Self-organized criticality in accretion disks

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The concept of self-organized criticality has gained substantial favor as a way of understanding many disparate physical systems containing fluctuations which lack characteristic length and time scales. A brief review of some astrophysical applications of this idea is given. The possibility that coherent structures and observable variability are produced in accretion disks through a self-organized critical state is considered. New simulations indicate that while this model does produce light-curves and power-spectra for the variability which fit observations, the universality of self-organized criticality is not achieved. This is because the character of the variations depend quite substantially on the ratio of the inner to outer radii of the unstable disk region.

1. Introduction

Coherent structures in accretion disks can be produced through the growth of specific hydrodynamical mechanisms. Vortices (e.g. Abramowicz *et al.* 1992; Spiegel, this volume) and shocks induced by companion objects (e.g. Sawada, Matsuda, & Hachisu 1986; Chakrabarti & Wiita 1993) provide such physical mechanisms that can yield significant long-lived (multi-orbit) perturbations on disks.

Such coherent structures can produce "bright-spots" on the disk surfaces which, when coupled with the disk's differential rotation and gravitational lensing of photons passing close to the central black hole, can reproduce many aspects of the variations seen in galactic black hole-candidates and active galactic nuclei (AGN). The X-ray light curves and power spectra (Abramowicz *et al.* 1989, 1991; Zhang & Bao 1991), optical/UV light curves and power spectra (Wiita *et al.* 1991; Mangalam & Wiita 1993) can both be understood within this bright spot model. The anti-correlation between X-ray variability and luminosity in AGN confirmed by Lawrence & Papadakis (1993) can also naturally be explained using this model (Bao & Abramowicz 1996). Finally, it has recently been demonstrated that the surprisingly strong coherence in the temporal variability seen between different X-ray energies for galactic black hole binaries (Vaughn & Nowak 1997) is also capable of being understood within the bright-spot framework (Abramowicz *et al.* 1997). This scenario also predicts energy dependent polarization variations in the X-ray bands (Bao, Wiita & Hadrava 1996; Bao *et al.* 1997) which could be detected by future X-ray satellites with polarimetric capabilities.

Another way to produce extended coherent structures in disks is available if accretion disks exhibit self-organized criticality (SOC). This concept arose from the realization that a huge range of physical systems display power-law or scale-invariant correlations over decades in time or space. "Flicker noise" or "1/f noise" is seen in a huge range of physical systems, ranging from stock market prices, through the height of the Nile river, to quasar light curves (e.g. Press 1978). Bak, Tang & Wiesenfeld (1987, 1988) showed extended systems with many metastable states can naturally evolve into a critical state with no fundamental length- or time-scales.

In § 2 we first discuss characteristics of the SOC state, and then we describe astrophysical applications. In particular, in § 3 we indicate how this might work in accretion disks, as first proposed by Mineshige, Ouchi & Nishimori (1994). In § 4 we display additional computations of modified SOC models for disks which display a goodly range of light curves and variability power spectra. We find that the slopes of the power spectra are quite sensitive to the ratio of the inner and outer disk radii.

2. Characteristics of self-organized criticality

The standard toy model for self-organized criticality is a sandpile, and the original papers by Bak *et al.* (1987, 1988) performed numerical simulations relevant to this physical model, as reduced to a cellular automata problem. The idea is to drop grains of sand randomly onto a pile until it builds up to a critical slope or "angle of repose". Once the pile is at SOC, the addition of a single extra grain of sand anywhere causes the local slope to be too steep. This leads to a small avalanche which readjusts the local shape to just below critical. But now the moved sand will steepen the neighboring slope, perhaps making it, in turn, too steep. Avalanches continue until the local slope everywhere adjusts to be at, or just below, the critical value.

The system becomes stationary when perturbations can just propagate the length of the system, and a full-fledged SOC state of any physical system has the following characteristics:

- a distribution of minimally stable regions of all sizes;
- small perturbations can yield avalanches of all sizes;
- the lack of any characteristic length scale produces a featureless power-law spectrum of avalanche sizes;
- the critical state is insensitive to initial conditions.

However, different physical systems have different power-law indices for the relation between the number of events (avalanches) that occur with different sizes or strengths. These indices depend on (Bak *et al.* 1988; O'Brien *et al.* 1991): the number of spatial dimensions; the symmetry of the system; and the attractor of the dynamics of the system.

