The contained mass is: $m(a) = 4\pi \int_0^a \rho(a')\, a'^2 da'$ (58)

So the mass element between Ψ & $\Psi + d\Psi$ is:

$$dm(a) = \rho(a)\, dV_a = 4\pi \rho(a)\, a^2 da \qquad (59)$$

Use (56) & (57) in (59) to find:

$$dm(a) = \rho \oint_{\mathcal{S}_a} dn\, dS = \rho\, d\Psi \oint_{\mathcal{S}_a} g^{-1} dS = \rho \langle g^{-1} \rangle \mathcal{A}_a\, d\Psi \qquad (60)$$

We can rewrite the cons. of mass as:

$$\frac{da}{dm(a)} = \frac{1}{4\pi a^2 \rho} \qquad (61)$$

Eq. (9) means we can write $\frac{dP}{dm(a)} = -\rho \frac{d\Psi}{dm(a)}$ (62)

Using (61) we are left with: $\frac{dP}{dm(a)} = \frac{-Gm(a)}{4\pi a^4} f_P$ (63)

with $f_P(a) = \frac{4\pi a^4}{Gm(a)\mathcal{A}_a} \langle g^{-1} \rangle^{-1}$ (64)

<u>N.B.</u> for a spherical star, $a = r$ & $f_P(a) = 1$.

Further assume our rotating ☆ is in thermal ⇌. Let $l(a)$ denote the energy passing/s through a level surface in the outward direction; then the increment the luminosity gains ($l(a)$) bet. Ψ & $\Psi + d\Psi$ is:

$$dl(a) = \epsilon\, dm_a \qquad [\text{ignore } \epsilon_\nu \text{ \& } \epsilon_g \text{ terms}] \qquad (65)$$

For a chemically homog. ☆, $\epsilon = \epsilon(\rho, T)$ & is const on $\Psi \Rightarrow$

$$\frac{dl}{dm} = \epsilon \qquad (66)$$

where we suppress a as the radial notation variable.

Now consider our ☆ in radiative ⇌. (10) ⇒

$$F_a = -\frac{4ac}{3\kappa\rho} T^3 \frac{dT}{dn} = -\frac{4acT^3}{3\kappa\rho} g \frac{dT}{d\Psi} \qquad (67)$$

<u>N.B</u> radiative flux varies $\propto g$ on a level surface.

Because, as we've shown in (26), meridional motions carry no net energy, we can write:

$$l_a = \oint_{\mathcal{S}_a} F_a \, dS = -\frac{4acT^3}{3K} \mathcal{A}_a^2 \langle g \rangle \langle g^{-1} \rangle \frac{dT}{dm}, \qquad (68)$$

as K is also const. over surfaces w/ Ψ const. Use (58) & (68) to find:

$$\frac{dT}{dm(a)} = \frac{-3K}{4acT^3} \frac{l_a}{(4\pi a^2)^2} f_T \qquad (69)$$

where

$$f_T(a) = \left(\frac{4\pi a^2}{\mathcal{A}_a}\right)^2 \langle g \rangle^{-1} \langle g^{-1} \rangle^{-1} \qquad (70)$$

& $f_T = 1$ if $\nexists$ rotation.

Finally, in regions where convective transport dominates one usually <u>assumes</u> Γ_2 is const. on level surfaces, so:

$$\frac{dT}{dm(a)} = \left(1 - \frac{1}{\Gamma_2}\right) \frac{T}{P} \frac{dP}{dm(a)} \qquad (71)$$

<u>Note</u>: (61) (62) (66) & (69) or (71) REPLACE USUAL EQNS of STELLAR STRUCTURE. Still need <u>constitutive rel'ns</u>

All we need are the purely geometrical $f_P(a)$ & $f_T(a)$ — based on shape of level surfaces.

