FOURTH ASSIGNMENT: ASTRONOMY 8100
STELLAR STRUCTURE and EVOLUTION: DUE 7 APRIL 2008
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1. For the piston problem discussed in class, show that the immediate disappearance of gravity from equation (3) [of our first "chapter" of the evolution portion of the notes], leads to the explosion time-scale (4b) [10]

$$\tau_{exp} \approx h\Big(\frac{\rho M^*}{P m^*}\Big)^{1/2}.$$

2. Assume the Pre-MS core contraction is halted when the temperature reaches a critical value, T_{cr}, where H fusion begins. Also assume that all the initial gravitational energy can go into thermal energy. Show that the greater the mass of the star, the smaller the density of the star at the point T_{cr} is reached, by finding an expression for ρ_{cr} in terms of T_{cr}, M, and constants. [15]

3. Now assume that the Pre-MS core contraction is halted by the onset of degeneracy pressure instead of H burning. For what type of objects is this the correct physical situation? Find an expression for that critical mass below which degeneracy pressure halts star formation and evaluate it in units of solar masses. [15]

4. Thanks to your efforts, several very good web-sites for stellar evolution calculations have been located. Here we shall consider two of the best: the Yale/Dartmouth code site of Brain Chaboyer: stellar.dartmouth.edu; the "advanced" Illinois code site (simpler versions are also available): rainman.astro.uiuc.edu/ddr/stellar/advanced.html. If you haven't already done so, you'll need to set up an "account" on the Dartmouth site. In this problem you'll use the Dartmouth code.

   a) Run evolutionary models for stars of 0.5, 1.0, 2.0 and 5.0 $M_\odot$. Use the output to plot two HR diagrams illustrating the evolution from ZAMS through the last possible stage available in the code that contains all 4 of these stars (use the default composition). One of the HR diagrams should be "theoretical" (L vs. $T_{\rm eff}$) while the other should be "observational" (M_V vs. color index, $B-V$).[10]

   b) What is the current age for the Sun for the default model? How old would a 1.0 $M_\odot$ star with the same Y but Z = 0.002 be when it has the same luminosity as does the sun today? Please give your best attempt at explaining this difference using physical arguments. For each of those stars) in part (a) find roughly at what times does $Y_c = 0.5$ and $Y_c = 0.9$? [10]

   c) Use the interior structure files for your four stars to plot the luminosity generation as functions of mass and radius for the variety of times given below. Explicitly locate the interior points (in mass and radius) by which 0.95 of the total luminosity has been generated at ages of 0.0 Gyr (ZAMS), 0.1 Gyr, 0.5 Gyr, 1.0 Gyr, 5.0 Gyr, and 10.0 Gyr. What nuclear reactions are producing the bulk of the energy at each of those times? [10]

Please turn over

5. Use the Illinois code for this problem.

   a) Compare the luminosities and surface temperatures, at the beginnings and/or ends of various MS and post-MS phases, for stars of 0.5, 0.8, 1.0, 2.0, 5.0, 10 and 25 $M_\odot$. Give the age of each star at each of those points, and plot them on an HR diagram with the stages identified. Note the amount of mass lost during each of those phases. For each star, assume the default (nominally solar) composition. What are the spectral types of each of those stars on the ZAMS? [10]

   b) Redo part (a) using $Z = 0.002$. Explain why the differences in these quantities arise from this change of metallicity. [10]

6. Now choose two masses, different from any you've considered in earlier problems on this assignment, such that both codes can handle both of those masses. Also choose two different compositions. Run both codes for all four models (2 masses each and 2 compositions for each mass). Compare the results and discuss how well these two codes agree. Can you explain (or at least speculate about) the differences between the results? [10]