Polarization Variability of Active Galactic Nuclei and X-ray Binaries

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In press in *The Astrophysical Journal*, 20 September 1997 issue

Abstract

In the innermost part of an accretion disk around a black hole electron scattering could provide the dominant opacity, and so the X-ray radiation originating from this hot region is expected to be partially linearly polarized. If strong magnetic fields are are present on or above the disk then synchrotron radiation from electrons may also contribute to the polarization, although different orientations of the magnetic field and Faraday rotation might reduce the effect. Both observational and theoretical studies suggest that the inner disk region is unstable and could appear "clumpy". In this paper, we investigate polarization features due to polarized orbiting clumps around a black hole. It is found that unlike in the Newtonian case, rapid polarization variability can be produced by these regions emitting extra radiation and that the variability amplitudes of both the degree of polarization and the angle of polarization plane are energy-dependent, i.e., the polarization variability amplitudes are larger at higher energy. This feature will not appear if the central object is not gravitationally strong even when polarized clumps rotate around the object; and this trend depends only weakly on the local physics such as the specific polarization mechanism, or optical depth of the sources.

Energy-dependent polarization variability is a direct result of near field bending of light rays by the central black hole and it is unique to black hole systems involving accretion disks. Since accretion disks around black holes are expected in both active galactic nuclei and X-ray binaries, we look for future X-ray polarimetry missions to confirm our prediction of this phenomenon.

Key words: accretion, accretion disks — black hole physics — galaxies: active — polarization — X-rays: binaries

1. Introduction

The study of the radiation from matter close to the event horizon of a black hole has always been of immense interest both in physics and in astronomy. This is because potentially it can provide information for understanding the geometry of the black hole and building blocks for developing astrophysical models. Since black holes have only three observable quantities, i.e., mass, charge and angular momentum (Carter 1973), a rigorous proof of the existence of a black hole should involve direct observation of matter in close orbits exhibiting peculiar properties. Very rapid X-ray and UV variability of both Galactic black hole candidates (e.g., Oda et al., 1971; Nolan et al., 1981; Makishima 1988) and active galactic nuclei (AGN) (e.g., Lawrence et al., 1985; Barr & Mushotzky 1986; Pounds & Turner 1987) with time scales comparable to the time that light takes to cross the holes implies that we are observing radiation emerging from the close vicinity of the black holes in these objects. Rapid X-ray variability in binary systems can also originate in disks around neutron stars, and for binary stellar systems it might be easier to discriminate between black hole and neutron star candidates through the presence of gamma-radiation from annihilation.

A rotating accretion disk around a black hole is currently the most popular model both for many X-ray binaries and for AGN (see e.g., Rees 1984; McClintock 1986). The innermost part of a disk is known to be subject to many kinds of instabilities (Pringle, Rees & Pacholczyk 1973; Lightman & Eardley 1974). It therefore appears "clumpy" (Day et al. 1990; George & Fabian 1991; Fabian & George 1991), yielding brightness fluctuations, perhaps involving physical processes such as orbiting stars, magnetic flares (Vries & Kuijpers 1992; Chagelishvili, Lominadze & Rogava 1989), vortices (Abramowicz et al. 1992) and spiral shocks (Wiita et al 1992a, b; Chakrabarti & Wiita 1993). A direct consequence of the "clumpy" disk is the rapid flux variability attributable to the relativistic rotation of the clumpy matter (Abramowicz et al 1989; Wiita et al. 1991), induced mainly by two effects: the Doppler effect and strong gravitational effects. Both effects depend strongly on the distance between the matter and the black hole, i.e., they are r-dependent (Bao 1992). Flux variability of this kind, from the optical to X-ray bands, has been extensively studied recently (see Bao & Abramowicz 1996 and references therein).

