

Astr 8100 Stellar Structure & Evolution

Introduction: H-R diagram

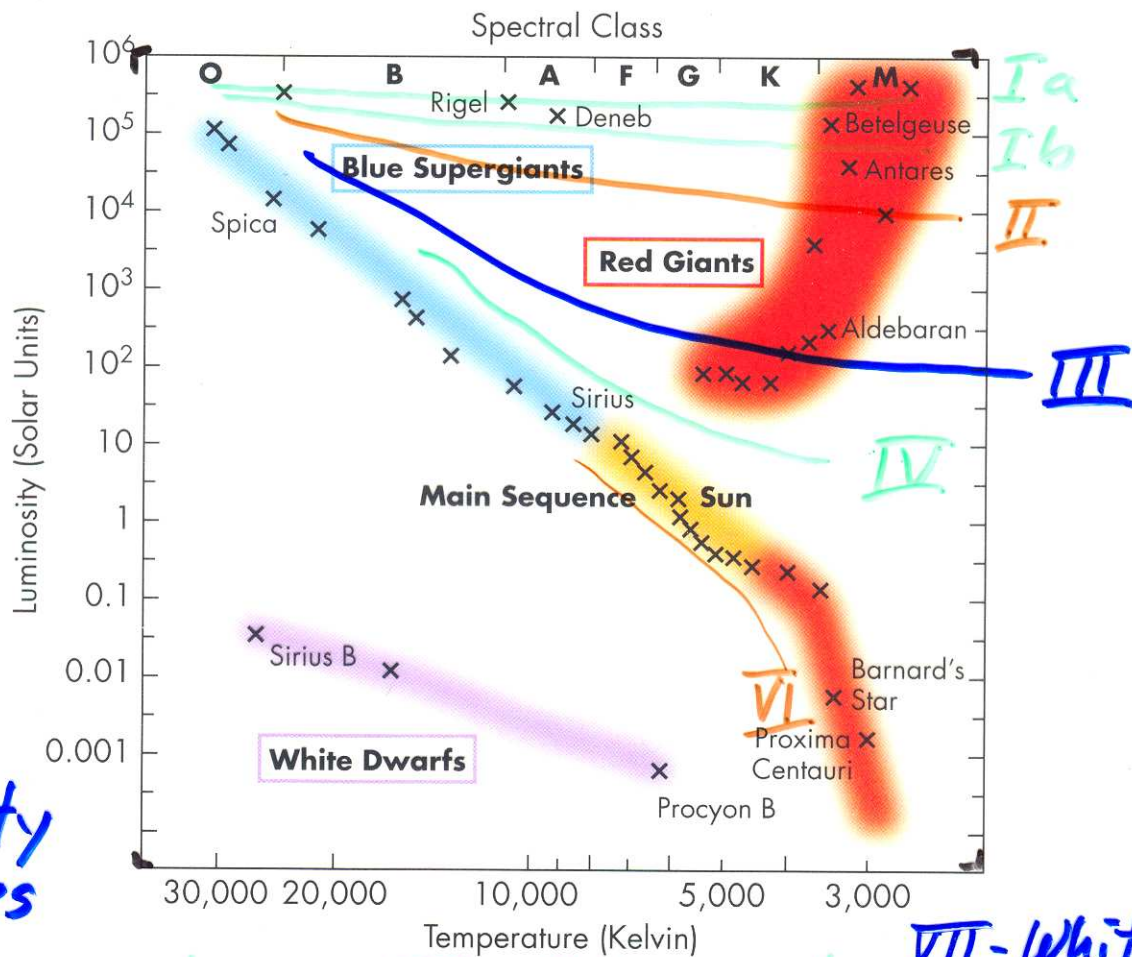
Spectral classes
Luminosity classes
Mass-luminosity relation

Three Sortings:

First: by evolutionary phase

- 1) Pre M-S : first in dynamical collapse,
then quasi-hydrostatic equilibrium
Strong IR source, dust reprocessing.
Variable, strong winds: T Tauri,
FU Orions, jets: Herbig Haro
- 2) M-S : H core burning - longest phase
of $\star$'s life $\Rightarrow$ most populated part
of H-R diagram.
- 3) Advanced phases - Red Giants :
H shell + He core burning - quiet mass ejection
Mira + Cepheid Variables
- 4) Late phases: C flash or core collapse
leading to Supernovae - Dynamical Evol.
- 5) Final phases: WDs, NSs, BHs