A full sol'n would solve: $\nabla^2 \Psi = 4\pi G\rho - 2\Omega^2$ (72)

w/ approx. BC's — slightly modified. Usual approach is to approx. total pot, $\Psi(a)$ as sol'n for Roche model (3) so that:

$$r(a,\mu) = a\left[1 - \frac{\Omega^2}{3GM} a^3 P_2(\mu)\right] \qquad (73)$$

or

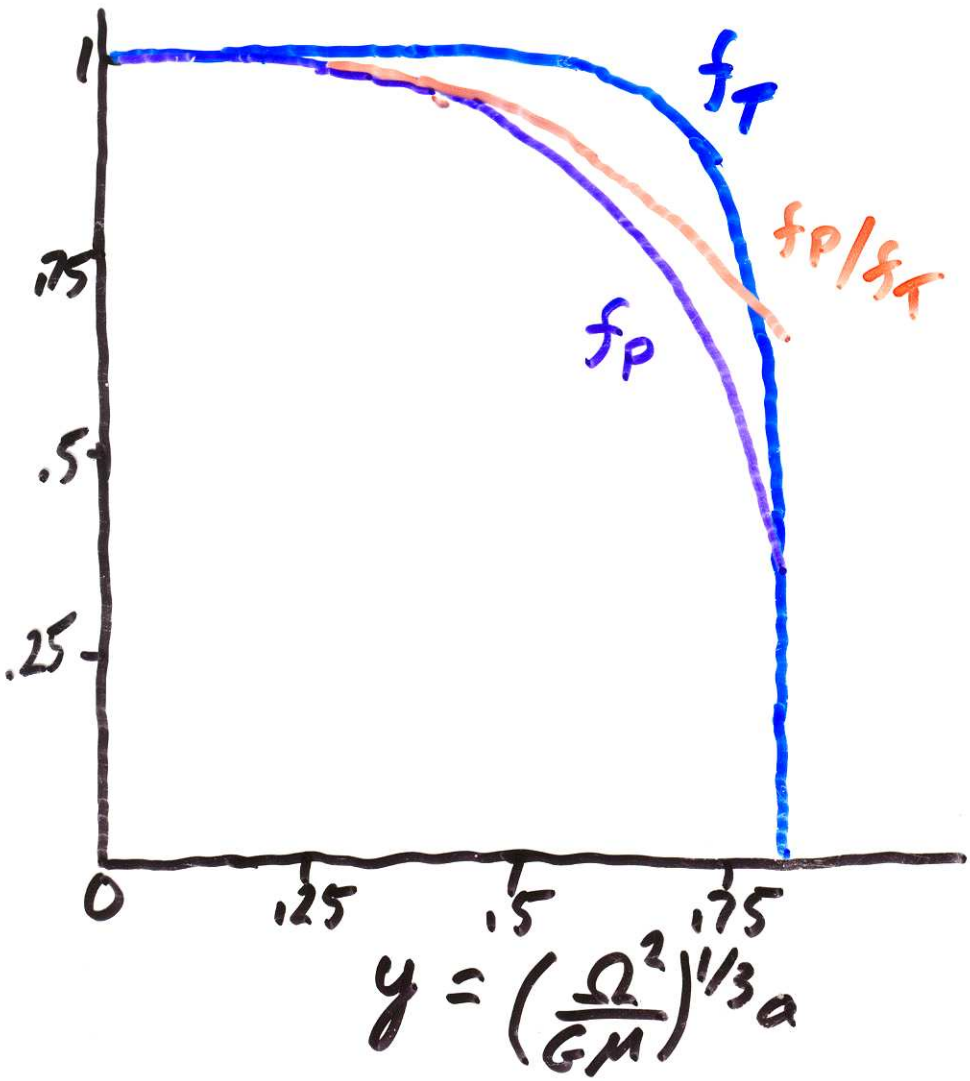
$$\Psi(r,\mu) = -\frac{GM}{r} - \frac{1}{2}\Omega^2 r^2 (1-\mu^2)$$

This method is very good $\forall$ uniformly rotating centrally condensed $\star$s.

<u>N.B.</u> $P_0(\mu) = 1$, $P_1(\mu) = \mu = \cos\theta$, $P_2(\mu) = \frac{1}{2}(3\cos^2\theta - 1)$

In general $P_n(\mu) = \frac{1}{2^n n!} \frac{d^n(\mu^2-1)^n}{d\mu^n}$

i.e. $P_4(\mu) = \frac{1}{4}\left[5\mu^4 + 10\mu^3 - 6\mu^2 - 6\mu + 1\right]$



Kippenhahn & Thomas, "Stellar Rotation"
ed. Slettebak, 1970, p. 24