FIRST ASSIGNMENT DUE: JANUARY 25, 2010

RELATIVISTIC ASTROPHYSICS, ASTRONOMY 8700

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Each question is worth 10 points. You may collaborate on questions 4, 5, 6, and 7, which are marked with an asterisk, but if you do so you must write down which other student(s) you worked with on each problem for which you collaborate. Note that you really should do more of the problems at the end of each chapter if you really want to learn the material well.

0. Read Chapters 1 and 5 of Hobson, Efstathou & Lasenby. Note the 1.5 pages of Errata at the very end of the book and use them to correct the actual text. As you read this first edition, please bring any other things you think are errors to my attention. I will check them and forward any errors you have caught to the authors for a second edition.

1. A straight rod lies in the xy plane of a Euclidean coordinate system. Draw a labeled diagram showing the rod in the xy plane and also label the projections of this rod on a rotated $x'y'$ system. Give an explicit analogy between the x components of the length of this rod as measured in the two rotated Euclidean frames and the different lengths of a rod measured in a lab frame and in a rocket frame in which the rod is at rest. Also give an explicit analogy between time dilation and the y components of the length of this rod as observed in the two rotated Euclidean systems. What are the Euclidean and Lorentz invariants?

2. Establish the fact that if v_{AB} and v_{BC} are small compared to c, then the composition law for velocities reduces to the standard additive law of Newtonian theory.

3.* At any instant there is just one plane in which clocks in both the laboratory and rocket frames agree. Show that the velocity of this plane in the lab frame is $\tanh(\theta_r/2)$, where θ_r is the relative velocity parameter (or rapidity parameter) between the two frames.

4.* Two γ-rays of different energies collide in a vacuum, yielding an e^+e^- pair. For what ranges of energies of the two γ-rays and for what range of angles between their initial directions can this pair-creation occur? *Please turn over!*

5.* A meter stick lies parallel to the x axis and moves in the y direction in the lab frame with speed β_y.

(a) Explain (without using any equations at first) why the stick is tilted upward in the positive x' direction in the rocket frame.

(b) Now let the center of the meter stick pass the point $x = y = x' = y' = 0$ at a time $t = t' = 0$. Calculate the angle θ' at which the meter stick is inclined to the x' axis in the rocket frame.

(c) When and where does the right end of the meter stick cross the x axis as observed in the lab frame? When and where does the right end of the meter stick cross the x axis as observed in the rocket frame?

6.* The Lorentz transformations have no meaning if $v > c$ or $\beta > 1$; this is a mathematical consequence of the physical fact that mass, energy, and information cannot go faster than light. But there are some apparent paradoxes you should consider, and here are two of them:

(a) *The scissors paradox.* A very long straight rod, inclined at an angle ϕ to the x axis, moves downward with a uniform speed β^y. What is the speed β_A of the point of intersection (A) of the rod with the x axis? Can this speed be greater than light? If so, can it be used to transmit a message from the origin to someone far out on the x axis? Explain.

(b) *The searchlight paradox.* A powerful searchlight is rotated rapidly so that its beam sweeps out in a plane. Observers A and B are on the plane and equidistant from the searchlight but not close to each other. How far from the searchlight must A and B be so that the searchlight beam will sweep from A to B faster than a light signal could travel from A to B? Before they went to their distant positions, A was told to fire a bullet at B as soon as she sees the searchlight beam; B was told to duck when he sees the searchlight beam because A has shot at him. Doesn't this imply a warning having gone from A to B at a speed faster than c? If not, why not?

7.* Problem 1.5 in Hobson, Efstathiou & Lasenby (HEL). Note that the (3,0) element of the transformation matrix should be $-\gamma\beta_z$, not $-\gamma\beta_y$.

8. Problem 1.6 in HEL

9. Problem 5.5 in HEL

10. Problem 5.9 in HEL