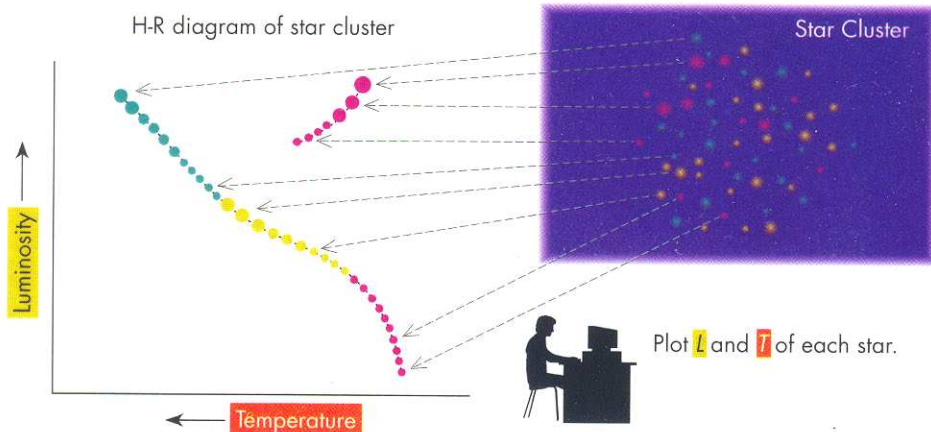
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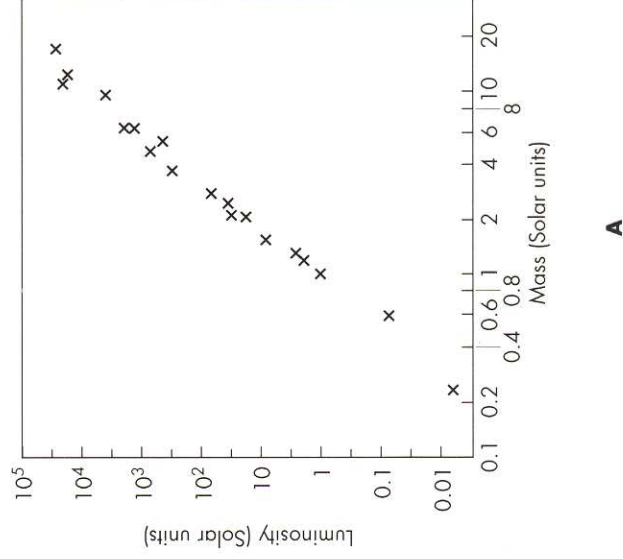
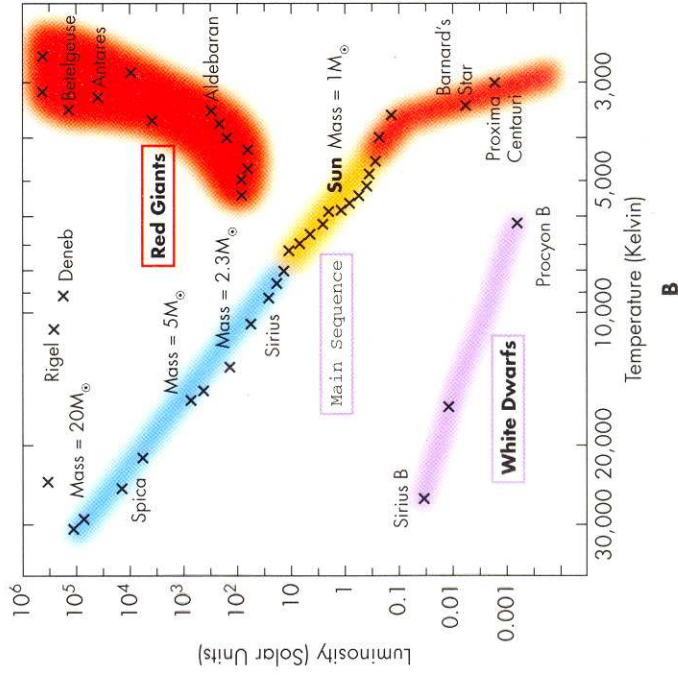
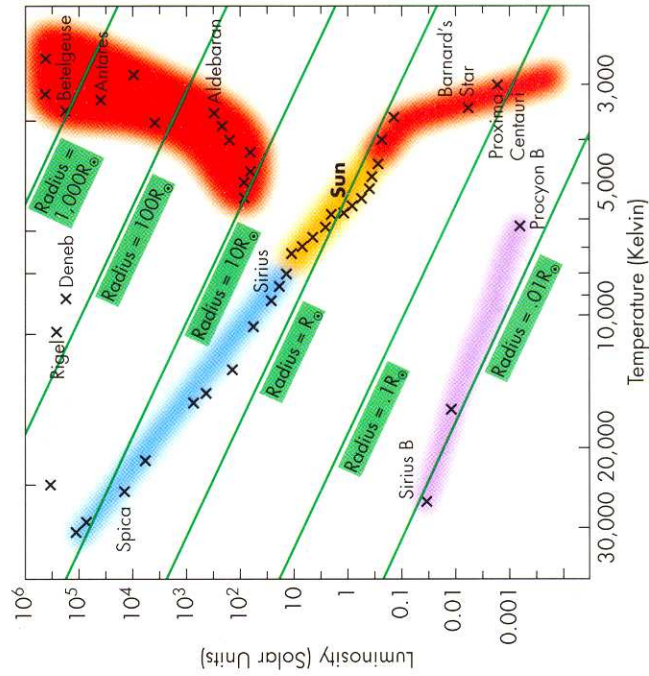


