Three-Dimensional Simulations of Extragalactic Jets Crossing ISM/ICM Interfaces

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ABSTRACT

The propagation and stability of light three-dimensional Mach 3 and Mach 6 jets of initially cylindrical cross-section are explored using the ZEUS-3D code with moderate resolution across the jet diameter. To mimic the behavior of powerful radio jets, which emerge from the centers of active elliptical galaxies through a hot interstellar medium (ISM) and later penetrate into a less dense intracluster medium (ICM), we have set up power-law atmospheres and ISM/ICM interfaces. The simulations are followed to lengths of 35 initial jet radii and axisymmetry is broken by examining the effects of various inclination angles of the ISM/ICM interface to the jet normal.

Increasing the inclination angle produces more asymmetric distributions of vortices in the cocoon which induce more wiggles and asymmetry in the propagating jet. Even inclination angles of 45° do not produce significant "refraction" of the jet, as the initial induced bend is counteracted by extended vortices in the cocoon on the side feeling the lower external densities first. Within the ISM, these 3-D jets exhibit rarefaction fans which reflect back from the contact discontinuity between the jet and the ambient medium, yielding compression waves which steepen along the jet axis to form quasi-biconical shocks. Larger inclination angles produce powerful asymmetric reconfining shocks in the jet while it propagates through the ICM. The Mach 3 jets apparently suffer faster growth of higher modes of instabilities. When examined at the ends of our simulations, the trailing portions of the cocoons have collapsed onto the jets.

Subject headings: galaxies: jets — hydrodynamics — intergalactic medium — radio continuum: galaxies

1. Introduction

The expansion of extragalactic jets through an ambient medium determines the gross morphology of powerful radio sources. Those radio sources surviving distances in excess of ~100 kpc must plow through their galaxy's interstellar medium (ISM) into a hot and diffuse intracluster medium (ICM). Most of these powerful sources are classified as Fanaroff-Riley Type II radio galaxies (FR-II). These sources, with small opening angles, often symmetrically placed knots, bright outer hotspots, and extended lobes, provide guidelines in seeking the parameters of hydrodynamic jets which may provide suitable models for them. Analytical and numerical analyses have adduced a link between the hydrodynamic instabilities in non-ballistic supersonic jets and the resolved features in FR-II sources (e.g., Norman et al. 1981; Norman & Winkler 1985; Hardee 1987; Chakrabarti 1988). Most of the literature has attributed the existence of hotspots to the formation of internal shocks in supersonic jets, but the effectiveness of internal shocks in maintaining a stable configuration for timescales of the order of a jet's life ($10^7 - 10^8$ yr) in a compressible medium has still not been proven conclusively. Recently available 3-D numerical codes have offered an alternative to the cumbersome and approximate analytical treatments of these problems and should allow better comparisons with observations. A number of controlled simulations using both periodic boundary conditions (e.g., Bassett & Woodward 1995) and free-outflow boundary conditions (Hardee, Clarke, & Howell 1995) which match reasonably well with the linear perturbative approaches (e.g., Payne & Cohn 1985; Hardee 1987; Birkinshaw 1991), provide an impetus to simulate the progress of numerical jets which pass through the hot atmosphere seen around elliptical galaxies (e.g., Forman, Jones & Tucker 1985) from which the powerful FR-II radio sources emerge.

The effect of differing atmospheres on the overall evolution of radio jets over cosmological history is clearly important (e.g., Gopal-Krishna & Wiita 1987). Support for such changes in the ambient medium can be inferred from the strong evolution in the X-ray luminosity function (XLF) (Loken et al. 1995 and references therein). The XLF evolution indicates that there are fewer high luminosity clusters at higher redshifts, thus implying higher densities of the ISM and ICM in the past. Such an increase in density can explain (Subramanian & Swarup 1990; Gopal-Krishna & Wiita 1991) the observed decrease in median linear size (MLS) of radio galaxies with redshift (Oort, Katgert, & Windhorst 1987; Neeser et al. 1995). And if the higher efficiency of conversion of jet power to synchrotron emission in a denser external medium (cf. Eilek & Shore 1989) is taken into account (Gopal-Krishna & Wiita 1991) the density evolution can also explain the apparent increase in MLS with radio power for sources of similar redshifts (e.g., Oort et al. 1987; Kapahi 1989).

Some two and three dimensional numerical simulations which take into account the non-static behavior of the external media, in particular the massive cooling flows observed in some of the rich clusters (e.g., Jones & Forman 1984; Edge, Stewart, & Fabian 1992), indicate they can be important in explaining the limited size of the associated radio source (Loken et al. 1995). In some of the simulations the effects of non-radial transonic cluster winds (e.g., Mitteldorf & Wiita 1988; Balsara & Norman 1992), perhaps arising from sub-cluster mergers (Loken et al. 1995), have been explored, and it was found that the interaction between the wind and the jet induces an oblique shock on the side which faces the cross wind and causes a bending which matches rather well with the morphology of Wide-Angle Tailed sources.

