SECOND ASSIGNMENT, DUE: 15 FEBRUARY 2010
ASTR 8700: RELATIVISTIC ASTROPHYSICS & COSMOLOGY

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Problems 1–4 are worth 10 points each. Problems 5–8 are worth 15 points each. You are not to collaborate on the first 4, but may do so on the last 4. If you do collaborate, be sure to list all members of your collaboration. (Collaboration includes asking or giving advice or sharing ideas; it doesn't mean copying someone else's work.)

0. Read Chapters 2, 3 and 4 of HEL if you haven't done so already.

1. A space traveler A' travels through space with uniform acceleration g (for maximum comfort). Find the distance covered in 24 years of A''s time. [Hint: first express g in units where time is measured in years and distances in light years.] On the other hand, A'' describes a straight double path, from location X (on the Earth) to Z and back to X, via location Y (i.e., XYZYX) with acceleration g on the XY and ZY legs, and decelerations of equal magnitude on the YZ and YX legs for six years each. Draw a space-time diagram for A'' as seen from the Earth, and find by how much someone living on the Earth would have aged in 24 years of A'''s time.

2. Exercise 4.2 in HEL

3. Prove that the Kronecker delta, δ^i_j, is a tensor. Also prove that it is a constant, or numerical tensor; i.e., it has the same components in all coordinate systems.

4. Prove the *quotient* law: If the following relation holds in all frames:

$$T_{ik} = A_i B_k$$

where T_{ik} is a tensor and A_i is an arbitrary non-zero covariant vector then B_i is also a covariant vector.

5. Write down the change of coordinates from Cartesian $(x^a) = (x, y, z)$ to spherical polar coordinates $(x'^a) = (r, \theta, \phi)$ in Euclidean 3-space, $\mathcal{R}^3$. Then obtain the transformation matrices $[\partial x^a / \partial x'^b]$ and $[\partial x'^a / \partial x^b]$ expressing them **both in terms of the primed coordinates**. Then find the Jacobians J and J' (the determinants of those two matrices, respectively). Where is J' zero or infinite?

6. (a) The line element of $\mathcal{R}^3$ in cylindrical polar coordinates is

$$ds^2 = dR^2 + R^2 d\phi^2 + dz^2.$$

Find g_{ab}, g^{ab} and g for this coordinate system and compare them with the values for Cartesian coordinates.

(b) Find the geodesic equation for $\mathcal{R}^3$ in cylindrical polar coordinates.

7. Exercise 3.6 in HEL

8. Exercise 4.9 in HEL