On the Variability Coherence Observed in Black Hole Candidates at Different X-Ray Energies

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ABSTRACT

The bright spot model explains in a natural way the variability coherence observed in black hole candidates at different X-ray energies.

Subject headings: accretion, accretion disks — black hole physics — X-rays: general — X-rays: stars

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1. Introduction

Recently, Vaughan and Nowak (1997, hereafter VN) have convincingly demonstrated that temporal variability is strongly coherent between different X-ray energies (channels) in some black hole sources. In particular, VN examined the *Ginga* X-ray variability data for two galactic black hole candidates, GX 339-4 and Cyg X-1. Using a powerful mathematical method to estimate the coherence function (Bendat & Piersol 1986), they demonstrated that a strong coherence between X-ray bands is indeed present in both sources over a fairly wide range of temporal frequencies.

For most of the current theoretical models for the X-ray variability, the presence of such a strong coherence is highly surprising. VN clearly demonstrated that the observed coherence could not be explained by shot noise, thermal flares, or simple models involving multiple flaring regions. They have shown that the coherence will be lost whenever there is a nonlinear transfer function between two channels or whenever there are multiple uncorrelated (linear or not) transfer functions between two channels. With the possible exception of a space-time separable model based on slowly inward propagating thermal waves on an accretion disk (Manmoto et al. 1996), at

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least one of these coherence destroying effects occurs for of all the X-ray variability models that have been discussed in their paper. Such behavior implies that none of these models could be physically correct: thay are all inconsistent with the observed coherence, even if they can provide fits to energy spectra and power-density spectra.

In this paper we show that the *bright spot model* naturally accounts for the observed coherence. This particular model was not discussed in the VN paper. The bright spot model was suggested by Abramowicz et al. (1989) for X-ray variability of AGN and independently by Wiita et al. (1991) in the context of rapid optical variability of AGN. The model is based on the fact that the surface of an accretion disk near the black hole is unlikely to be smooth and steady. Instead, the variable, non-axially symmetric surface brightness distribution should resemble a collection of transient "bright spots". The irregular intrinsic emission from these spots, when convolved with the rapid, relativistic rotation of the disk surface, yields variability in the observed brightness of the disk emission. Orbital motion of a single spot with a lifetime longer than the orbital period at the spot location produces a periodic pattern of variability due to the Doppler effect. At high inclination angles of the line-of-sight to the disk normal, gravitational lensing and self-eclipsing add substantially to the observed variability. The resulting light curves appear erratic because differential rotation of the disk mixes individual patterns together. The model explains quantitative details of the variability energy spectra and power density spectra (Abramowicz et al. 1991; Zhang & Bao 1991; Bao 1992; Chakrabarti & Wiita 1993; Green, McHardy & Lehto 1993; Lawrence & Papadakis 1993; Mangalam & Wiita 1993). The observed anti-correlation between X-ray luminosity and the variability amplitude (McHardy 1989; Green et al. 1993) can also be explained using the bright spot model (Bao & Abramowicz 1996), although other explanations for this anti-correlation are possible (Green et al. 1993).

Using the same terminology as did VN, one may say that some coherence should be expected in the bright spot model, because different energy bands are influenced exactly the same way by Doppler effects, gravitational lensing and eclipses: for each of these physical pieces of the model the transfer function is linear. However, for the realistic situation where many spots are present and contribute to the observed fluxes, one should demonstrate that the superposition of these effects can be described by a correlated (between channels) transfer function.

2. Variability Coherence in the Bright Spot Model

We briefly review a few points on the coherence function and its related formulas (Bendat & Piersol 1986) that are necessary to discuss the variability coherence between energy bands in the bright spot model. Let $x(t)$ and $y(t)$ be two noiseless time signals. We denote by $G_{xx}(f)$ the autospectral density function of $x(t)$, by $G_{yy}(f)$ the autospectral density function of $y(t)$, and by $G_{xy}(f)$ the cross-spectral density function between $x(t)$ and $y(t)$; f denotes the temporal frequency. In the standard notation of Bendat & Piersol (1986), the *coherence function* is,

$$\gamma_{xy}^2(f) = \frac{|G_{xy}(f)|^2}{G_{xx}(f)G_{yy}(f)}, \tag{1}$$

and the *cross-spectral inequality* reads

$$|G_{xy}(f)|^2 \le G_{xx}(f)G_{yy}(f). \tag{2}$$

Therefore, $0 \le \gamma_{xy}^2(f) \le 1$. From the above relations it follows that if $G_{xx}(f)$ and $G_{yy}(f)$ are related by a linear transform, then $\gamma_{xy}^2(f) = 1$. Coherence is therefore a measure of the fraction of the root-mean-square amplitude of one process at a given frequency that can be predicted from the other process by a linear transform.

Because in the bright spot model these processes occur at the rotating disk surface, one should take into account how the disk rotation influences the *observed* processes $x(t)$ and $y(t)$ at different energy channels. Here we give only a very simple estimate of this effect to show that strong coherence is indeed expected in the bright spot model for a particular, self-similar, distribution of spots. The same self-similar distribution was previously found to fit other observed properties of the X-ray variability: the range of the observed power indexes in the variability power spectra (Zhang & Bao 1991; Mangalam & Wiita 1993), and the luminosity – variability amplitude anticorrelation (Bao & Abramowicz 1996). Physically, such a self-similar distribution of spots could be generated as a result of self-organized criticality, of the types discussed by Abramowicz et al. (1992) and Mineshige, Ouchi & Nishimori (1994; also Xiong et al. 1997).