The mathematics of the one-dimensional sandpile model can be expressed very simply (Bak *et al.* 1988). This formulation can be easily generalized to two and three dimensions and to more complex phenomena, so it is worthwhile summarizing here how such a simple simulation is performed.

Let $h(x)$ be the integer height at a lattice site, x, and the slope,

$$z(x) \equiv h(x) - h(x+1). \tag{2.1}$$

Add a grain at x and then: $z(x) \longrightarrow z(x)+1$, while: $z(x-1) \longrightarrow z(x-1)-1$.

If z exceeds z_c then the slope is too steep and the site is unstable. Then a grain moves to the next position, so that:

$$z(x) \longrightarrow z(x) - 2, \tag{2.2a}$$

$$z(x \pm 1) \longrightarrow z(x \pm 1) + 1. \tag{2.2b}$$

Now, if the site $x+1$ becomes unstable after the transfer (with $z(x+1) > z_c$), then the process is iterated until either all locations are stable or the end of the grid is reached. For two-dimensional sandpile models, Bak *et al.* (1988) found the power spectral density for the Fourier transform of the time-series to be $S(f) \propto f^{-1.57}$; for three-dimensional models they obtained $S(f) \propto f^{-1.08}$.

2.1. *Application to solar flares*

The first astronomical application of this idea of which we are aware was the proposal by Lu & Hamilton (1991) that the solar coronal magnetic field is in a SOC state. They noted that observations of hard X-ray bursts in solar flares have a distribution that is a power-law in peak photon flux with logarithmic slope 1.8 over up to five decades (e.g.

Dennis 1985). This relation, $N(P) \propto P^{-1.8}$, is essentially independent of solar cycle phase.

The Lu & Hamilton model assumes that the flares are avalanches of smaller magnetic reconnection events and that all flares arise from the same physical process. Therefore the size of an observed flare depends on the number of small reconnection events of which it is comprised.

A lattice picture in three dimensions assumes the local field gradient is destroyed by reconnection when the local magnetic gradient is too large, as argued by Parker (1988). The magnetic gradient vector of a lattice point which exceeds this limit is eliminated at that particular location, but the gradient is shared with neighboring lattice points. In order for this model to work, changes in magnetic field must have a directionality (i.e., photospheric motions should on average increase B in a particular direction, a not unreasonable assumption over substantial areas).

Lu & Hamilton's computations were performed on a 30^3 grid. The results can be summarized as follows: energy release goes as $N(E) \propto E^{-1.4}$; peak flux goes as $N(P) \propto P^{-1.8}$ (as observed). Lu & Hamilton (1991) argue that these power-law indices are rather insensitive to the critical field gradient. While this picture is not clearly well motivated physically, it does incorporate some viable assumptions and does produce a way to understand an otherwise unexplained relation between the size and frequency of solar flares. Additional physical conditions and implications of this idea have been explored by Lu (1995).

2.2. *Application to gamma-ray bursts*

The possibility that a similar "avalanche" or "chain reaction" may play an important role in understanding details of gamma-ray bursts has been proposed by Stern & Svensson (1996). They note that while the time profiles of γ-ray bursts (GRBs) show a very wide range of behaviors, there are several statistical properties that have specific mathematical structures. In particular, the average peak-aligned profile and the autocorrelation function show stretched exponential behaviors. As for other SOC-type processes, this could imply that the wide range of GRB characteristics could arise from different random realizations of a simple stochastic process which is scale invariant in time.

The Stern & Svensson (1996) model is based upon a pulse avalanche, where a trigger event induces many additional flares by setting off a "chain reaction" in a near-critical regime. Such a basic picture was shown to fit both the diversity of GRB time-profiles and important average statistical properties of GRBs, such as their third-moment, their autocorrelation function and their duration distribution function. Similarly to Lu & Hamilton (1991), Stern & Svensson (1996) suggest that magnetic reconnection in a turbulent medium is a plausible underlying mechanism, one that can fit within an exploding fireball model for GRBs.

3. Applications to Accretion Disks

Mineshige, Ouchi & Nishimori (1994) first proposed that accretion disks around black holes could be in an SOC state. This could provide physical connections between bright spots (or determine the size thereof) and thereby engender coherent structures for interesting lengths of time. This was done in an attempt to produce the steeper than $\sim 1/f$ fluctuations seen in X-ray power-spectrum densities (PSD) for BH candidates such as Cyg X-1 (e.g. Miyamoto *et al.* 1993).

