Strehl Ratio and Visibility in Long Baseline Stellar Interferometry

T. A. ten Brummelaar & W.G. Bagnuolo Jr.
Center for High Angular Resolution Astronomy (CHARA)
Georgia State University

S.T. Ridgway
National Optical Astronomy Observatories, Tucson

Optics Letters, Vol 20, p521-523 (1995)

Abstract

The relationship between Strehl ratio (S), the parameter most often used by those in the adaptive optics field, and visibility (V), the parameter of most interest to those working in interferometry, is investigated. It is found that if the atmospheric turbulence is assumed to be Kolmogorov $V \simeq S$ for high Strehls. At low Strehls, both simulations and an analytical formulation show that the Strehl ratio underestimates the visibility.

During the last few years there has been rapidly growing interest and activity in the development of large aperture optical interferometers for astronomy[1]. In planning an interferometer with an aperture of order r_0 or greater, and in using adaptive optics, it is necessary to make design tradeoffs to optimize performance. One way to characterize the efficiency of an interferometer is with a coherence transfer factor η. If the visibility magnitude of an object is $\gamma_{\rm obj}$ the measured visibility will be $\gamma_{\rm meas} = \eta\ \gamma_{\rm obj}$. In this work we shall use the definition of visibility given by Tango and Twiss[2].

For accurate predictions of the coherence transfer factor, especially at large D/r_0, simulation will give the best results. However, it is not always convenient or necessary to invest this much effort. It is very useful to have an approximate predictor of η based on the Strehl ratio, which is also a standard indicator of the performance of classical and adaptive imaging systems.

Several authors have mentioned this possibility, but none have yet explicitly justified the use of the Strehl in this role. Tango and Twiss[2] state that the Strehl ratio and coherence loss should only differ by a scale factor, but they do not give an evaluation of the scale factor. Rousset et al[3] give a set of simulations for several cases which show that the Strehl is close to a quantity they call the relative coherent energy, but they do not discuss the range over which Strehl can be used to estimate coherence.

In this paper we show that the Strehl is a useful measure of the coherence transfer factor in situations of most interest to optical interferometry.

The Strehl ratio is the ratio of peak power in the center of the image plane compared to that of an equivalent unaberrated system. Many approximations for the Strehl ratio have been investigated[4, 5, 6] and the best approximation found was empirical and of the form

$$S \simeq \exp(-\sigma_\phi^2) \tag{1}$$

where σ_ϕ^2 is the variance of the phase aberration across the pupil.

A more precise way to look at Strehl ratios is in terms of the optical transfer function of the system. If the optical transfer function of the optical system is $T(\boldsymbol{\nu})$ and the optical transfer function of the phase aberrations caused by the atmosphere is $B(\boldsymbol{\nu})$, the intensity of the aberrated image is the Fourier transform of the combined optical transfer function $T(\boldsymbol{\nu})B(\boldsymbol{\nu})$ and that of the unaberrated image the Fourier transform of $T(\boldsymbol{\nu})$. The tilt corrected, or short exposure, Strehl ratio will be given by the ratio these transforms at the origin and is therefore

$$S = \frac{\int T(\boldsymbol{\nu})B(\boldsymbol{\nu})\, d\boldsymbol{\nu}}{\int T(\boldsymbol{\nu})\, d\boldsymbol{\nu}}. \tag{2}$$

If the effect of the atmosphere is small, Tango and Twiss show that we can write the coherence transfer factor as

$$\overline{|\eta|^2} = 1 - (\sigma_{\phi_1}^2 + \sigma_{\phi_2}^2) - (\sigma_{\chi_1}^2 + \sigma_{\chi_2}^2) \tag{3}$$

where $\sigma_{\phi_i}^2$ and $\sigma_{\chi_i}^2$ are the variance of phase and log amplitude fluctuations across the ith input aperture. Since we have assumed that the the phase variations dominate we ignore the log amplitude variations. Furthermore, if we assume that the two apertures have the same phase variance, or equivalently the same Strehl ratio, we get

$$\overline{|\eta|^2} \simeq 1 - 2\sigma_\phi^2. \tag{4}$$

By assuming that the turbulence is stationary, uncorrelated at the two apertures and Kolmogorov in nature, Tango and Twiss go on to state that the coherence transfer factor can be written

$$\langle |\eta|^2 \rangle = \frac{\int T(\boldsymbol{\nu})B^2(\boldsymbol{\nu})\, d\boldsymbol{\nu}}{\int T(\boldsymbol{\nu})\, d\boldsymbol{\nu}}. \tag{5}$$

