been determined using the BW method. Note that the difference in term of formal error: ℓ Car is more than 3 mas in angular diameter, whereas the 6 others are of order 1 mas. Original plot published in Kervella et al. (2004[9]).

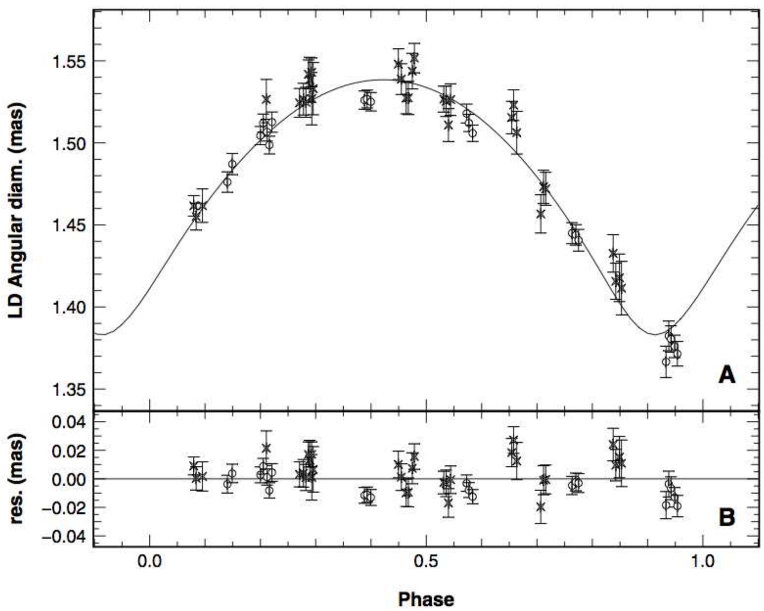

Figure 5. δ Cep angular diameter with respect to pulsation phase (A), as seen by CHARA/FLUOR in the K band using a 310m (circles) and a 250m (crosses) baseline (Mérand et al. 2005[13]). The fine continuous line is the integration of the pulsation velocity. The lower panel (B) displays the residuals. The formal precision on the distance using the IBW method is 1.6% here.

4.2. The projection factor

As mentioned above, the parallax of δ Cep has been measured by the HST (Benedict et al. 2002[12]), using the Fine Guidance Sensor (an interferometer, actually). Thus, the interferometric measurements of this star, due to their high quality, can be used to verify the two methods.

There is at least one good reason to consider this, because of the projection factor. Spectroscopy does not have access directly to the pulsation velocity: the star is a sphere and current spectrographs do not have sufficient spatial resolution. Thus, the radial velocity, as measured by the spectrograph, is an average over the stellar surface. The projection factor, or p-factor, is defined as $p = v_p/v_r$, where v_p is the pulsation velocity and v_r the radial velocity. The IBW equation is then:

$$\theta(t) - \theta(0) = -\frac{2}{d}\int_0^t p \times v_r \, dt \qquad (7)$$

The projected velocity at the center of the projected stellar disk is the true pulsation velocity, while the limb has a null projected velocity toward the observer. The problem would be easy if Cepheids were solid spheres, but there are not. First of all, the CLD introduces a weight correction in the simple spherical problem, so, in order to computer p, one should be aware of this effect. This problem was recognized early by Wesselink.[3] The second problem is the possibility of velocity gradients. In the same manner CLD is sensitive to different layers of the photosphere, which have different temperatures, the p-factor can be sensitive to velocity gradients by accounting for the highest layers while going toward the limb. Another effect deals with shock waves: because a p-factor value is only valid for a given absorption line, if this line is somewhat sensitive to a shock wave (which are known to occur in Cepheid photospheres), then it can be highly disturbed. There are other phenomena, but the problem is already very complicated since it requires photospheric modeling, including the hydrodynamic aspect.