Our simulations begin to investigate the stability of 3-D propagating jets against the growth of the Kelvin-Helmholtz (K-H) and Rayleigh-Taylor (R-T) instabilities on the contact surfaces between the jet, the cocoon, and the shocked ambient medium. To allow for some of the effects of the origin within galaxies of the jets forming FR-II sources, density gradients are supplied within the ISM. To break the axisymmetry in the direction of propagation of the jet, the normals to the ISM/ICM interfaces are inclined with respect to the jet axis, but we do not force any driving wiggles at the origin (e.g., Hardee et al. 1992, 1995). Both the variable density ISM, and the ICM, a constant density external medium (CDEM), are assumed isothermal and in pressure equilibrium with each other. Simulations with different ratios of the jet to the ambient density are compared to explore the differences in morphologies and rate of advancement of the jets.

In §2 the features of the numerical hydrodynamic code ZEUS-3D are summarized. One dimensional analytical results of some relevance to these simulations are given in §3. Our results are analyzed in §4, and conclusions and caveats are given in §5.

2. Numerical Approach

In our simulations we have used a well tested general purpose three dimensional code ZEUS-3D, v.3.4 (see the user manual, Clark & Norman 1994). This code is an extension of an earlier, two dimensional code, ZEUS-2D, which is described in detail in Stone & Norman (1992a,b), and Stone, Mihalas, & Norman (1992).

The ZEUS-3D code is an Eulerian finite-difference code which can employ either the second-order monotonic upwind advection scheme of van Leer (1977) or the piece-wise parabolic advection method (PPA) of Collela & Woodward (1983). Shock waves are treated using the usual von Neumann-Richtmyer artificial viscosity. The dynamical equations are time-differenced in an explicit fashion, with the exception being the use of a time-implicit internal energy equation for flows with strong cooling, which is not necessary for our efforts. The algorithms used in these codes have been tested extensively against a variety of flow problems where the solution is either known analytically or has been determined experimentally (Stone & Norman 1992a). Aside from giving small errors for shocks inclined at angles to the grids, the code works well in most situations.

Our 2-D simulations of interface crossing jets (Wiita, Rosen & Norman 1990, hereafter WRN; Wiita & Norman 1992, WN; Hooda, Mangalam & Wiita 1994, HMW) used both cylindrical and spherical-polar coordinates (with the latter best suited to initially conical jets, as was assumed in the relevant analytical work). Since our extensive intercomparisons have shown no significant coordinate dependent differences, for improved speed our 3-D simulations employ Cartesian coordinates; the code runs more slowly in curvilinear coordinates. For greater accuracy we have employed the PPA scheme, albeit at a cost in CPU time.

The simulations were carried out on the Pittsburgh Supercomputer Center Cray Y-MP C90 and postprocessing was performed using Spyglass TRANSFORM and NCAR interactive graphical software packages. The initiating subroutines for the atmospheres and interfaces were written separately to allow for the density gradient in an isothermal ISM. The density within the ISM follows

$$\rho(x,y,z) = \frac{\rho_0}{\{1.0 + [(\frac{z}{z_{\rm scale}})(\frac{z_{\rm H}}{z_{\rm H} - x\tan\theta})]^2\}^b}.$$

On crossing the interface, $z_{\rm H} - x\tan\theta$, where $z_{\rm H}$ is equal to 10 times the initial jet radius, R_0 (which defines the length scale), the density drops to $0.2\rho(z_{\rm H})$, but the temperature rises by a factor of 5 to provide a pressure matched boundary; the density and pressure of the external ambient medium are then kept constant for $z > z_{\rm H} - x\tan\theta$. The key parameters are: $\eta \equiv \rho_{\rm jet}/\rho_0$; the jet Mach number $M = u/a_{\rm in}$, with $a_{\rm in}$ the jet sound speed; and the angle θ at which the interface is tilted with respect to the normal to the jet. For all our simulations we assume the jet's initial pressure matches that of the ISM at $z = 0$. We have chosen $z_{\rm scale} = 8.9$, and b=0.75, corresponding to a power-law decline of $\sim$1.5 in density with distance in the ISM (cf. Forman et al. 1985; WRN). For a canonical value of $R_0 = 1$ kpc, our galaxy extends 10 kpc in radius, and if $T_{\rm ISM} = 10^7$K, then one time unit corresponds to 2.6×10^6yr.

Our 3-D simulations were run on $150\times50\times50$ active zones, extending out to $35R_0$ along the initial direction of propagation, z, and to $\pm12R_0$ in both directions perpendicular to it. To provide adequate resolution in the most important region, 14 uniform zones span the jet diameter. The central 16×16 zones, across the x and y directions, are equally spaced over distances of $2.4R_0 \times 2.4R_0$; whereas the remaining 34 zones (17 on each side, from $\pm1.2R_0$ to $\pm12.0R_0$) are ratioed with the smallest zone equal to $0.15R_0$. Outflow boundary conditions are used everywhere except for the zones at which the jet fluid is introduced.