If the phase variance is small, we see that both equation (1) and equation (4) can be expanded as a Taylor series yielding

$$\eta_{\mathrm{rms}} \approx S \approx 1 - \sigma_\phi^2 + \frac{(\sigma_\phi^2)^2}{2} + O\left((\sigma_\phi^2)^3\right) \tag{6}$$

showing that, to second order, coherence transfer factor and Strehl ratio should match for high Strehl ratios. If the phase variance is close to or larger than 1 these expressions no longer hold. Since a phase variance of 1 implies an aperture size of r_0, Fried's coherence

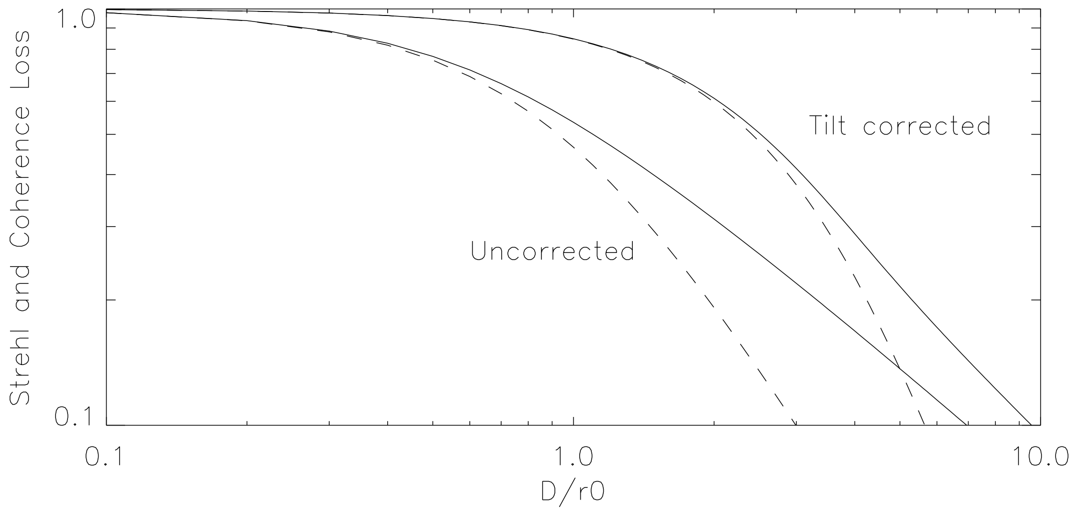

Figure 1: The theoretical coherence loss (solid lines) and Strehl ratio (dashed lines) for the uncorrected and tip/tilt corrected cases.

length, we can say that Strehl ratio and coherence transfer factor should track each other well when $D/r_0 << 1$ where D is the aperture diameter.

A more precise way to compare Strehl ratio and coherence transfer factor is to use equations (2) and (5) directly. The optical transfer function of a single circular aperture is given by

$$T(\boldsymbol{\nu}) = \begin{cases} \frac{2}{\pi}\left[\cos^{-1}\left(\frac{\lambda f \nu}{D}\right) - \frac{\lambda f \nu}{D}\sqrt{1-\left(\frac{\lambda f \nu}{D}\right)^2}\right] & \nu < \frac{D}{\lambda f} \\ 0 & \nu > \frac{D}{\lambda f} \end{cases} \tag{7}$$

where $\nu = |\boldsymbol{\nu}|$, λ is the wavelength, f is the focal length of the optical system and D is the aperture diameter. The optical transfer function of the atmosphere is derived by Fried[7] to be

$$B(\boldsymbol{\nu}) = \exp\left(-3.44\left(\frac{\lambda f \nu}{r_0}\right)^{5/3} \times \left[1 - \alpha\left(\frac{\lambda f \nu}{D}\right)^{1/3}\right]\right) \tag{8}$$

where α takes the value of 0 for what Fried terms the long exposure case, and which we will call the uncorrected case, and has a non-zero value for the short exposure or tilt corrected case. The non-zero value depends on whether one is interested in the near field, for which $\alpha =$ 1, or the far field, for which $\alpha = 0.5$. For application to long baseline stellar interferometry we will use the near field and set $\alpha = 1$. The near field is also more relevant to compensated imaging systems because adaptive optics systems simply vary the phases with a deformable mirror. The results of using equations (2) and (5) with (7) and (8) are shown in figure 1. The solid lines in figure 1 are exactly the same as figure 4 of Tango and Twiss.