One should add that the p-factor formalism is only valid if spectrographs were true velocity measurement instruments, which they are not. A given absorption line experiences the Fizeau-Doppler effect, so if one wants to really compute a useful p-factor, one should synthesize the line (from elementary lines coming from each part of the projected stellar disk), as seen by the spectrograph, taking into account all the effects described earlier, then extract v_r by the chosen method (line minimum, Gaussian fit, etc.) and compare it to the pulsation velocity, known from the model. The difference between Gaussian fit and minimum of line leads, for example, to p values different by 2.5% (Nardetto et al. 2004[14]). One should be aware that p depends on the actual instrument (spectral resolution for example) and the data reduction scheme as well. Yet another aspect of p is the difference in wavelength between spectroscopy and interferometry. The simple answer is that this effect should be included in p. A greater contribution to p surely comes from the fact that spectroscopy measures radial velocity in a single line whereas interferometry observations often take place in the continuum. Therefore, these two techniques are probably not sensitive to the same layer in the photosphere. According to Nardetto et al. (2004), this corresponds to δp of 0.05, or 4%. It makes p an even more complicated quantity to use and compute from models.

Wesselink[3] calculated p using a simple CLD model. Latter on, refinements were added and the first full calculation, including instrumental effects, was done by Burki et al. (1982[15]), but still lacking detailed photospheric models. The adopted value was then $p = 1.36$. It remained the most widely adopted value, until much more complete models were developed: Sabbey et al. (1995[16]) and then Nardetto et al. (2004[14]). These two works implemented hydrodynamic or pseudo-hydrodynamic treatments for the pulsation. The problem is that they came up with opposite conclusions: according to Sabbey et al. (1995[16]), p should be higher ($p = 1.45$), whereas Nardetto et al. specifically modeled δ Cep, with IBW in mind, and found $p = 1.27$, lower than the usual value $p = 1.36$.

CHARA/FLUOR observations are exactly what one needs to tackle this problem: the interferometric and spectroscopic (Bersier et al. 1994[17]) data are of the best quality as is the distance estimation (Benedict et al. 2002[12]).

The value observationally for p is (Mérand et al. 2005[13]):

$$p = 1.27 \pm 0.06 \, (\pm 0.007_{\mathrm{spectro.}} \pm 0.020_{\mathrm{interf.}} \pm 0.050_{\mathrm{astrometry}})$$

which is equal to the value predicted by Nardetto et al.,[14] marginally compatible with the traditional value (1.36) and 3 sigmas from the Sabbey et al.[16] estimation. Another straightforward comment on this result is the different contributions of spectroscopy, interferometry and astrometry to the uncertainty: because of the wonderful radial velocity data set, spectroscopy does not introduce much error. The uncertainty on the astrometric distance accounts for most of the final error on p. On the contrary, if one assumes a perfectly known p, the error on the distance becomes dominated by the interferometric measurements.

Finally, one should argue that the correction we apply to p, moving it from 1.36 to 1.27, can be put on k. Indeed, k and p play a similar role in the implementation of the IBW method we used:

$$\theta_{\rm UD}(t) - \theta_{\rm UD}(0) = -\frac{2pk}{d}\int_0^t v_r dt \tag{8}$$

However, a change of 7% in k is enormous since a UD has $k = 1$ and we used $k \approx 0.97$ for δ Cep. In other words, that would mean that the Cepheids are more that three times further from the UD than what we thought. A less dramatic point of view could be that p and k share the discrepancy. Because p and k play a symetric role in the IBW equation (Eq. 8), a definitive conclusion is yet possible.

4.3. CLD and circum stellar environment

The Cepheids' CLDs play an important role in the IBW method: interferometrists need it to reduce visibilities to angular diameters, and spectroscopists need it to compute the p-factor. Thankfully, SI offers a unique opportunity to directly measure the CLD of stars, assuming the instrument is able to access the second lobe of the visibility curve.

The current long baseline interferometers (CHARA, VLTI, PTI or NPOI) already have difficulties to properly resolve Cepheids in order to measure their pulsation with a great accuracy. CLD measurements require greater angular resolution ($1.2 \lesssim B\theta/\lambda \lesssim 2.2$). A limb darkened stars appears almost as a uniform disk in the first lobe, only the second lobe (especially its height) tells us something about the CLD.