3. Input and Expectations

3.1. Analytical Considerations

The stability of non-ballistic hydrodynamic jets, such as those in which we are interested, rests squarely on the non-linear coupling between the jet, the shear layer, and the surrounding cocoon. Linear perturbative analyses of 3-D hydrodynamic jets transversely pressured matched with the ambient medium have confirmed that the rate of growth of the dominant Fourier modes inside or on the surface of the supersonic jets, with fluid speeds more than the sum of the internal and external sound speeds, can be traced to the energy amplification from their interaction with the vortex layer (Gill 1965; Ferrari et al. 1981, 1982; Cohn 1983; Payne & Cohn 1985; Hardee 1987). These modes are speculated to develop into the familiar criss-cross pattern (or conical shocks), seen in the laboratory jets and numerical simulations when their perturbative velocities approach the local sound speeds (Courant & Friedrichs 1976). Among the debatable issues, it is still unclear whether the dominant growing mode does or does not suppress the growth-rate of more slowly growing modes; if it does not, then in higher resolution simulations more and more small scale structures — beside the originally dominant large scale structures — will show up. Recently Bassett & Woodward (1995) have performed some high resolution 2-D simulations (with 150 or more computational zones across the jet width) of Mach-2 and Mach-4 gaseous jets and found that the higher resolution runs support different modes of the *kink* instabilities than do the low resolution runs. They also found an enhancement of small scale turbulence at higher resolution which indicates that the high frequency modes do play important roles at small scales and may disrupt or slow down the jets at distances shorter than those found in lower resolution runs.

In the small amplitude perturbative analysis of a cylindrical jet which is pressure matched with the surrounding medium, a dispersion relation allows the growth of Fourier modes which can be categorized into surface modes or fundamental modes ($m = 0$) and body modes or reflection modes ($m \geq 1$). Here the parameter m represents the number of nodes in the radial direction of the Fourier perturbation $\mathbf{f}(r, \phi, z) = \mathbf{g}_m(r) \exp\, i(kz + n\phi - \omega t)$, and the parameter n defines the rotation of the wave around the azimuthal direction as it propagates along the z direction; $n = 0$ represents an axisymmetric/pinching mode and $n \geq 1$ represents non-axisymmetric/fluting modes ($n = 1$, helical; $n = 2$, elliptical; $n = 3$, triangular; and so on). The complex roots of either ω (temporal instabilities) or k (spatial instabilities) in the dispersion relation indicate the growth of these perturbations as they reflect and refract from the vortex layer. The reflection modes result only for $u > a_{\rm ex} + a_{\rm in}$, and are also referred to as supersonic resonances; the maximum growth rates of the helical reflection modes exceed that of the helical fundamental mode, but for higher m the fundamental modes still can grow more rapidly. In the non-relativistic limit, the analytical approximations of the spatial growth rates for a given reflection mode (see Hardee 1987) yield upper limit to the imaginary part of the wave number and an estimate for the real part of the wave number:

$$k_{\rm im} < \frac{1}{2MR} \ln\left(4\frac{\omega^* R}{a_{\rm ex}}\right), \tag{1}$$

$$k_{\rm re} \approx \frac{\omega^*}{M a_{\rm ex}}(1 + \sqrt{\eta}); \tag{2}$$

where,

$$\omega^* \approx \frac{\pi a_{\rm ex}}{2R}\left[\left(n + \frac{1}{2}\right) + 2m\right]. \tag{3}$$

Here M is the Mach number, $a_{\rm ex}$ is the external sound speed, and R is the radius of the jet.

In the presence of a shear layer, the analytical studies done for 2-D jets (Ferrari, Trussoni, & Zaninetti 1981) and for plane-parallel shear layers (Ray 1982; Choudhury & Lovelace 1986) show that the higher frequency modes stabilize faster than do the low frequency modes and so the overall jet behavior is governed by the low frequency modes. If the scale height of shear layer is represented by h, then modes with wavenumber $k > 1.28h^{-1}$, are shown to be stabilized. Hardee (1987) examined the implications of the above results for cylindrical jets and showed that the higher order reflection modes of higher order fluting modes are suppressed and the maximum number of non-suppressed reflection modes belongs to the pinching mode. On the other hand, among the non-suppressed modes, the fundamental helical mode grows to the largest amplitude before converging into a shock. Using a different approach, Birkinshaw (1991), numerically solved several linear analytical models assuming a jet with a top-hat type velocity profile surrounded by a shear layer containing a linear velocity profile, matching the inner and outer velocities. For light supersonic jets, Birkinshaw's results indicate that the presence of a shear layer reduces the influence of the reflection modes relative to the fundamental modes. Among the solved cases of the $n = 0, 1, 2$ modes, the $n = 2$ fundamental mode shows a higher growth rate over a wide range of wavelengths; however, for the longest wavelengths the $n = 1$ fundamental mode dominates. None of these studies have addressed the complications introduced by the extremely sensitive *critical layer*, which forms within the shear layer of a supersonic jet at a distance from the jet's axis where the local fluid velocity becomes equal to the local phase speed of the perturbation; thereby, exposing the singular nature of the linear perturbation equation (e.g. Maslowe 1986). Thus, the inclusion of even the lowest order non-linear terms may significantly modify these results (Hooda, in preparation).