In addition, in the innermost part of an accretion disk where electron scattering provides the main opacity in standard disk models (e.g., Novikov & Thorne 1973) we expect polarization to be linear, with the electric vector lying in the plane of the disk (Rees 1975). If the inner part of an accretion disk is dominated by magnetic events, such as coronal flares (e.g., Krishan & Wiita 1994), buoyant flux tubes (e.g., Chakrabarti & D'Silva 1994), or some classes of vortices (e.g., Abramowicz et al. 1992), then polarization due to synchrotron radiation from relativistic electrons (and possibly positrons) may play a key role in the observed polarization. In this paper, we present a study of polarization signatures due to the linearly polarized matter orbiting the central black hole, i.e., we find the variability of the degree of polarization and the variability of the angle of the polarization plane. We first use a fully relativistic approach to solve photon trajectories from each source. Then we use a numerical Monte Carlo method for calculating Stokes parameters describing polarization properties and the variability amplitude of collections of sources on the disk. We demonstrate that, unlike the rapid flux variability produced by changes in the special relativistic Doppler effect due to the rotation of the source, or by shocks, polarization variability is essentially generated by the strong gravitational field of black hole. This gravitational bending of radiation by the central black hole is a strongly r-dependent phenomenon. Since the temperature distribution of radiating matter is likely to be a strong function of the distance from the central black hole (if it is at all similar to the case of the standard accretion disk models), the observed polarization variation will then be energy-dependent (or, equivalently, temperature-dependent), and changes in both the degree and the angle of the plane of polarization from the system are energy-dependent. These types of fluctuations appear to be unique to a black hole system involving a relativistic accretion disk and thus could serve as a sensitive signature of black holes.

The basic idea on which this paper is based is described in Bao, Wiita, & Hadrava (1996), and some of the formulae are also given there. In this paper we expand upon that work by including studies of intrinsic polarization due to both electron scattering and synchrotron radiation. Some results based on different emission laws are also presented. Here we also give more general derivations of the required formulae, provide additional numerical examples, and discuss in substantially more detail the implications of our results. In §2 of this paper, we present the formulae needed for calculating both the degree of polarization and the angle of polarization for a single source and in §3 we obtain them for collections of sources. All relativistic effects are included with no approximations for a non-rotating central object. In §4 we verify analytically that the Newtonian approximation shows no polarization variation, while the radial dependence when only special relativity is included is also very weak. In §5, we present the results and related discussions, and demonstrate that the general property of polarization variability with frequency rarely depends on the specific mechanism chosen, although of course, absolute variability amplitudes are different for alternate sources of polarization. A summary of our study and its astrophysical implications are given in the final section.

2. Polarization variability of a clumpy disk around a black hole

In our model, the disk is approximated by a system of individual sources, e.g., blobs, flux tubes. Generally, the sources are moving independently on eccentric orbits in the equatorial plane of the Schwarzschild metric around the central object.

In the case where polarization arises from electron scattering, each source is supposed to be a homogeneous plane-parallel slab of gas, the geometrical thickness (perpendicular to the equatorial plane) of which is – in its proper comoving frame – much smaller than its dimensions in the directions of the disk plane. These dimensions are in turn negligible compared to the semimajor axis of the orbit. Some distributions of both the orbital parameters and physical characteristics (i.e., temperatures and dimensions) of the sources must be assumed. This model may correspond (mostly for zero eccentricities) to local increases of the temperature or density of a continuous accretion disk, or, approximately, to a disk composed by individual blobs of matter. It does not directly correspond to a local enhancement of emissivity which may be caused, e.g., by shock waves moving with respect to the disk. We then consider the thermal radiation from each source, which is linearly polarized by the electron scattering within the slab.

On or just above the accretion disk there may exist features dominated by magnetic fields, e.g., magnetic flares, flux tubes, or vortices (Abramowicz et al., 1992; Krishan & Wiita 1994). If this is true, then synchrotron radiation may play an important role in the emitted radiation, and thus it may also be a source of polarization and its variability. We discuss below the idealized situation where the magnetic field of each source is parallel to the normal of the orbital plane. Obviously a realistic picture could be far more complicated than what we assume here, but this is a plausible first approximation. Moreover, one can expect that the differing orientations of magnetic field in each source and any Faraday rotation or depolarization may eventually diminish the observed polarization. Nevertheless, it is clearly worth considering this probable source of polarization.

Due to aberration and the Doppler shifts, radiation which is assumed to be emitted axially symmetrically in the proper rest frame of the source, appears to be highly anisotropic in the static coordinate frame. The vector of the polarization for each ray must be also transformed by the Lorentz transformation between these two frames (details are given in the following sections). In the special relativistic (SR) approximation with straight rays, the contributions of all sources to the total light seen by an observer at infinity could be directly summed for each polarization. However, in the correct general relativistic (GR) treatment, the rays are bent by the gravitational field of the central object. Consequently, contributions of some sources are amplified through gravitational lensing and the vector of polarization, which parallelly propagates along the ray, must be taken into account for each source. To include these general relativistic effects into a numerical model, it is thus necessary first to solve (e.g., in terms of the impact parameter) for the appropriate ray which joins the instantaneous position of the source asymptotically with the position of the observer at infinity. This solution also determines the direction into which the photon has to be emitted with respect to the static frame, and, thus its inclination in the comoving frame, on which the intrinsic intensity and polarization of the radiation depend. Because this is the crucial point of the numerical modeling for the appearance of the disk, we will describe it in detail in the following subsections.