Their cellular automata model is based on the following prescription. The outer disk is smooth enough to not produce any variability; however, the inner disk is subject to some, not yet physically constrained, instability. (Mineshige *et al.* suggest this could be to flares in the corona, presumed to lie above and below the disk, in analogy with Lu & Hamilton's picture.) The unstable inner disk is divided into zones (i, j) characterized by radial (r_i) and azimuthal (ϕ_j) coordinates. To model ordinary viscous accretion, the outer disk feeds a mass particle, m, to a zone in the outermost ring of the inner disk, r_1, with random ϕ. In lieu of more developed physics, they assume $M_{\mathrm{crit},i} \propto r$. If the mass contained in the $(i,j)^{th}$ zone, $M_{i,j} > M_{\mathrm{crit},i}$, they then assume enough mass is dumped from (i, j) to bring $M_{i,j} \leq M_{\mathrm{crit},i}$. This excreted mass is taken to spread to the three nearest zones one ring interior to the destabilized zone (for simulations corresponding to rigid rotation) or to three trailing interior zones (as a simple way of mimicking differential rotation).

Once the mass has been dumped from an overloaded zone into others closer to the central mass, one must test those adjacent interior zones for $M_{i+1,(j-1,j,j+1)} > M_{\mathrm{crit},i+1}$ (these second subscript choices pertain to rigid rotation). If this condition is fulfilled, one continues the avalanche by dumping mass into next ring inwards. One estimates the luminosity for each dumped m by:

$$L_x \simeq (GM_{BH}m)\Big(\frac{1}{r_f} - \frac{1}{r_i}\Big), \tag{3.3}$$

where M_{BH} is the mass of the central black hole, and r_i and r_f are the initial and final radii experienced by that particular mass unit during that particular flow inwards. Then one sums up for the energy released by all avalanching masses. Finally one reinjects mass at the outer edge of the inner disk and repeats the above procedures. For their canonical disk model, Mineshige *et al.* (1994) discover approximate scalings where the number of events vary with their energy as $N(E) \propto E^{-1.35}$, the lifetime distribution varies as $N(\tau) \propto \tau^{-1.7}$, and the PSD follows $S(f) \propto f^{-1.8}$. They showed these power-law indices, or slopes, to be nearly independent of the amount of differential rotation artificially implemented in their simulations.

3.1. *Diffusion Plus Avalanches in Disks*

A modification of this picture was made by Mineshige, Takeuchi & Nishimori (1994), who included gradual mass inflow in every timestep, even if all zones were stable, with $M_{i,j} < M_{\mathrm{crit},i}$. By doing so Mineshige *et al.* (1994b) were able to model continuous accretion through viscosity in the disk, even in the absence of substantial flow (and energy release) through instabilities. In their standard model they transfered $3m$ if a zone is unstable (as before) and also $0.01m$ from one cell (randomly chosen) in each ring during each fundamental timestep.

The key result was that the PSD index β for $S(f) \propto f^{-\beta}$ was reduced from ~1.8 to ~1.6. They note that this is in better agreement with AGN (Lawrence & Papadakis 1993) and galactic BH candidate data (Miyamoto *et al.* 1993). Raising the diffusion rate substantially lowers β somewhat further. Other results from these models are that diffusion produces more small-scale avalanches in the inner regions, and decreases the chance for bigger flares. Small shots (energy releases) usually appear randomly in time since the mass is deposited into the outer ring at random points. On the other hand, big shots could have a non-random temporal distribution because a big event drains many cells below their critical values and therefore a longer time is needed to build back up to the critical state. Careful studies of the temporal distribution of various size flares could therefore, in principle, provide a test of this SOC-type model. However, this effect is reduced by diffusion, and we are unaware of additional studies that have performed this type of analysis on usefully large observational datasets.

3.2. *Extrinsic, relativistic modifications*

General relativistic effects, primarily Doppler boosts and gravitational light bending, also lower β (Abramowicz & Bao 1994). This work combined the basic idea of the bright-spot model (§ 1) with the SOC concept. The extent to which these extrinsic effects modify the PSD index is a strong function of the viewing angle, i, to the normal to the disk plane; larger values of i produce shallower slopes, with β reduced to ~1.4 from ~1.8 for $i = 80^\circ$ (Abramowicz & Bao 1994).

4. New variations on SOC in accretion disks

We have recently considered changes in various parameters for simple models such as those suggested by Mineshige *et al.* (1994a,b) to see if the light curves and power spectra for the fluctuations were actually as independent of initial and boundary conditions as they suggested. We examined models with different values of the:

- inner and outer radii of the unstable inner portion of the disk;
- initial deviations from critical density;
- accretion rate;
- amount of mass dumped when a cell exceeded the stability limit;
- amount of differential rotation;
- and diffusion to sideways and outward as well as inward zones.