Besides the non-linear effects of shear layers there are other factors which are known to govern the behavior of radio jets, one of them being the atmospheric gradient. In 2-D axisymmetric numerical simulations HMW found that the low Mach number jets, on crossing a sufficiently strong atmosphere gradient, eventually disrupt from a K-H type instability. On the other hand, high Mach number jets show more temporal and spatial stability than do low Mach number jets; most of the difference lies in the smaller Mach angle of the internal shocks with respect to the stream lines for higher Mach number jets, which slows the growth of higher frequency sideways spatial resonances compared to low Mach number jets. For example, it can be seen from the upper limit on the growth rate of supersonic resonances (Eq. 1) that a Mach-3 jet disrupts essentially twice as fast as a Mach-6 jet.

A steady state jet, with initial velocity $u_0 = u(0)$, radius $r_0 = R(0)$, density $\rho_{\rm in}(0)$, Mach number M_0, and adiabatic index γ, pressure matched with an isothermal ambient medium with a 1-D density gradient (for $0 < z < z_{\rm H}$) given by,

$$\rho_{\rm ex}(z) = \rho_{\rm ex}(0)\left[\frac{1}{[1+(\frac{z}{z_{\rm scale}})^2]^b}\right], \tag{4}$$

slowly expands in pressure balance with the external medium as:

$$\frac{R(z)}{R(0)} = \left[\frac{[1+(\frac{z}{z_{\rm scale}})^2]^{b/\gamma}}{[1+\frac{2}{{M_0}^2(\gamma-1)}\{1-(1+(\frac{z}{z_{\rm scale}})^2)^{-b(\gamma-1)/\gamma}\}]^{1/2}}\right]^{1/2}. \tag{5}$$

The conservation of the mass and momentum flux, in the one dimensional approximation, also result in the approximate velocity and density variations along the jet for $z < z_{\rm H}$:

$$\frac{u(z)}{u(0)} = \frac{\rho_{\rm in}(0)}{\rho_{\rm in}(z)}\left[\frac{R(0)}{R(z)}\right]^2; \tag{6}$$

$$\rho_{\rm in}(z) = \rho_{\rm in}(0)\left[\frac{1}{(1+(\frac{z}{z_{\rm scale}})^2)^{b/\gamma}}\right]. \tag{7}$$

In the presence of the cocoon, the effect of external density variation $\rho_{\rm ex}(z)$ can be used in defining the effective area of the working surface, $A_{\rm h}$; whereas at the same time, the jet fluid cross section, $A_{\rm j}$, may be assumed to be essentially constant. This crude approximation results in the velocity of the working surface or the head of the jet being

$$V_{\rm h}(z) \approx u(z)\frac{[R(0)/R(z)]\eta(z)^{1/2}}{1+[R(0)/R(z)]\eta(z)^{1/2}}, \tag{8}$$

where $\eta(z) = \rho_{\rm in}(z)/\rho_{\rm ex}(z)$.

The time taken by the jet to reach the interface is

$$t^{\rm a}_{\rm H} \;=\; \int_0^{z_{\rm H}} \frac{dz}{V_{\rm h}(z)},$$

where $V_{\rm h}(z)$ is given by Eq. (8).

3.2. Input Parameters

In WRN detailed discussions of the reasons for choosing various parameters were given; here we just quote the values we have used. The observationally determined values of various parameters are: jet radii of $\sim$1 kpc, $a_{\rm j} \approx 500$ km s^{-1}, and $b = 0.75$ (e.g., Bridle 1986; Forman et al. 1985). All of the 3-D runs have a typical observationally determined value for the power-law index of $b = 0.75$, although other values were used in some of our earlier 2-D work. These allowed us to confirm the prediction (WRN) that steep power-laws lead to accelerated conical beams in the halo. Across an ISM/ICM interface a suitable factor by which the density drops (and temperature rises) will be of the order of 2–10 (e.g., Sarazin 1988); whereas, at an ISM/intergalactic medium (IGM) interface the changes could be in the range 10–50 (Guilbert & Fabian 1986; Rosen & Wiita 1991). Using the fairly well determined ICM value in preference to the putative IGM value to govern large scale morphologies, we have used a density drop down to $0.2\rho(z_{\rm H})$ at the interface for all of our 3-D simulations.