It should be noted that the use of Fried's expressions in this way has been criticized by Buscher[8] because it assumes that the residual fluctuations are homogeneous. He goes on to calculate the coherence loss in the tilt corrected case by using a computer simulation resulting in values slightly higher than those predicted by equation (5). Similar simulations by Shaklan et al[9] also result in coherence loss estimates that are close to those of Tango and Twiss only this time lower.

The plot demonstrates that, as predicted by the simple Taylor expansions, Strehl ratio and coherence transfer factor are approximately equal for low D/r_0 values and diverge as D/r_0 increases. This result holds for a greater range of D/r_0 values in the tilt corrected case than in the uncorrected case. Since all large baseline interferometers must employ a tip/tilt servo we will henceforth only use the tilt corrected case. Unfortunately, since very few large baseline interferometers have been constructed there is a severe lack of instruments capable of directly measuring these parameters. We are forced to use computer simulations to test this conclusion.

Before forming fringes or calculating Strehl ratios one must have a series of simulated wavefronts. In our case these data were supplied by SAIC under contract to CHARA. The SAIC simulator is a computer model of a complete adaptive optics system including both diffraction and photon noise effects. It is better described elsewhere[10] but basically consists of an atmospheric model employing several phase screens assuming Kolmogorov turbulence and translated at various wind speeds. The adaptive optics system is modeled using either a Hartmann or shearing interferometer detector and includes a fast steering mirror and a selection of deformable mirrors. The simulations supplied by SAIC to CHARA consist of several realizations of the complex wave reaching a 1 meter aperture for 10cm and 20cm seeing, adaptive correction of 0, 3, 6 and 21 Zernike modes and high and low light level cases. Each realization includes 200 frames at a sample rate of 5ms per frame.

The maximum Strehl ratio found in these files was 0.55 and so in order to model lower D/r_0 values, corresponding to higher Strehls, sub-apertures of the wavefronts corresponding to 20cm seeing and 21 orders of correction were used. These sub-apertures were formed by selecting a subset of pixels across the wavefront and regridding them to the size of the entire aperture using linear interpolation.

These modeled wavefronts were then directly added together to measure visibility and Fourier transformed to measure Strehl ratio. The visibilities were calculated using the method outlined by Tango and Twiss and used in the Sydney University Stellar Interferometer[11]. Thus the visibilities used here are the root mean square visibility. The simulations were also repeated using the mean visibility directly and the results did not differ significantly. The Strehl ratio was calculated for each frame and averaged over the entire run of 200 frames.

In order to compare the two sets of data the relative difference between Strehl ratio and coherence was plotted against Strehl ratio in figure 2. The simulation results match the theoretical curve well and, when they differ, show a better correlation than predicted. This is probably a result of the fact that in the theoretical calculation wavefront tilt is artificially set to zero while in the simulations a real tip/tilt servo is modeled. In order to check that these results are not an artifact of the wavefront model, simulations using a different wavefront generator[12] were used producing a similar correspondence of Strehl and visibility.

There are two ways of applying this result to an optical array. One is as an aid in specifying the requirements of an adaptive optics system to be used with the array and the other as a potential calibrator of the visibilities measured by the array[13].

