FINAL EXAMINATION: ASTRONOMY 8100

STELLAR STRUCTURE and EVOLUTION

Spring 2008

Prof. Paul J. Wiita **Due 10:30 AM, Monday April 28th**

READ THESE DIRECTIONS CAREFULLY AND FOLLOW THEM!

This is an open book take-home examination. While you have nearly 12 days to do this exam, I expect that (aside from the last question, which should take about 3 hours by itself), the other questions should be completed within about 8 hours of work. In case this is an underestimate for you be sure to start soon, so as to not run short of time to complete it, and to have time to study for any other, non-take-home, exams. Collaboration between students (current and current or current and former) is not allowed. If you have a question, you must address it to me. I will be in town except for the 18th-20th, and if not in my office, can almost always be reached on my cell phone: 609 273-7177.

Please write on the top of the first sheet you hand in: "I have neither given nor received unauthorized assistance in this examination", followed by your signature.

If you use a fact, equation, etc., that is not from the text or the notes, please be sure to cite the book or journal title, author(s), and (volume and) page numbers. Please start each full question on a separate sheet and put your name on each sheet handed in, even though you will have stapled them together in the correct order. Clearly state any assumptions you make, and clearly label any diagrams you draw.

Each question is worth 10 points, with the exception of No. 10, which is 15 points. Thus there are a total of 105 points, but I will grade this on the basis of 100, so you effectively get a 5 point bonus. Partial credit will be awarded, so do try some of each question even if you cannot complete it correctly and note that legibility is important; if I can't read it, it can't be right! In other words, please recopy your work unless the first version is very neat. **THERE WILL BE NO EXTENSIONS** because I leave for Shanghai on the morning of May 1st and therefore must have the grades posted by April 30th. **If this examination is handed in after 10:30 on April 28th I will deduct 15 points for every hour (or part thereof) it is late. No examination will be accepted after 4:30 PM on April 28th, and you will get a zero for the final if I do not have it by that time;** i.e., if you are going to become ill, or your computer is going to crash, finish the exam early, before any of that happens! If you can give it to me on the 25th (or even earlier) that will be appreciated.

1. Give good estimates for L, $T_{\rm eff}$, T_c, and ρ_c for a 5 $M_\odot$ star at the following times: (a) the maximum luminosity phase of its pre-main sequence life; (b) ZAMS; (c) exhaustion of H at the very center; (d) ignition of He in the core; (e) steady He core fusion on the horizontal branch.

2. Complete the following nuclear reactions: A is the atomic mass and Z the atomic number of each isotope (X or Y). More than one answer will almost always be possible, but choose one that is likely to be the dominant reaction channel. For each case, explicitly state how baryon number, charge, lepton number, and energy-momentum are conserved. Also for each case, write a specific reaction you should be familiar with that is an example of the generic reaction listed.

   (a) ${}^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Y + ...$

   (b) ${}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + ...$

   (c) ${}^{A}_{Z}X + e^{-} \rightarrow$

   (d) ${}^{A}_{Z}X + p \rightarrow$

   (e) ${}^{A}_{Z}X + e^{+} \rightarrow$

3. (a) Consider a body made of a collection of atoms in which both atomic binding and gravitational energies are important. As the body becomes larger, gravity will start to play a bigger role. Show that self-gravity is important if the number of atoms in a body is larger than a critical number,

$$N_{\rm cr} \simeq \alpha^{3/2}(Gm_p^2/\hbar c)^{-3/2},$$

   where the fine structure constant, $\alpha \equiv (e^2/\hbar c)$, and the other symbols have their usual meanings. [7]

   (b) Estimate the corresponding critical mass at which gravity becomes important and express it in units of both grams and $M_{\odot}$. [2]

   (c) What type of astronomical object does this critical mass (and size) most closely resemble? [1]

4. Follow and expand upon — by filling in every step carefully — the discussion in §25.2.2 of K&W for a detailed treatment of vibrational stability for the piston problem. More explicitly, starting from their (25.7) [our stellar evolution eq. 10] derive their (25.14) [our 12, with the σ for $i\omega$ substitution].

5. Suppose a star of radius R has an inactive shell at radius r_s with envelope mass M_e. Denote the envelope's average density by $\overline{\rho} = (\rho_0 + \rho_s)/2$, with ρ_0 the density at the base of the photosphere. Assuming $r_s \ll R$, show that a homologous expansion or contraction throughout the core produces the same type of change in the envelope. Specifically, show that if $\Delta r_s/r_s < 0$, then $\Delta R/R < 0$ too.