Luminosity Classes

- I - Supergiants
- II - Bright giants
- III - Giants

- IV - Sub-giants
- V - Main-sequence (dwarfs)
- VI - Pop II - subdwarfs
- VII - White dwarfs





92 Constant radii lines, and mass-luminosity relation

Thomas T. Arny, *Explorations: An Introduction to Astronomy*, 2e. Copyright © 1998 McGraw-Hill Companies, Inc. Dubuque, Iowa. All Rights Reserved.

Second sorting: by mass

- 1) Featherweight: too light to fuse H:
 P can stop grav. indep of T. - degenerate
 or solid state forces: $M \lesssim 0.075 M_{\odot}$
 Brown dwarfs (Planets if $< 0.013 M_{\odot} \approx 13 M_J$)
- 2) Light weight: burn H (up to $\sim 0.3 M_{\odot}$)
 or both H + He (up to 6 or 7 $M_{\odot}$)
 Leave behind white dwarfs.
- 3) Middle weight: can burn C but probably
 degenerate $\therefore$ C flash probable - could $\Rightarrow$
 fully disrupted SN or could leave $N\star$ + type I SN
 $(6-7) M_{\odot} < M < (8.5-9) M_{\odot}$
- 4) Heavy weight: C burns non-degenerately -
 many heavy elements produced. Likely that
 many $\Rightarrow$ Type II SN & $N\star$ remnant
 $(8.5-9) M_{\odot} < M < (\sim 30-40) M_{\odot}$
- 5) Superheavy weight: $(\sim 30-40) M_{\odot} < M < (80-120) M_{\odot}$
 Again C burns & onwards to Fe. Should $\Rightarrow$ BTH
 Upper limit - rad'n pressure, pulsational instabilities,
 electron-positron pair instabilities
 $L_{\text{Edd}} \approx 1.3 \times 10^{38} \text{ ergs}^{-1} / M_{\odot}$ while $L_{\star} \propto M_{\star}^{3.5}$. Theory
 says VML $\lesssim 80 M_{\odot}$ but some $\star$ s (e.g. η Carinae) $\sim 100 M_{\odot}$

$$F_{\text{rad}} = \frac{L}{4\pi r^2 c} \quad \text{Force}_{\text{rad}} = \frac{L \sigma_T}{4\pi r^2 c} \sim \frac{8\pi}{3} \left(\frac{e^2}{m_e c^2} \right)^2 = 6.7 \times 10^{-25} \text{ cm}^2$$

$$\text{Force}_{\text{grav}} = \frac{m_H M G}{r^2} \quad \therefore L_{\text{Edd}} = \frac{4\pi c m_H G M}{\sigma_T}$$

3 Third sorting: Chemical Composition (Populations)

- 1) Extreme Pop I - O, B $z \approx 0.025$
- 2) Intermediate Pop I - $\sim 10^8 \gamma$, Pleides $z \approx 0.022$
- 3) Disk Population (I) - $10^9 - 10^{10} \gamma$, most $\star$'s $z \approx 0.020$
- 4) Intermediate Pop II ($\leq 10^{10} \gamma$, High $v \star$'s) $z \approx 0.002$
- 5) Extreme Pop II $\geq 10^{10} \gamma$, very hi $v \star$'s ($\geq 100 \text{ km/s}$) $z \approx 0.0003$
- 6) Pop III - postulated 1st generation $z \sim 10^{-9}$

These are only averages for Pop I + Disk
I variations by ~ 2 depending on position
in galaxy, with composition gradient $\rightarrow$
higher z towards center.