While several stars have seen their CLDs by interferometry,[18–20] there has been only one attempt on a Cepheid, Polaris (Mérand et al. 2006[21]). The result was unexpected, since the visibility curve does not follow a UD curve in the first lobe, nor in the second lobe either. We stated that this is because Polaris is probably surrounded by a circumstellar envelope (CSE). At the same time, we[22] found a similar feature around ℓ Car. The CSE appears in the K band data as a visibility deficit at baselines corresponding to $V^2 \approx 50\%$ (see Fig. 6). These CSE, according to our simple models, has an angular diameter three times larger than the star itself, and accounts for a few percent of the stellar flux in the K band. The presence of ℓ Car's CSE has been confirmed by VLTI/MIDI interferometric observation: at these wavelengths (8 to 12 microns) the CSE dominates, so the object is resolved (visibility less than 1) and should not be if the star does not have a CSE (see Fig. 6, lower panel). Finally, complementary observations of δ Cep,[21] using smaller baselines, shows a deficit in visibility as well, compatible with the CSE characteristics of Polaris and ℓ Car ($\theta_{\rm CSE}/\theta_{\rm star} \approx 3$ and a few percent of the stellar flux).

One should note that the CLD model used in the composite star+CSE model is from an hydrostatic model. Nonetheless, it seems that it matches Polaris observations in the second lobe. Polaris, however, is a low amplitude Cepheid (less than a percent in diameter), whose photosphere is probably closer to a static atmosphere than a full amplitude Cepheid (20%): it is not yet easy to conclude anything regarding the Cepheids' CLDs.

4.4. Discussion on Cepheids CSE

A number of Cepheids have shown moderate infrared excesses that are related to the presence of circumstellar matter. An excessive brightness at mid-infrared wavelengths is characteristic of a relatively warm, dusty environment. Deasy[23] compared the IRAS excess ratios F (60 μm)/F (12 μm) to predicted values for several bright Cepheids. Using IRAS data in the mid-IR, McAlary & Welch[24] have studied a broad sample of Cepheids in order to detect infrared excesses. They obtained a clear detection in the case of RS Pup and SU Cas (two classical Cepheids). For ℓ Car however, Kervella, Mérand et al.[22] confirmed that no significant mid-infrared excess is present in ℓ Car (using the absolute calibration of MIDI spectra).