In Table 1 the input parameters for our total of seven 3-D simulations are listed in columns (2)–(4) after the identifier given in column (1). The time taken by the jets to reach the interfaces

($t_{\rm H}$) and the end of the grids ($t_{\rm end}$) are given in columns (5) and (6) respectively. The times expected to be taken by the heads of the jets (from the one-dimensional analytical treatment, $t^a_{\rm H}$) to go from $z = 0$ to $z = z_{\rm H}$ are calculated in column (7). The last column indicates the maximum swings of the jets in the x-direction (which always exceeds that in the y-direction, since the tilt is in the $x - z$ plane and there is no direct symmetry breaking in the y-direction), about the original axes.

The morphological results of numerous previous simulations are consistent with FR-II radio jets having $M \geq 3$ and $\eta \ll 1$ (e.g., Norman & Winkler 1985; cf. §4.2), though Hardee et al. (1995) have recently argued that $M \approx 5$ is too low to effectively reproduce correct morphologies. The lower values of M (column 2) used in some runs (F & G) were chosen to examine the difference in growth rates of various instability modes. Two different values of the initial ratio of jet density to ISM density, $\eta = 0.0316$ and 0.0100, respectively, were used to investigate the dependence of the shock strength on density and to check simple analytical integral estimates for the propagation time (column 7) on different η's for the same Mach number jets. Such values of η still guarantee that the jets remain under-dense with respect to the ICM, and avoid the ballistic propagation investigated in some 2-D simulations by WRN and WN. Even smaller values of η and larger values of M would probably produce better simulations of FR-II radio morphologies, but either of these changes require substantially greater computational resources for the same size grids, and will be postponed to later efforts.

4. Results

4.1. Evolution within the ISM

Examination of density and velocity maps made during the earliest stages of a jet's evolution show that the shear layers separating the jet from the back-flow (or the cocoon) and the back-flow from the shocked ambient medium quickly develop a number of concentric vortex rings around the jet's axis. Soon after their formation the vortex rings, in turn, influence the fluid flow across the jet by engendering conical oblique shocks. The Mach disk (MD) retains its radius essentially equal to the jet's radius only until the jet gets out to a distance of around one radial unit (R_0); after that the MD comes under the influence of a conical oblique shock emanating from the foremost axisymmetric vortex ring. Axisymmetric rarefaction fans also arise from near the orifice and reach the jet core at a distance $R_0 \cot A$ from the origin; here A is the Mach angle of the jet. Flanked by the innermost rarefaction fans, a stationary and quasi-parallel flow exists in a conical cavity (also known as potential core) around the origin (with its base at orifice and apex at $R_0 \cot A$). These features are common to transverse pressure-matched flows close to an inlet boundary and are not dramatically affected by the presence of an atmosphere of decreasing density.

The conical shock dissolves into rarefaction fans as soon as the vortices lose their strength, owing to entrainment and various other non-linear processes (e.g, at small-scales, the transition to turbulence). The Mach-3 jets maintain the primary conical oblique shock (and also have a consequent reflected shock, not seen in Mach-6 jets for our ISM conditions) for longer durations than do the Mach-6 jets. During this phase the MD is supported by an annular shock which converts the oblique supersonic streamlines into subsonic ones and also refracts them into the surrounding vortices. Once the jet enters into the CDEM, the rarefaction and compression fans take the place of the oblique shocks. The rarefaction fans cause the jet to expand into the, now, somewhat lower pressured surrounding cocoon (see Courant & Friedrichs 1976). The lower pressure arises essentially through the thermal expansion of the cocoon into the low density CDEM and the pressure gradient in the ISM; so while the jet is pressure matched with the ISM at the orifice, its pressure can exceed the cocoon's pressure soon after the cocoon thermally expands.

The total transverse expansion of the jet can be treated as a combination of a steady state expansion due to the pressure gradient in the ISM (see Eq. 5), and oscillations about this state arising from the non-linear growth of acoustic modes (e.g., Norman et al. 1981). After passing the potential core, the influences of rarefaction and compression fans assist the MD in losing the stable configuration of a pure disk, and its outer portion extends into an annular shock. The stability of the annular shock seems to be linked with the strength/dominance of the vortex ring in the vicinity of the MD; stronger vortical motions generate stronger annular shocks and vice-versa. In the compression zone the expanding symmetric acoustic waves from the earlier rarefaction zone reflect back from the contact surface between the jet and cocoon and thereby allow the asymmetry present in the ambient medium to affect the jet dynamics. The convergence of these asymmetries affects the momentum flux across the MD and thus contributes to the asymmetric expansion of the cocoon. The momentum flux is also affected by the asymmetric compression between the bow shock and the MD; this develops due to the asymmetric gradient in the ISM, which is scaled in parallel with the ISM/ICM inclination angle. Small variations in the MD inclination with respect to the jet flow arise as a result of the ram pressure exerted by the vortices from the sides; the bending of the jet may also be a direct result of these perturbation from the vortices. The vortex rings around the jet are also linked through a number of superimposed counter-rotating secondary vortex tubes; their helical axes extend and wrap around the jet (Arnold & Arnett 1986; Bassett & Woodward 1995). These secondary structures could be produced by non-linear roll-ups of the shear layer under the influence of the fundamental helical mode, which is expected to dominate soon after the axisymmetric body modes reach a point of saturation.

