THIRD ASSIGNMENT: ASTRONOMY 8100

STELLAR STRUCTURE AND EVOLUTION: DUE 19 MARCH 2008

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You are allowed to form groups of 2 or 3 people to do problem 5. If you do, hand in separate write-ups with a list of collaborators. The other questions should be done independently.

1. Consider the situation in nuclear fusion when all the PP chains operate in equilibrium but CNO reactions can be ignored.

(a) Define

$$\alpha(T, {}^4\text{He/H}) = \frac{\lambda_{34}^2}{\lambda_{pp}\lambda_{33}}\Big(\frac{{}^4\text{He}}{\text{H}}\Big)^2,$$

Show, that with this definition of α, that [5]

$$\frac{PPI}{PPII + PPIII} = 0.25[(1 + 2/\alpha)^{1/2} - 1].$$

(b) Express $\frac{d^4\text{He}}{dt}$ in terms of $\frac{d\text{H}}{dt}$ in this situation. [5]

(c) Show that the production rate of Helium-4 is given by [10]

$$\frac{d^4\text{He}}{dt} = \frac{1}{2}\lambda_{pp}\frac{\text{H}^2}{2}\Big(1 + \frac{2\lambda_{34}{}^3\text{He}_{,eq}{}^4\text{He}}{\lambda_{pp}\text{H}^2}\Big),$$

where the rate due to PP-I is modified by second term within the parentheses due to PP-II and PP-III and ${}^3\text{He}_{,eq}$ is given in Eq. (21) of the nuclear portion of your notes.

(d) Show that by inserting the value of ${}^3\text{He}_{,eq}$ into the previous expression that the ${}^4\text{He}$ production rate can be written as [8]

$$\frac{d^4\text{He}}{dt} = \frac{1}{2}\lambda_{pp}\frac{\text{H}^2}{2}\Phi(\alpha),$$

where $\Phi(\alpha) = 1 - \alpha + \alpha(1 + 2/\alpha)^{1/2}$

2. Recall that during the pp-chains, when ${}^3\text{He}$ is in equilibrium, the ratio of reaction rates is given by:

$$\frac{PPI}{PPII + PPIII} = \frac{-\lambda_{34} + [\lambda_{34}^2 + 2\lambda_{pp}\lambda_{33}(\text{H}/{}^4\text{He})^2]^{1/2}}{4\lambda_{34}},$$

where the λ's were defined in class and α was defined in Problem 1.

You've already (I hope) shown that $PPI/(PPII + PPIII) = [(1 + 2/\alpha)^{1/2} - 1]/4$.

(a) With the following values for the nuclear form factors, $S_{11} = S_{pp}(E_0) = 3.78 \times 10^{-22}$ keV-barns, $S_{33} = 5.0 \times 10^3$ keV-barns, and $S_{34} = 4.7 \times 10^{-1}$ keV-barns, show that: [6]

$$\alpha(T, {}^4\text{He}/H) \approx (S_{34}^2/S_{11}S_{33})({}^4\text{He}/H)^2\text{exp}(-100T_6^{-1/3}) \approx 1.2 \times 10^{17}\text{exp}(-100T_6^{-1/3})$$

(b) Find the value of α for which the production of ${}^4\text{He}$ by the PP-I chain equals the sum of the production rates by PP-II and PP-III. If X = Y, to what temperature does this correspond? [6].

3. (a) Starting from eq. (34) for $\kappa_{ff}(\nu)$ of our lectures on energy transport, verify that the general form of the free-free opacity is Kramer's opacity (our Eq. 35 in the Energy Transport section): [8]

$$\overline{\kappa_{ff}} = \kappa_0 \rho T^{-3.5}.$$

(b). Give physical arguments motivating the dependence of κ_0 on X and Z in the following expression: $\kappa_0 = 3.7 \times 10^{22}(1+X)(1-Z)\overline{g_{ff}}$. [5]

(c). Show (somewhat more explicitly than in the notes leading to Eq. 47 of the Energy Transport section) that

$$\kappa_{bf} = \kappa_0 \rho T^{-7/2},$$

where $\kappa_0 = 4.3 \times 10^{25} Z(1+X)\overline{g_{bf}}/t_{bf}$. [7]

4. Consider the following sources of opacity: free-free; bound-bound; free-bound; electron scattering.

(a) In which types *and/or* parts of stars are each of them *most* important? [5].

(b) Quantitatively compare their contributions to the opacity at the center of the Sun at its current age. [10]

5. Locate at least one stellar structure (and hopefully) evolution program on the Web. Run at least three models on it for main sequence stars of different masses; more (up to 6) would be even better.

(a) Write down the URL and a brief description of your favorite code, including input parameters required and output results provided. Also give the URL of a second option. Do these codes deal with post-main sequence evolution? Do they attempt to deal with pre-main sequence evolution? [10]

(b) Give the results (graphical, tabular, or preferably both) of the three interior models you like the best of those you have run for as many output results that the code you prefer provides. [10]

(c) Discuss the ease of use and, if possible, the accuracy of this code. If you do this for two codes that would be even better. [5]+[5 bonus points for a second code]