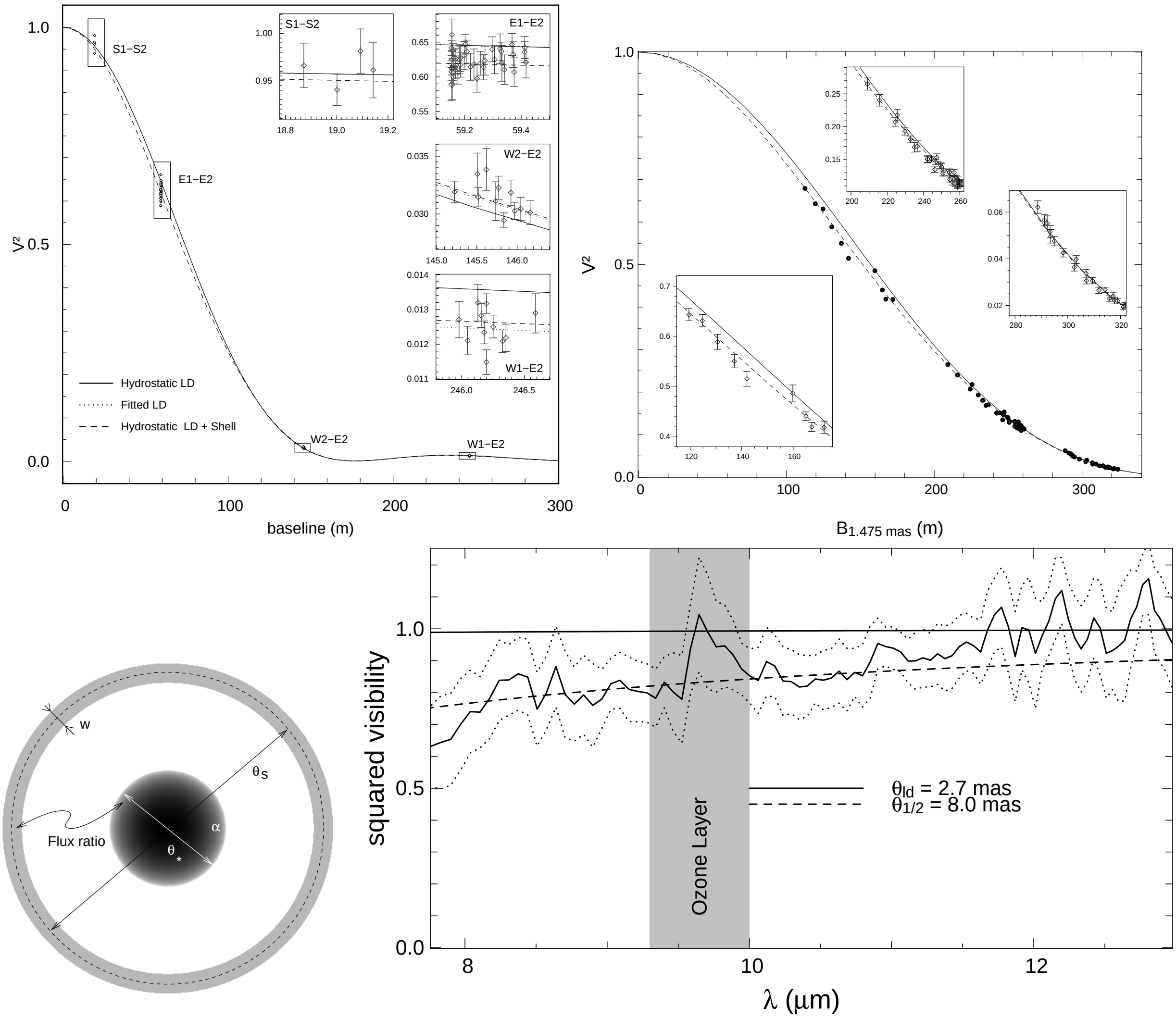

Figure 6. CSE around Cepheids. *Upper left*: squared visibility as a function of baseline for Polaris as seen by CHARA/FLUOR (Mérand et al.[21]). Three models are plotted (lines): the hydrostatic CLD, the adjusted CLD and a composite model using the a limb darkened star and a CSE (*lower left*). Insets show a zoom in for each of the baselines (named after the telescopes pairs). The CSE appears as a visibility deficit at intermediate baselines. Similar results have been obtained on δ Cep[21] with CHARA/FLUOR (*upper right*, see Polaris panel for explanations). *Lower right*: interferometric flux of ℓ Car with the mid-infrared VLTI/MIDI instrument: the fine straight line is the expected stellar disk contribution, whereas the dashed line is a 8 mas Gaussian. Obviously, the star is more resolved than expected, confirming the presence of a very strong CSE in the mid-infrared (Kervella, Mérand et al.[22]). A similar deficit in the K band visibility has been recorded on ℓ Car using VLTI/VINCI and δ Cep using CHARA/FLUOR.

We[21] tested the best star+CSE fit geometry (Fig. 6, upper right panel) with a physical model, such as the one used by Perrin et al.[25] for Mira stars: this model is a simple radiative transfer calculation for a single layer shell surrounding a star. The shell is a self emitting black body, like the star itself. This type of model and our Polaris' star+CSE model lead to similar geometries for the object, as seen by the interferometer. In the model described by Perrin et al.,[25] the shell temperature can be computed using a simple radiative equilibrium model, such as presented in Ireland et al.[26] Using silicate opacities from Suh[27] and a black body spectrum for the Cepheid ($T_{\rm eff} \approx 6000$ K), we found an equilibrium temperature of the order of 2500 K at 3 stellar radii, which does not allow silicate dust grains to survive. Based on this test, the observed circumstellar emission around Cepheids is unlikely to be due to thermal emission from a silicate dust shell. This conclusion does not apply to Mira stars ($T_{\rm eff} \approx 2800$ K), for which the equilibrium temperature is much lower for a shell at the same distance.