Models must feed individual stellar
evolution into galactic structure
& evolution.

Since lightweight $\star$'s dominate the mass
(most numerous) but not necessarily the
light M/L ratios require calibration:
In some spiral galaxies this is particularly true.
In most S & all E galaxies the bulk of light
comes from red (K) $\star$'s - both RGs + MS
so M/L comes from two classes.

Timescales

4

1) Free-fall or dynamical: dimensional analysis shows: $t_{ff} \sim \left(\frac{R^3}{GM}\right)^{1/2}$

Crudely: $\frac{d^2r}{dt^2} = -\frac{GM}{r^2}$ $\therefore$ over average time of collapse $\frac{d^2r}{dt^2} \approx -\frac{R}{t^2} = -\frac{MG}{R^2}$, again $\Rightarrow t_{ff} = \left(\frac{R^3}{GM}\right)^{1/2}$

For the $\odot$ $t_{ff} \approx 1.6 \times 10^3 \left(\frac{M}{M_{\odot}}\right)^{-1/2} \left(\frac{R}{R_{\odot}}\right)^{3/2} \text{ s}$
 $= 26.6 \text{ min}$ as $M_{\odot} = 1.989 \times 10^{33} \text{ g}$
 $R_{\odot} = 6.96 \times 10^{10} \text{ cm}$

N.B.: Pulsational periods of g-modes have similar periods $\Rightarrow P \bar{\rho}^{1/2} = G^{-1/2}$

2) Kelvin-Helmholtz - extraction of gravitational energy: $W = -\frac{Gmm'}{r}$, integrate: $W \approx -\frac{M^2 G}{R} \approx 4 \times 10^{48} \text{ erg}$

Now $L_{\odot} = 3.86 \times 10^{33} \text{ erg s}^{-1}$ so,

$t_K = \frac{W}{L_{\odot}} \approx 10^{15} \text{ s} \sim 3 \times 10^7 \text{ yr}$ - lifetime of

$\odot$ on stored energy $\Rightarrow$ Not chemical or stored energy.

3) Fusion, w/ $\Delta E \sim 1 \text{ MeV} \sim 10^{-6} \text{ erg} \Rightarrow t_{ev} \approx 10^{10} \text{ y}$

Total energy, $E = M_{\odot} c^2 = 1.79 \times 10^{54} \text{ erg}$,

but efficiency ≈ 0.007 . But only the inner 10% by mass typically can fuse.

Binary Systems ~60% of $\star$ s in binaries

$\Rightarrow$ Fundamental quantities: $M, L, R!$

Visual: real doubles, large separations ("Optical" are false superpositions)

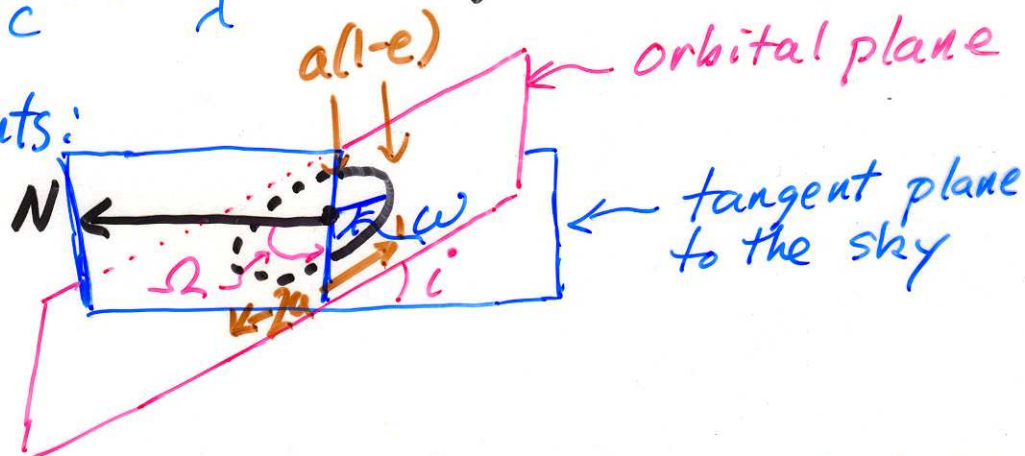
Speckle: closer separations

Eclipsing: large enough inclination & close enough so one $\star$ blocks another $\Rightarrow$ light curves