To portray the early influence of the ISM/ICM inclination angle, in Fig. 1 we display several velocity vector cross sections of run G. These snapshots are taken at a time when the jet has nearly reached the end of its ISM phase; the last panel for the simulation shows a cross-section through the bow-shock. A careful look at the velocity-vector directions in the jet core, at $t = 2.4$ (Fig. 1), shows that an oblique conical shock is present, while the rarefaction wave still exists in the Mach-6 case (not displayed).

Thanks to the cocoon's hindrance, a secular steady state expansion, as predicted from the analytical expression (Eq. 5), is not an easily identifiable feature in these simulations; in the

Mach-3 jet a slight expansion is visible only near the end of the simulation. The analytical expression, however, gives a reasonably good estimate for the effective cross section of the Mach disk; from the conservation of momentum the confinement of the Mach disk results from a direct interaction with the ambient medium. Comparisons between two Mach-6 (C and E) and two Mach-3 (F and G) cases with $\eta = 0.0316$ and $\eta = 0.01$, respectively (but all with a 30° tilt), indicate that within the ISM the lower η run propagates about 1.23 and 1.30 times faster respectively than does the higher η run. This trend can be basically understood by comparing the jets' thrusts, which scale as $\rho_j v_j^3$, and therefore as $\eta^{-1/2}$, since we have assumed pressure matches with the ambient medium; so higher η for fixed M implies a lower $v_j \propto \eta^{-1/2}$. But the simple analytical integrated times (Eq. 8) indicate that the low η runs are expected to be only 1.07 times faster than the corresponding high η runs for both the Mach numbers investigated. This discrepancy may arise from the jet's increase in immunity against K-H instabilities for smaller η (cf. Gill 1965; Cohn 1983), which could produce stronger internal shocks in the higher η non-ballistic jets and thereby reduce their effective thrusts in comparison to the lower η runs. The numerical differences between the ratios for Mach-3 and Mach-6 jets may have been caused by a still stronger growth in the shock strengths for the low Mach number jets (see Eq. 1). The difference between the numerical and analytical times mostly results from the simplifications and lower dimensionality of the analytical work, but some of the difference may be attributable to the relatively low resolution of the 3-D simulations, and will be addressed in future work.

4.2. Crossing the Interface and into the CDEM

The effect of the ISM/ICM inclination angle, although not very pronounced while the jet is still within the ISM, is quite strong once it enters the ICM. In Fig. 2 we compare the density contours for two Mach-6 simulations (runs B and D) with the same parameters except for inclination angles, at two different times. As discussed above, the inclination angle influences the shock strength across the MD and causes an uneven flow of energy in the surrounding vortex ring, which gives rise to an asymmetric inflation of the cocoon. The cocoon on the steeper side of the gradient (i.e., the side first exposed to the ICM) bursts into the low density ICM sooner than the cocoon on the far side, and then its expansion accelerates. This trend reverses as soon as the MD crosses the interface and gives an opportunity for the undeveloped cocoon to expand into the low density ICM; the expansion halts soon after the cocoon's transient pressure becomes equal to the ICM pressure. During the cocoon's expansion, the jet also experiences a transverse compression which stops soon after the jet's transverse pressure exceeds the cocoon's pressure. As a result, an asymmetric quasi-conical compression wave is engendered; this narrows the jet through a reconfining shock. For example, the cross-sectional radii of the jet, in simulation E at $t = 4.0$ (Fig. 5) along the x and y directions measured at $z = 0.0$, $14.4R_0$, $20.0R_0$, $22.4R_0$, and $26.6R_0$ are (in units of R_0), respectively: $1.00 \times 1.00; 1.33 \times 1.28; 0.64 \times 0.71;\ 0.39 \times 0.89; 0.72 \times 0.94$. Recall that the canonical physical values are $R_0 = 1$ kpc, and $t = 4.0$ corresponds to 1.04×10^7yr.