4.5. Effect of the CSE on the IBW method

The CLD effects have been discussed earlier: it affects the distance estimation linearly, and is not function of the baseline as long as the observations are done in the first lobe. The three Cepheids we looked at all have visibility deficits we interpret as the presence of a faint CSE, slightly (by a factor of three) larger than the star. If the IBW is to be applied in the next future to several stars, there will probably be not much observing time dedicated to measure the full $V^2(B)$ profile: one would rather use a single setup and rely on a visibility model to extract the angular diameter. Hopefully, with the advent of multi way beams combiners, this will not be a problem.

If we have to face the single baseline case, we should consider the bias introduced by ignoring the CSE. If the differences in term of visibility are small (i.e. the CSE effects are small on the visibility), the multiplicative bias in term of diameter is:

$$\beta(B) = \frac{\theta_{CLD} - \theta_{CLD+CSE}}{\theta_{CLD+CSE}} \tag{9}$$

$$= \frac{V^2_{CLD}(B) - V^2_{CLD+CSE}(B)}{V^2_{CLD+CSE}(b)} \times \frac{1}{A_f(B)} \tag{10}$$

where V^2_{CLD} is the visibility of the model without CSE and $V^2_{CLD+CSE}$ the visibility of the model with CSE. In other words, the visibility deficit is amplified by the visibility function local sensitivity to a change in angular diameter. We can plot this bias with respect to the baseline. In the single baseline observation context, this should help choosing the best baseline, for which the bias is the smallest. This bias should be compared to the formal diameter error using the amplification factor: $\sigma\theta/\theta = \sigma V^2/V^2 \times 1/A_f$. Doing so (see Fig. 7), one can see on the one hand the effects of the amplification factor described earlier: the formal error on the diameter is decreasing at large baselines. On the other hand, the bias due to the CSE is only important when the visibility deficit is the greatest (in the middle of the second lobe for Cepheid). The position of this deficit is linked to the ratio of the angular dimensions of the CSE and the star. The conclusion of this preliminary and simplistic approach is that δ Cep observations that led to the p-factor estimation[13] are probably safe from bias.

5. WORK TO DO IN THE FUTURE

The latest interferometric observations of Cepheids suggest that the application of the IBW method can be done with a great formal precision. However, it seems that current instruments (such as CHARA/FLUOR) and, most likely, facilities with higher resolving power (CHARA with shorter wavelengths instruments, VLTI/AMBER in J band, Magdalena Ridge Observervatory Interferometer, etc.) are entering a regime where the IBW is no longer limited by the formal precision of angular diameter measurements, but by the basics assumptions underlying the method itself, as with the recent δ Cep observations.

In that respect, the greatest uncertainty on the pulsation parallax comes from the projection factor. This factor includes so many parameters, from the physics of the pulsating photosphere to the algorithm used to extract the radial velocity from the spectra. Several workers have modeled the problem, and found different values, often contradictory. There is no general agreement for a value of p, especially because instrumental effects must be taken into account. That is why efforts should be put on modeling these complex objects, in