We have also modified the gravitational potential from the Newtonian r^{-1}, to the pseudo-Newtonian type, $\propto (r-2)^{-1}$, which reproduces many of the leading effects of the Schwarzschild geometry quite accurately (Paczyński & Wiita 1980; Artimova, Björnsson & Novikov 1996). With these expressions we begin the use of notation where distances are given in units of $r_g = GM_{BH}/c^2$.

All of the results we display and discuss below illustrate only the intrinsic variability in the source rest frame. Nearly equivalently, one can say that these computations are restricted to the fluctuations triggered by avalanches as viewed face-on to the disk. One modification we have not yet performed is to incorporate Doppler and general relativistic effects (cf. Abramowicz & Bao 1994), which would allow for the additional extrinsic changes engendered by the observer's viewing angle; as noted above in § 3.2, this would tend to produce less steep PSD slopes.

Our first simulations used what appeared to be the standard model of Mineshige *et al.* (1994a), with 64 rings of unit radius extending inward from $r_1 = 100$ and with the individual zones starting just slightly below critical densities; given those parameters we do indeed reproduce a quasi-power law PSD with slope $\beta \approx -1.8$. These slopes depend very weakly upon most of the parameters needed to characterize the simple model. Of course, the early stages of the light curves are very sensitive to the initial deviations from the critical density. Only very small fluctuations are seen at early times if $m_{\rm dump}/3 < M_{\rm crit} - M_{\rm init} \equiv \Delta M$; however, the disks eventually approach the critical values in most zones and the later stages of the lightcurves appear similar to each other. We note that stable portions of the PSDs (those pertaining to higher frequencies) have very similar slopes even if there is a substantial delay before the critical state is reached throughout the disk (see Figs. 1 and 2).

Smaller values of the accretion rate tend to yield larger numbers of flares of specific energies; the light curves can retain a banded (or "quantized") appearance for significant times (see Fig. 3); nonetheless, values of β are quite independent of $\dot{M}$. In modeling differential rotation, not only did we consider simple extra zone shifts from ring to ring (as was done by Mineshige *et al.*) but we looked at models where the amount of trailing

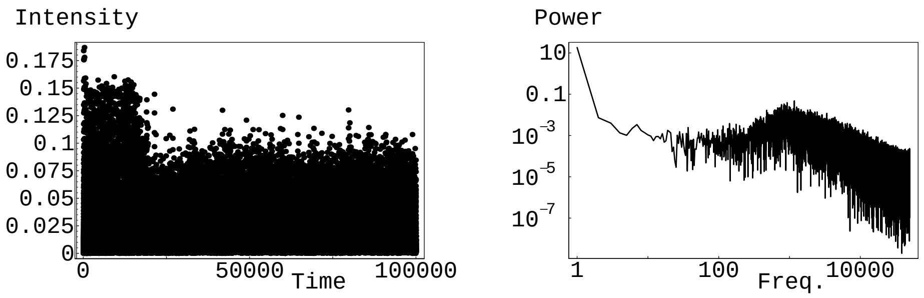

FIGURE 1. The left panel displays a light curve for a simulation with the following parameters: $r_1 = 200$; $r_N = 27.2$; $\Delta M = 0.5$; $\dot{M} = 1$; $m_{\rm dump} = 2$. The right panel shows the corresponding PSD; the best fitting slope, $\beta = 1.54$.

FIGURE 2. As in Fig. 1, with identical parameters except for a larger initial deviation from criticality, $\Delta M = 5.0$; for this case $\beta = 1.50$.

between the i^{th} and $i+1^{th}$ rings was a function of r_i. This allowed for a more realistic approximation to Keplerian or sub-Keplerian rotation curves. Regardless of our approach, we found that changes in the degree of differential rotation also made for only very small differences ($\Delta\beta < 0.04$) in the PSDs, confirming their independence of the amount of rotation noted by Mineshige *et al.* (1994a). This is not surprising given that extrinsic orientation effects, which more strongly depend on differential rotation, have not been included.