These results confirm that the Strehl ratio is a useful estimate for the coherence transfer factor in the case of atmospherically aberrated, tilt-corrected wavefronts. The cases investi-

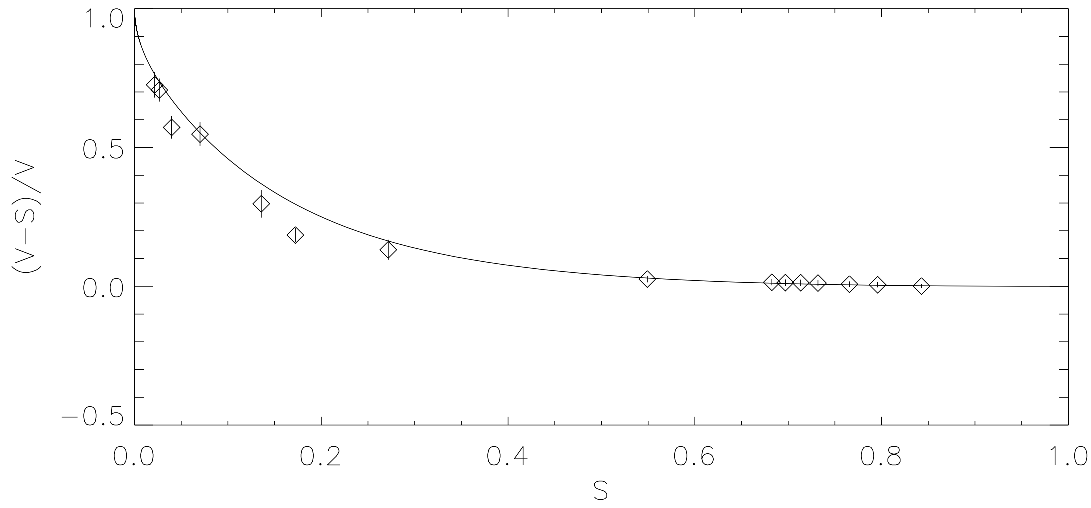

Figure 2: A comparrison of the theoretical and simulated results where the relative difference between Strehl ratio and coherrence loss has been plotted against Strehl ratio. The solid line represents the tip/tilt corrected case and the points are the results of the simulation.

gated by Rousset et al suggest that this result can be extended to wavefronts with a higher order of adaptive correction as well. This result will be useful in predicting the performance of arrays with and without adaptive optics. We also recall the suggestion of Tango and Twiss that the observed Strehl can be used to calibrate observed visibilities. Figure 2 illustrates how the measured Strehl could be used for calibration in the example simulated here. It would be interesting to investigate the sensitivity of the calibration function to the assumptions about the wavefront statistics and degree of adaptive correction.

Acknowledgments

This work was supported by the Center for High Angular Resolution Astronomy at Georgia State University during the design phase for the CHARA Array. CHARA research has been funded by the National Science Foundation through NSF Cooperative Agreement AST90-08941.

References

[1] M. Shao and M.M. Colavita, Annu. Rev. Astron. Astrophys., **30**, 457-498 (1992)

[2] W.J. Tango and R.Q. Twiss, Progress in Optics, **Vol XVII,** 239 - 277 (1980)

[3] Rousset , Madec and Rabaud , *SPIE Proc: High Resolution Imaging by Interferometry II,* Ed J.M. Beckers & F. Merkle (1991), pp. 1095-1104.

[4] V.N. Mahajan, JOSA, **72,** 1258-1266 (1982)

[5] V.N. Mahajan, JOSA, **73,** 860-861 (1983)

[6] R. Herloski, JOSA-A, **2,** 1027-1030 (1985)

[7] D.L. Fried, JOSA, **56,** 1372-1379 (1966)

[8] D. Buscher, Mon. Not. R. astr. Soc., **235,** 1203-1226 (1988)

[9] S.B. Shaklan, M.M. Colavita and M. Shao, *SPIE Proceedings: High-Resolution Imaging by Interferometry II,* Ed J.M. Beckers & F. Merkle (1991), pp. 1271 - 1283.

[10] D.J. Link, *Proc: SPIE conference on laserbeam propagation and control,* 2120 (in press)

[11] J. Davis, W.J. Tango, A.J. Booth, R.A. Minard, S.M. Owens and R.R Shobbrook, *Proc: SPIE conference on Advanced Telescopes & Instrumentation,* 2200 (1994), pp. 231-241.

[12] W.G. Bagnuolo Jr., *Proc: NOAO-ESO conference on High Resolution Imaging by Interferometer,* Ed F. Merkle (1988), pp 981

[13] The idea of using of the Strehl ratio as a visibility calibrator was first introduced by Tango and Twiss.