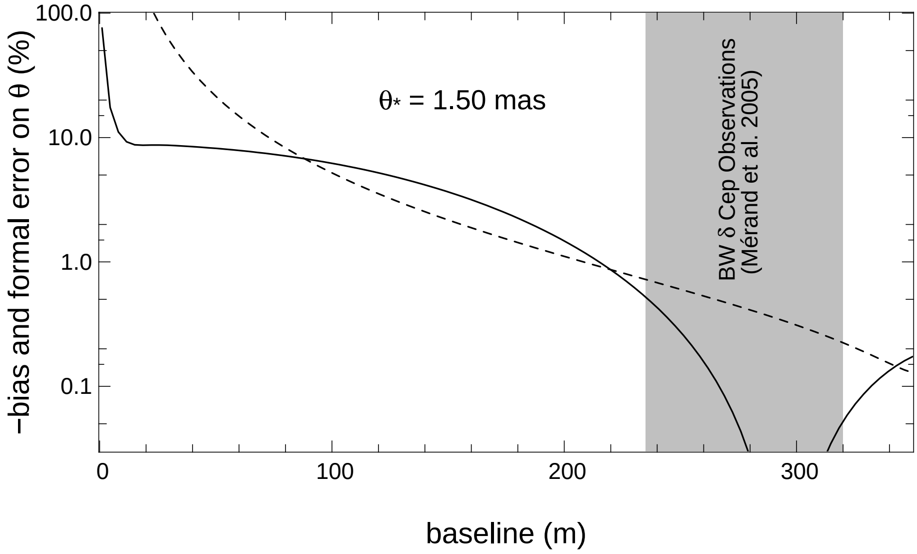

Figure 7. Expected angular diameter bias for a single baseline measurement, due to the presence of the CSE around δ Cep as a function of baseline in K band, compared to the formal error on the angular diameter. The continuous line is the bias (-β actually), the dashed line is the formal error $\sigma\theta/\theta = \sigma V^2/V^2 \times 1/A_f$. The parameters of this plot relate to the δ Cep (1.5 mas) observations by CHARA/FLUOR ($\sigma V^2/V^2 = 2\%$).[13] The actual baselines used for the IBW method were in the gray band (225 to 313 meters in length).

order to estimate at least the effects of the different phenomena contributing to p. On the other hand, at least one object allows a direct calibration, thanks to the direct parallax measurement: δ Cep. This has been done and the result, $p = 1.27 \pm 0.06$ is barely compatible with the traditional value, $p = 1.36$. It turns out that the same HST team obtained HST astrometric data for a dozen more Cepheids (Barnes, priv. comm. 2005). Thus, once interferometric observations are carried out on these targets, it will be possible to estimate p for different stars (different periods).

Another aspect of the uncertainties related to the IBW method is, generally speaking, the morphology of the Cepheids: center-to-limb darkening and circumstellar environments. These effects directly impact the angular diameter estimation derived from interferometric measurements. This observational field is just at its very beginning, and most of the work has to be done. Both CSE and CLD interferometric studies will benefit from upcoming instruments: multi-way beam combiners (3+) offer unique snapshot capabilities that will ease the future investigations.

6. CONCLUSION

Just five years ago, stellar interferometry was able to detect the angular diameter pulsation of a Cepheid and therefore measure its distance using the BW method for the first time,[7] leading a few years later to the calibration of the P-L realtion[9] by measuring the distance of half a dozen Cepheids. Nowadays, because angular resolution has increased and precision has improved, we are reaching a point where the BW is limited by its basic assumptions, i.e. our knowledge of Cepheids. First of all, the projection factor has been calibrated for one star.[13] The computation of this factor is not trivial and reauires the modeling of a pulsating photosphere.[14,16] The calibration[13] and the latest models[14] seem to agree, yet a confirmation on other targets (with different periods) is highly desirable. Another aspect stellar interferometry recently contributed is the study of the center to limb darkening (CLD) and circumstellar envelopes (CSE). Whereas the first aspect (CLD) has been recognized as a potential source of uncertainty or bias for the BW method for a long time,[3] stellar interferometers are able to directly measure this quantity. However, the first attempts[21] led to the discovery of CSE, faint in the near infrared but dominating in the mid-infrared.[22] Such a detection holds for the three Cepheids interferometers carefully looked at, with similarities (roughly same size and same flux ratio). It is too early to conclude that all Cepheids have CSEs. CSE and CLD must be addressed by new interferometric instruments: multi-way beam combiners will add 'snapshot' capabilities (multiple spatial frequencies at once) whereas high spectroscopic dispersion associated with the highest angular resolution will be nice for CLD studies — and more generally photospheric interferometric studies.