The asymmetric nature of the reconfinement shock also produces bending, with amplitude roughly proportional to the steepness in the ISM/ICM inclination angle. The average of the *maximum transitory* bendings taken from Table 1, which mostly occur shortly after crossing the interface, for Mach-6 and Mach-3 jets are $0.4R_0$ and $1.3R_0$, respectively. These oscillations usually damp out, but variations in the position of the jet head remain throughout each simulation. The two velocity vector sections in Fig. 3, taken at the end of simulation G, partially illustrate structures in the cocoon. There are filamentary structures which are generated by the secondary counter-rotating helical vortex tubes (see last section). These tubes snake through the cocoon, and also link with the toroidal vortices surrounding the MD as can be seen in more finely sliced cross-sections. When we compare cross sectional and longitudinal images made at several close time-slices we see that while these structures naturally fluctuate, they are surprisingly stable, and their enhanced density (and probably enhanced magnetic field strength) may provide an effective site for the filamentary radio emission seen in the cocoons in many extragalactic sources (cf. 3-D simulations by Clarke 1995).

The distance between the bow shock and MD immediately after crossing the interface increases and stabilizes to a new value in accordance with the change in MD velocity and density differences between ISM and ICM, for which analytical and 2-D numerical simulations are in good agreement (WN, HMW). One can also interpret this change in terms of the distance between a standing shock and a blunt projectile, immersed in a supersonic flow; the distance increases with a decrease in supersonic velocity (Courant & Friedrichs 1976; Landau & Lifshitz 1987). A comparison of Mach-6 and Mach-3 jets, simulations C and F, respectively, progressed to the same distance (see Fig. 4), indicates that the Mach-3 jet, after crossing the interface and reconfinement shock, goes through an expansion; but at the same time it suffers from the rapid growth of K-H instabilities. The roughly measured expansion appears to be in agreement with the expected width of a Mach-3 steady-state jet that is pressure matched to the ICM. On the other hand, the Mach-6 jets do not show expansion after crossing the reconfinement shock, indicating that the relaxation length for a Mach-6 jet is significantly larger than for the Mach-3 jet. If we consider the interface as a nozzle venting the jet into a low pressure CDEM then the relaxation lengths for higher Mach number jets should indeed be higher than those for low Mach number jets. The average distance between the internal shocks (including the MD) in our Mach-3 jet is found to be $\approx 3/4$ that of the Mach-6 case. The filamentary structures in Mach-3 jets are also stronger than in Mach-6 jets. This is seen to some extent in Fig. 5 which shows two perpendicular cuts through run E, although multiple slices in various directions are needed to clearly see these structures. The stronger filamentary structures also imply higher velocity shears (or Reynolds stresses) which usually cause higher entrainment of the laminar flow (e.g., Oster & Wygnanski 1982). Jets with stronger filamentary structures may thus represent a transitional state between the laminar and turbulent flows. These results are in accord with the expectation (e.g., Bicknell 1984; De Young 1986) that the entrainment rates are higher in lower Mach number light jets and so the extended synchrotron emission arising from denser and higher pressure regions (which are at least partly due to the filamentary tubes within the cocoon) should increase with a decrease in Mach number.

Comparisons between different Mach-6 (3) simulations, with different η's, run E (G) and run C (F), at the end of the computations, show that the $\eta = 0.0100$ run propagates about 1.27 (1.18) times faster than the $\eta = 0.0316$ run. Fig. 6 gives gray scale representations of the density for runs E and C, at the end of these simulations, illustrating the difference in jet and cocoon thicknesses and oscillations for different η.

4.3. Discussion

Examination of pressure images during various stages of jet propagation shows that pressure inhomogeneities (usually the sites of active vortices) within the cocoon are highly variable and play a decisive role in constraining the morphology of a given jet. Depending upon the strength of their motion, the vortices exert a localized ram pressure on the surface of the jet which sends either an oblique shock or a compression wave downstream. Because they eventually shed from the jet, the vortices lose their strength with time, and likewise their associated oblique shocks assume the form of compression waves.

Since the working surface of the jet feeds only the vortices closest to its location, the tail of the cocoon collapses through the R-T instabilities which develop along the contact discontinuity between the shocked ambient medium (screen) and the cocoon (see Chakrabarti 1988). The collapse is asymmetric for the inclined interfaces we consider here and may result in some bending at times longer then those reached in our simulations. The collapse causes a secular entrainment of the cocoon material along the jet and exposes the jet's core in the similar way. This loss of the cocoon near the base of the jet was also seen in 2-D slab jet simulations propagating through an atmosphere (Hardee et al. 1992).

During the earlier phases of the simulations the Mach-3 jets show strong reflections of the conical oblique shocks (which are generated by the axisymmetric localized vortices from the cocoons) off the jet axes and later from the surface discontinuities. This result is not seen in the Mach-6 jets, because they reach the ISM/ICM interface faster than the reflections can travel. The Fourier mode analysis of the stability of the jet may be applicable only in this regime; otherwise, the reflections of the modes from the jet surface are lost among the disturbances created by the local inhomogeneities induced by the cocoon.