In agreement with Mineshige *et al.* (1994b) we do find significant dependence of the power-law on diffusion. They showed that if the diffusion is only inward, then the slope of the PSD becomes shallower. This is a reasonable approximation to standard viscosity in disks, which transports matter inwards and angular momentum outwards. We also performed simulations which allow for a more general diffusion of energy, which might be more realistic if reconnection events provide the physical trigger. To allow for propagation of a disturbance partially sideways and back outwards as well as inwards (which was still presumed to dominate) we also considered cases where the mass outflow from an unstable zone would go 1/5 to each of three interior zones (60% inwards), 1/8 to each of the neighboring zones in the same ring (25% sideways) and 1/20 to each of the three exterior zones (15% outwards). We find that if the triggering zone can send some matter back outwards and sideways, then the slope can become steeper by up to $\Delta\beta \approx 0.15$–0.25 (compare Figs. 1 and 4).

FIGURE 3. As in Fig. 1, with identical parameters except for: $r_1 = 198$; $r_N = 6.0$; here $\beta = 1.07$.

FIGURE 4. As in Fig. 1, with identical parameters; however, here the backwards and sideways diffusion discussed in the text has been included, and $\beta = 1.70$.

We have found a substantial sensitivity of the power spectrum density index β to the location of the disk's inner radius (r_N) with respect to its outer radius (r_1). The power spectrum is steeper (higher values of β) if the inner unstable zones do not get too close to the BH. Even without including diffusion, $1.1 < \beta < 1.8$, is found, with the shallowest slopes corresponding to $r_N \approx 6$, i.e., at the innermost stable orbit for a non-rotating black hole, when the outer radius is fixed at $r_1 = 200$. The results obtained by fixing $r_N = 6.0$ and allowing r_1 to vary show that the dominant source of the variation in β is the ratio of r_1/r_N; these changes are illustrated in Figs. 3, 5 and 6. An analogy with advection dominated accretion flows (e.g. Chakrabarti 1996; Chen *et al.* 1995; Lasota, this volume; Narayan, this volume) may well exist here: if the flow loses its disk-like morphology within a larger inner radius (the "transition radius" for such advection dominated flows) then weaker fluctuations and higher values of β might be favored.

Relatively small changes are induced by changing the energy released in Eq. (3.3) from a pure Newtonian, and apparently scale free, r^{-1}, dependence to the pseudo-Newtonian, $(r-2)^{-1}$, dependence. In cases where the inner radius was set at $r_N = 6$ so this effect would be maximized, we found the slope to become flatter by only $\Delta\beta \approx 0.1$ (compare Figs. 3 and 7). Therefore, we conclude that the differences in PSD slopes produced by the change in bounding radii are not due predominantly to the breaking of linear-scale

FIGURE 5. As in Fig. 3, with the only different parameter being: $r_1 = 31.6$; the resulting $\beta = 1.62$.

FIGURE 6. As in Fig. 3, with the only different parameter being: $r_1 = 18.8$; the resulting $\beta = 1.80$.

invariance by the approximate inclusion of general relativistic effects, as one might at first have assumed. To further test this conclusion we performed simulations where the energy released was independent of r, but where the triggering of avalanches otherwise remained the same. In this toy model, which is closer to a standard sandpile picture, we found $\beta \approx 2.0$ with no dependence on the ratio r_1/r_N.

5. Conclusions

Our simulations lead us to argue that a true state of self-organized criticality is not likely to be established in accretion disks because the fundamental units of energy release are probably not all the same; rather, energy released deeper in the gravitational well is more substantial for a given mass traversing a given distance. A realistic physical model might modify the statement made in the previous sentence, but need not do so. For example, consider magnetic flares based on reconnection events: if the energy density in the magnetic fields is close to equipartition with the pressure in the matter, then higher energy releases would still be expected closer to the inner edge of the disk. Therefore, even when the disk is operating right at the "hairy edge" of stability, so that flares of many different sizes can be randomly triggered by a single infalling mass unit (or

FIGURE 7. As in Fig. 3, with the only difference being the use of the pseudo-Newtonian potential instead of the standard Newtonian potential; $\beta = 0.96$.

reconnection event) at the outer edge of the unstable portion of the disk, thus implying that an approximate SOC state exists, we do not see all the properties of self-organized criticality. Rather, the key observables, i.e., the light curves and PSD, still depend on the region over which the instability operates; they are not independent of boundary conditions, as one might hope would be the case for a system that is a full-fledged SOC. Nonetheless, it is intriguing that these models seem capable of only reproducing variability power spectrum slopes between -1.0 and -2.0, as this also seems to span the range of observed PSD slopes seen in both galactic and extragalactic variable sources.

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