ACKNOWLEDGMENTS

author would like to thank Jason Aufdenberg for his careful reading of this manuscript. The author also thanks the organizing committee, especially J.D. Monnier, for inviting him to give this talk.

REFERENCES

1. F. A. Lindemann, "Note on the pulsation theory of Cepheid variables," *Royal Astronomical Society, Monthly Notices* **78**, pp. 639–+, June 1918.
2. W. Baade, "Über eine Möglichkeit, die Pulsationstheorie der δ Cephei-Veränderlichen zu prüten," *Astronomische Nachrichten* **228**, pp. 359–+, Nov. 1926.
3. A. J. Wesselink, "Discussion of radial velocities and photometric observations of δ Cephei," *Bulletin of the Astronomical Institutes of the Netherlands* **10**, pp. 88–+, Jan. 1946.
4. W. P. Gieren, P. Fouque, and M. Gomez, "Cepheid Period-Radius and Period-Luminosity Relations and the Distance to the Large Magellanic Cloud," *Astrophys. Journal* **496**, pp. 17–+, Mar. 1998.
5. P. Fouque and W. P. Gieren, "An improved calibration of Cepheid visual and infrared surface brightness relations from accurate angular diameter measurements of cool giants and supergiants.," *Astron. Astrophys.* **320**, pp. 799–810, Apr. 1997.
6. D. Mourard, D. Bonneau, L. Koechlin, A. Labeyrie, F. Morand, P. Stee, I. Tallon-Bosc, and F. Vakili, "The mean angular diameter of δ Cephei measured by optical long-baseline interferometry.," *Astron. Astrophys.* **317**, pp. 789–792, Feb. 1997.
7. B. F. Lane, M. J. Kuchner, A. F. Boden, M. Creech-Eakman, and S. R. Kulkarni, "Direct detection of pulsations of the Cepheid star ζ Gem and an independent calibration of the period-luminosity relation," *Nature* **407**, pp. 485–487, Sept. 2000.
8. P. Kervella, N. Nardetto, D. Bersier, D. Mourard, and V. Coudé du Foresto, "Cepheid distances from infrared long-baseline interferometry. I. VINCI/VLTI observations of seven Galactic Cepheids," *Astron. Astrophys.* **416**, pp. 941–953, Mar. 2004.
9. P. Kervella, D. Bersier, D. Mourard, N. Nardetto, and V. Coudé du Foresto, "Cepheid distances from infrared long-baseline interferometry. II. Calibration of the period-radius and period-luminosity relations," *Astron. Astrophys.* **423**, pp. 327–333, Aug. 2004.
10. P. Moskalik and N. A. Gorynya, "Mean Angular Diameters and Angular Diameter Amplitudes of Bright Cepheids," *Acta Astronomica* **55**, pp. 247–260, June 2005.
11. M. Marengo, M. Karovska, D. D. Sasselov, C. Papaliolios, J. T. Armstrong, and T. E. Nordgren, "Theoretical Limb Darkening for Classical Cepheids. II. Corrections for the Geometric Baade-Wesselink Method," *Astrophys. Journal* **589**, pp. 968–975, June 2003.
12. G. F. Benedict, B. E. McArthur, L. W. Fredrick, T. E. Harrison, C. L. Slesnick, J. Rhee, R. J. Patterson, M. F. Skrutskie, O. G. Franz, L. H. Wasserman, W. H. Jefferys, E. Nelan, W. van Altena, P. J. Shelus, P. D. Hemenway, R. L. Duncombe, D. Story, A. L. Whipple, and A. J. Bradley, "Astrometry with the Hubble Space Telescope: A Parallax of the Fundamental Distance Calibrator δ Cephei," *AJ* **124**, pp. 1695–1705, Sept. 2002.
13. A. Mérand, P. Kervella, V. Coudé Du Foresto, S. T. Ridgway, J. P. Aufdenberg, T. A. Ten Brummelaar, D. H. Berger, J. Sturmann, L. Sturmann, N. H. Turner, and H. A. McAlister, "The projection factor of δ Cephei. A calibration of the Baade-Wesselink method using the CHARA Array," *Astron. Astrophys.* **438**, pp. L9–L12, July 2005.
14. N. Nardetto, A. Fokin, D. Mourard, P. Mathias, P. Kervella, and D. Bersier, "Self consistent modelling of the projection factor for interferometric distance determination," *Astron. Astrophys.* **428**, pp. 131–137, Dec. 2004.