Spectrum: see two superposed spectra
 $\therefore$ comparable magnitudes but no visible orbital motion

Spectroscopic: shifts in lines due to l-o-s motion
$$\frac{V_{rel}}{c} = \frac{\Delta\lambda}{\lambda}$$
 single- or double-line

Orbital elements:



P = orbital period (days or hours) spectroscopic/eclipsing
(years) visual

i = inclination of orbital plane to tangent plane of Celestial Sphere

a = semi-major axis

e = eccentricity

T = time of periastron

Ω = λ of nodes: position λ of line of nodes
(intersection of orbital & tangent planes)
measured in tangent plane from N through E

ω = angle bet. direction of ascending node
(when $\star$ crosses tangent plane while receding)
and point of closest approach of $\star$'s (periastron)

Visual: ω of secondary; Spectroclipping: ω of primary

$$\text{SO } \omega_s = \omega_p + \pi$$

Absolute orbit: each $\star$ about CM of system, ellipses

Relative orbit (or true orbit): primary @ one focus

Apparent orbit: projected on plane of sky - also
elliptical but primary no longer @ the focus.

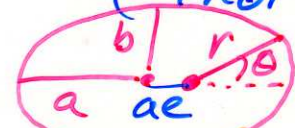
Kepler's 3rd Law: $M_1 + M_2 = \frac{a^3}{P^2}$ with P in yr,
a of true orbit in AU & M's in $M_\odot$

Since most visual & spectroscopic binaries are
close to $\oplus$ their parallaxes can be found

$\Rightarrow d$ (in pc) so $a'' \rightarrow a$

$$P = \frac{365.25 (a/a_\odot)^{3/2}}{(M/M_\odot)^{1/2}} = \frac{0.116 (a/R_\odot)^{3/2}}{(M/M_\odot)^{1/2}} \text{ days} \quad (a_\odot = 1 \text{ AU} = 1.496 \times 10^{13} \text{ cm})$$

Circular orbit: $v = \left(\frac{GM}{a}\right)^{1/2} = 437 \frac{(M/M_\odot)^{1/2}}{(a/R_\odot)^{1/2}} \text{ km s}^{-1}$

Elliptical case: $r = \frac{a(1-e^2)}{1+e\cos\theta}$  for a

single-line system, best is the mass function:

$$f(M_1, M_2, i) = \left(\frac{M_2}{M_1 + M_2}\right)^2 M_2 \sin^3 i = \frac{4\pi^2}{G} \frac{(a_1 \sin i)^3}{P^2}$$

is known $\Rightarrow M_1 + M_2$ known $\Rightarrow$ rel. bet. M_2 & i & bound on M_2
 If $a = a_1 + a_2$ limit on $i \Rightarrow$ bound on M_2

Information Obtainable from Binary Systems

Quantity	Visual	Spectroscopic Single	Spectroscopic Double	Eclipsing
P	✓	✓	✓	✓
a	apparent a''	$a_1 \sin i$	$a \sin i \begin{cases} a_1 \sin i \\ a_2 \sin i \end{cases}$	no
e	✓	✓	✓	✓
ω	✓	✓	✓	✓
T	✓	✓	✓	✓
i	✓	no	no	✓
Ω	✓ Ambiguous w/o radial vel. measurements	no	no	no
M_1	{ Σ is parallax } is known	$f(m)$	$M_1 \sin^3 i$	no
M_2			$M_2 \sin^3 i$	no
R_1	no	Estimated from knowledge of spectrum & luminosity		$r_1 = R_1/a$
Fractional (L ₁) luminosity (L ₂)	✓	Estimated from spectrum		✓
Spectral types	✓	✓	✓	If several colors available
limb darkening u_1	no	no	no	In principle - but only small piece of light curve is affected
u_2	no	no	no	
ellipticity	no	no	no	In principle - in close binaries since distorted star displays more area at quadrature than conjunction

For spect. binaries we measure:

V_0 = radial vel. of CM of system
 K_1, K_2 = half-total range of radial-vel variations of brighter (fainter) star [as long as fainter's spectrum is visible]