6. Follow and expand upon — by filling in every step carefully — the class discussion for a detailed treatment of the core-mass-luminosity relation for larger core masses where the shell is not degenerate but radiation pressure is important. More explicitly, starting from equation (74) on p. 65 of the evolution notes,

$$P = \frac{\mathcal{R}}{\mu}\rho T + \frac{a}{3}T^4 \equiv \frac{1}{\beta}\frac{\mathcal{R}}{\mu}\rho T,$$

and assuming $a = b = 0$, derive equations (75) on p. 66

$$\phi_1 = \frac{4-\nu}{D} \qquad \phi_2 = \frac{\nu - 12 + 6\beta}{D}$$

$$\psi_1 = \frac{1+n}{D} \qquad \psi_2 = \frac{2\beta - n - 3}{D}$$

$$\tau_1 = \beta\phi_1 + (4-3\beta)\psi_1 \qquad \tau_2 = \beta\phi_2 + (4-3\beta)\psi_2$$

$$\sigma_1 = \frac{4n+\nu}{D} \qquad \sigma_2 = \frac{3-\nu-3n}{D}\beta$$

where $D = (4-3\beta)(n+1) + (1-\beta)(\nu-4)$, and

$$\rho \propto M_c^{\phi_1} R_c^{\phi_2}, \quad T \propto M_c^{\psi_1} R_c^{\psi_2},$$

$$P \propto M_c^{\tau_1} R_c^{\tau_2}, \quad \ell \propto M_c^{\sigma_1} R_c^{\sigma_2}.$$

This basically requires following the procedure on pp. 47–49 of the evolution notes.

7. A particular stellar model has a single active shell source whose minimum radius is $r_s = 0.0321 R_\odot$. If its shell and envelope density varies as

$$\rho(r) = \rho_s (r_s/r)^3,$$

and the shell's temperature varies as $T(r) = (r_s/r)T_s$, *find the shell's luminosity*, supposing that

$$\epsilon(r) = 3.98 \times 10^{-26} \rho X^2 T^{3.5} \mathrm{erg/g/s},$$

for $r \geq r_s$, that the hydrogen abundance, $X = 0$ in the core, but that $X = 0.73$ in the shell (to begin with). Assume $T_s = 2.13 \times 10^7$K and $\rho_s = 930$ g cm^{-3}.

(a) What is the thickness of the region from which 99% of the shell's energy originates? [4]

(b) What is the mass in that dominant part of the shell? [4]

(c) Roughly how long can the star provide luminosity in this fashion? (For this last question, do not worry about the gradual decline in X; just estimate the shell burning time based upon the initial shell luminosity and fuel supply.) [2]

8. (a) All stars lose mass via their production of energy through fusion. Express the mass loss rate for the sun from this process (while on the MS) in terms of $M_\odot$ yr^{-1} and g s^{-1} [2].

(b) First consider two stars which currently have equal masses, M_2, but where one has always had this constant mass while the other has lost mass at a constant rate $dM/dt = -\zeta M_\odot$ yr^{-1}, starting from $M(t = 0) = M_1$. (Ignore the minimal mass loss arising directly from special relativity discussed in (a).) The latter clearly has evolved faster than the former since it started with a higher mass. At some time τ when $M(\tau) > M_2$ both stars will have converted equal amounts of H into He, so that they have radiated equal total amounts of energy in their lifetimes. Assume that you can use a power-law MS mass–luminosity relation with index $\alpha = 3.7$ to relate the luminosities at any time, and take $\zeta = 1.0 \times 10^{-7} M_\odot$ yr^{-1}.

Find the initial mass M_1 that would evolve down to $5M_\odot$ after $t = 6.8 \times 10^7$ yr, the approximate lifetime of a $5M_\odot$ star on the MS. Then find the time τ required for that mass-losing star to radiate the same total amount of energy as would be emitted by the constant mass $5M_\odot$ star during its entire MS lifetime. [8]

9. Assume that the energy generation rate in a late stage of stellar evolution is given by: $\epsilon_{\rm Si} = 3 \times 10^9$ erg s^{-1} g^{-1}, and that this Si-core contains 2.0 M$_\odot$ and can be approximated by a constant density of 7.0×10^6 g cm^{-3}.

(a) What is the core luminosity due to this nuclear fusion? [3]

(b) Assuming all of this energy could be used to expel a 7.0 M$_\odot$ envelope consisting of all the material above this core, how long would this reaction have to continue? [5]

(c) What is the most important loss process ignored in the assumption you were asked to take in (b), and how realistic is your answer for (b)? [2]

10. Carefully read at least three of the term papers submitted by your colleagues in this class. They will be posted to my website as soon as I have electronic versions of them, which had better be no later than this Friday, April 18th.

For any three of these papers, write a one page critique of each, emphasizing both what you thought were the best aspects of the paper and what you found to be obscure or unenlightening. You may also comment upon style, grammar or anything else that strikes your fancy, but do not exceed one typed page (single spaced) per paper.

Finally, give a grade to each of those three papers (on a 1-100 scale, with 90 the floor for an A, 80 a B, 70 a C, and 60 a D). The person who comes closest to my evaluation of each paper (for which there are at least two discussants) will get a bonus point, so if you are clairvoyant (and/or a good judge of quality) you could conceivably earn three bonus points.