15. G. Burki, M. Mayor, and W. Benz, "The peculiar classical Cepheid HR 7308," *Astron. Astrophys.* **109**, pp. 258–270, May 1982.
16. C. N. Sabbey, D. D. Sasselov, M. S. Fieldus, J. B. Lester, K. A. Venn, and R. P. Butler, "On Spectral Line Formation and Measurement in Cepheids: Implications to Distance Determination," *Astrophys. Journal* **446**, pp. 250–+, June 1995.
17. D. Bersier, G. Burki, M. Mayor, and A. Duquennoy, "Fundamental parameters of Cepheids. II. Radial velocity data.," *Astron. Astrophys. Suppl.* **108**, pp. 25–39, Nov. 1994.
18. L. Bigot, P. Kervella, F. Thévenin, and D. Ségransan, "The limb darkening of α Centauri B. Matching 3D hydrodynamical models with interferometric measurements," *Astron. Astrophys.* **446**, pp. 635–641, Feb. 2006.
19. J. P. Aufdenberg, H.-G. Ludwig, and P. Kervella, "On the Limb Darkening, Spectral Energy Distribution, and Temperature Structure of Procyon," *Astrophys. Journnal* **633**, pp. 424–439, Nov. 2005.
20. G. Perrin, S. T. Ridgway, V. Coudé du Foresto, B. Mennesson, W. A. Traub, and M. G. Lacasse, "Interferometric observations of the supergiant stars α Orionis and α Herculis with FLUOR at IOTA," *Astron. Atrophys.* **418**, pp. 675–685, May 2004.
21. A. Mérand, P. Kervella, V. Coudé du Foresto, T. Ridgway, S., P. Aufdenberg, T. ten Brummelaar, A. McAlister, H, L. Sturmann, J. Sturmann, N. Turner, and D. Berger, "Extended envelopes around Galactic Cepheids. II. Polaris and δ Cep from near-infrared interferometry with CHARA/FLUOR," *Astron. Astrophys.* (in press) , 2006.
22. P. Kervella, A. Mérand, G. Perrin, and V. Coudé Du Foresto, "Extended envelopes around Galactic Cepheids. I. ℓ Carinae from near and mid-infrared interferometry with the VLTI," *Astron. Astrophys.* **448**, pp. 623–631, Mar. 2006.
23. H. P. Deasy, "Observational evidence for mass loss from classical Cepheids," *Royal Astronomical Society, Monthly Notices* **231**, pp. 673–694, Apr. 1988.
24. C. W. McAlary and D. L. Welch, "Detection of Cepheid variables by the Infrared Astronomical Satellite," *Astron. Journal* **91**, pp. 1209–1220, May 1986.
25. G. Perrin, S. T. Ridgway, B. Mennesson, W. D. Cotton, J. Woillez, T. Verhoelst, P. Schuller, V. Coudé du Foresto, W. A. Traub, R. Millan-Gabet, and M. G. Lacasse, "Unveiling Mira stars behind the molecules. Confirmation of the molecular layer model with narrow band near-infrared interferometry," *Astron. Astrophys.* **426**, pp. 279–296, Oct. 2004.
26. M. J. Ireland, P. G. Tuthill, J. Davis, and W. Tango, "Dust scattering in the Miras R Car and RR Sco resolved by optical interferometric polarimetry," *Royal Astronomical Society, Monthly Notices* **361**, pp. 337–344, July 2005.
27. K.-W. Suh, "Optical properties of the silicate dust grains in the envelopes around asymptotic giant branch stars," *Royal Astronomical Society, Monthly Notices* **304**, pp. 389–405, Apr. 1999.