Consider two spots located, respectively, at radii r_i and r_k. (Generalization of the following discussion to a distribution of many spots is straightforward.) Both of them emit X-rays at two energies E_1 and E_2. At the energy E_1 the signal is $x(t) = x_i(E_1) + x_k(E_1)$, and at the energy E_2 the signal is $y(t) = y_i(E_2) + y_k(E_2)$. For the coherence function to be unity, $\gamma_{xy}^2(f) = 1$, it is sufficient that

$$x(t) = Ay(t) \tag{3}$$

be true for the whole observed time series where A is a constant. The above condition requires that the radiation received from each source satisfies

$$x_i = Ay_i, \quad x_k = Ay_k, \tag{4}$$

or, in another expression,

$$\frac{x_i}{y_i} = \frac{x_k}{y_k} = A. \tag{5}$$

First, let us consider the case where the two sources have equal intrinsic intensities at all energies of interest, and the variability is purely due to Doppler (rotation) and gravitational (bending, lensing) effects. The Doppler effect is proportional to factors of $1 \pm v/c$, and it does not depend on the energy of the photons radiated by the source. Photon behavior in gravitational fields is also energy independent; thus we have

$$x_i = y_i, \quad x_k = y_k, \tag{6}$$

with $A = 1$. Condition (5) is naturally satisfied. So, the radiation from the simplest relativistic "spots" at different energies has unity coherence.

Second, if each source radiates thermal photons then we have

$$x_i = B\frac{E_1^3}{e^{E_1/kT_i} - 1}, \quad y_i = B\frac{E_2^3}{e^{E_2/kT_i} - 1}, \tag{7}$$

$$x_k = B\frac{E_1^3}{e^{E_1/kT_k} - 1}, \quad y_k = B\frac{E_2^3}{e^{E_2/kT_k} - 1}, \tag{8}$$

where B is a constant. Therefore,

$$\frac{x_i}{y_i} = \left(\frac{E_1}{E_2}\right)^3 \frac{e^{E_2/kT_i} - 1}{e^{E_1/kT_i} - 1}, \tag{9}$$

and

$$\frac{x_k}{y_k} = \left(\frac{E_1}{E_2}\right)^3 \frac{e^{E_2/kT_k} - 1}{e^{E_1/kT_k} - 1}, \tag{10}$$

Obviously, condition (5) is not satisfied in this case and thus the radiation at different energies from thermal spots (e.g., Mangalam & Wiita 1993) will not show unity coherence (cf. VN), even when modulated by Doppler and gravitational effects.

Third, suppose that the source radiates X-ray photons in a power-law distribution. Then we have

$$x_i = g(r_i)E_1^{\alpha(r_i)}, \quad y_i = g(r_i)E_2^{\alpha(r_i)}, \tag{11}$$

for the first source, and

$$x_k = g(r_k)E_1^{\alpha(r_k)}, \quad y_k = g(r_k)E_2^{\alpha(r_k)}, \tag{12}$$

for the second source. Here $\alpha(r)$ is the power index of the radiation and $g(r)$ is a coefficient that may depend on radius. In the case described by (11) and (12) one has,

$$\frac{x_i}{y_i} = \left(\frac{E_1}{E_2}\right)^{\alpha(r_i)}, \quad \frac{x_k}{y_k} = \left(\frac{E_1}{E_2}\right)^{\alpha(r_k)}. \tag{13}$$

Thus, there are two possible sub-cases: (i) if α is a function of r then, in general, there is no coherence; (ii) but, if α does not depend on r there is coherence with $\gamma^2_{xy}(f) = 1$. While it is unlikely that α is completely independent of r, it is certainly plausible that this dependence be very weak and therefore strong coherence between nearby X-ray bands can be retained. Models of X-ray emission with α being independent of r are often considered in modeling black hole condidates (e.g., George & Fabian, 1991). We note that having α independent of r is also natural in the self-similar clumpy disk discussed by Mineshige et al. (1994).

3. Conclusions

The variability coherence observed in several black hole candidates at different X-ray energies can be explained in terms of the bright spot model. The model assumes a non-axially symmetric distribution of the intrinsic brightness at the accretion disk surface. Differential rotation of the disk induces, through the Doppler and gravitational lens effects, variability of the observed brightness. The Doppler and gravitational effects (as well as possible self-eclipses of inner regions by a disk whose thickness grows faster than linearly with its radius) influence the brightness in the same way in all X-ray energy channels emerging from the same region. Variability coherence should be expected if there is no strong radial dependence of the energy spectrum emitted by such spots. We show that, in fact, strict coherence is preserved for spots emitting X-rays according to a power law, $I(E) \propto E^{\alpha}$, as long as the power-law index α is independent of radius. This type of emissivity is predicted for many physical models of the disk.

As the observed high coherences between bands are inconsistent with most alternative models, the data on Cyg X-1 and GX 339−4 analyzed by Vaughn & Nowak (1997) may be taken as supporting the bright spot model.

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