The highly pressurized cavity between the MD and the bow shock is stable against R-T instabilities and confirms the findings of Chakrabarti (1988) for light jets. Using the one dimensional Bernouli's equation and the conservation of momentum across the working surface, the pressure between the jet and the cavity are approximately related by (see Landau & Lifshitz 1987),

$$p_{\rm c}(z) \approx p_{\rm j}(z)\left[\frac{\gamma+1}{2}\right]^{(\gamma+1)/(\gamma-1)} \frac{[M_{\rm h}(z)]^2}{\{\gamma-(\gamma-1)/2[M_{\rm h}(z)]^2\}^{1/(\gamma-1)}}, \tag{9}$$

where $M_{\rm h}(z) = [u(z) - u_{\rm h}(z)]/a_{\rm in}(z)$. Due to the high pressure in the cavity, a nearly axisymmetric radially outward flow emerges from behind the bow shock and flows along the cocoon–screen discontinuity. This flow also causes the development of K-H instabilities along the discontinuity and leads to the finger like protrusions of the shocked ambient medium inside the cocoon's body. The continuous non-linear growth of these instabilities starts the collapse of the cocoon along the jet axis and later may also influence the morphology of the jet.

5. Conclusions

Our analysis of these simulations indicates that:

1. Decreasing the jet density produces a broader cocoon.

2. The stronger internal shocks in the higher density (η) jets make them slower than the lower η jets when compared to the steady state (shock-free) analytical results.

3. The shocks are usually induced by the localized pressure perturbations of the vortices. Owing to the interaction, both the shocks and the vortices decay with time; the shocks decay into rarefaction and compression zones.

4. The high pressure cavity behind the bow shock drives the flow predominantly along the discontinuity between the cocoon and the shocked ambient medium.

5. An annular shock forms around and in front of the Mach disk (MD).

6. The toroidal cocoon near the MD is comprised of a number of eddies with opposite helicities in the adjoining eddies.

7. The finger like growth of the Rayleigh-Taylor instabilities along the discontinuity of the cocoon and the shocked ambient medium dominate the eventual collapse of the cocoon and the consequent exposure of the jet.

8. Lower Mach numbers yield more internal structure for simulations compared at the same distances of propagation. The cocoon's interactions with the jet play a larger role in the slower jets.

9. Increasing θ induces more wiggles and asymmetry in the jet propagation. However, no significant long-term deflection of the jet occurs up to $\theta = 45°$, the maximum we have investigated. This is not unexpected, as we have not included an actual sideways pressure gradient or wind needed to drive a large bend.

10. None of these jets have completely broken up or become clearly unstable in these simulations; however, even the no-tilt simulation (A) does show some breaking of axisymmetry by the end of the run. perhaps due to our attempt to model a cylinder using a rectilinear grid. But a recent higher resolution simulation reported by Norman (1996) for an unperturbed $M_j = 10$, $\eta = 0.01$ jet propagating through a CDEM for a distance somewhat longer than we simulated here indicated

that a very unstable working surface eventually develops.

Several of these conclusions (1, 4, 5, 7, 8) had been established earlier for 2-D simulations and have merely been confirmed by these new 3-D investigations.

Although these 3-D HD simulations have given us some new insights concerning the stability and morphology of radio jets, they are clearly inadequate in spatial resolution and temporal extent to yield a thorough understanding of the physics of the expansion and stability of extragalactic hydrodynamical jets. Some of the recent higher resolution 2-D and 3-D simulations (Hardee et al. 1992, 1995; Bassett & Woodward 1995; Clarke 1995; Norman 1996) have reported a significant increase in entrainment and turbulence at smaller scales, causing the jets to slow down and disrupt at times earlier than those found in similar low resolution simulations. We have performed two higher resolution simulations with 30 zones across the jets diameter, and a preliminary analysis indicates that they are indeed more unstable and slower than those reported here although no dramatic differences are seen. We expect that these, and additional higher resolution results with parameters otherwise identical to our current simulations, will allow us to better estimate the contributions of numerical artifacts and the effects of entrainment within propagating jets (as opposed to the equilibrium jets considered by some other workers).

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Table 1: Input Parameters and Summary of Results

Run (1)	M (2)	η (3)	θ (4)	$t_{\rm H}$ (5)	$t_{\rm end}$ (6)	$t^{\rm a}_{\rm H}$ (7)	Max bend (8)
A	6	0.0316	0°	1.30	4.20	1.99	
B	6	0.0316	10°	1.30	4.18	1.99	$-0.2R_0, +0.5R_0$
C	6	0.0316	30°	1.30	4.19	1.99	$-0.4R_0, +0.4R_0$
D	6	0.0316	45°	1.30	3.98	1.99	$-0.2R_0, +1.1R_0$
E	6	0.0100	30°	1.06	3.29	1.86	$-0.6R_0, +0.0R_0$
F	3	0.0316	30°	3.0	10.0	3.93	$-0.6R_0, +0.4R_0$
G	3	0.0100	30°	2.3	8.5	3.67	$-0.7R_0, +1.0R_0$