2.1 Intrinsic polarization and the intensity of a single source

2.1.1. Electron scattering

Following Chandrasekhar (1960), the equation of radiative transfer in the plane-parallel slab can be written in the form

$$\mu \frac{d}{d\tau} I(\tau,\mu) = I(\tau,\mu) - (1-\varepsilon)B(\tau)\begin{pmatrix}1\\1\end{pmatrix} - \varepsilon \int_{-1}^{+1} P(\mu,\mu')I(\tau,\mu')\frac{d\mu'}{2}\,, \tag{2.1.1}$$

where μ is the direction cosine, the intensity $I = \begin{pmatrix}I_l\\I_r\end{pmatrix}$ has components linearly polarized in the meridional plane (I_l) and perpendicularly to it (I_r), $\varepsilon = \sigma/(\sigma+\kappa)$ is the ratio of the scattering to the total (i.e., scattering + true absorption) opacity, B is the source function of the thermal radiation, and the kernel due to the Rayleigh's scattering reads

$$P(\mu,\mu') = \frac{3}{4}\begin{pmatrix}2(1-\mu^2)(1-\mu'^2)+\mu^2\mu'^2 & \mu^2\\ \mu'^2 & 1\end{pmatrix}\,. \tag{2.1.2}$$

Unlike the semi-infinite atmosphere, in our case the optical depth $\tau = -\int(\sigma+\kappa)dz$ varies from $-t$ to $+t$ between the surface of the slab emitting the ray (with $\mu > 0$) and its back side.

If the electron scattering is negligible ($\varepsilon = 0$), the solution

$$I_0(\tau,\mu) = \begin{pmatrix}1\\1\end{pmatrix}\int_{\tau'=\tau}^{t(\mathrm{sign}\mu)} B(\tau')\exp\left(\frac{\tau-\tau'}{\mu}\right)\frac{d\tau'}{\mu} \tag{2.1.3}$$

of Eq. (2.1.1) corresponds to unpolarized radiation. Assuming the general solution of Eq. (2.1.1) to be in the form of power series in the small parameter ε,

$$I(\tau,\mu) = \sum_k I_k \varepsilon^k \tag{2.1.4}$$

(the generalization to the case $\varepsilon = \varepsilon(\tau)$ is relatively straightforward), we obtain a the recurrence relation for the coefficients I_k,

$$I_k(\tau,\mu) = \int_{\tau}^{t(\mathrm{sign}\mu)}\left[\int_{-1}^{+1} P(\mu,\mu')I_{k-1}(\tau',\mu')\frac{d\mu'}{2} - \begin{pmatrix}1\\1\end{pmatrix}B(\tau')\delta_k^1\right]\exp\left(\frac{\tau-\tau'}{\mu}\right)\frac{d\tau'}{\mu}\,, \tag{2.1.5}$$

with $\delta_k^1 = 1$ if $k = 1$ and 0 otherwise.

Due to the symmetry, $I(\tau,\mu) = I(-\tau,-\mu)$ provided $B(\tau) = B(-\tau)$, and we can choose $\mathrm{sign}\mu = 1$ in this case. $B(\tau)$ is given by the inner structure of the slab, which is determined by the energy release and its transport to the surface. This is rather model dependent, but for a reasonable first approximation, we can suppose B to be independent of τ. The solution of Eq. (2.1.3) is then

$$I_0(\tau,\mu) = \begin{pmatrix}1\\1\end{pmatrix}B\left[1-\exp\left(\frac{\tau-t}{\mu}\right)\right]\,. \tag{2.1.6}$$

We have numerically solved Eq. (2.1.5) for several higher terms of Eq. (2.1.4) for large ranges of the parameters t, μ and ε. We found the fractional linear polarization varies around 0.1 if the contribution of electron scattering to the overall opacity is non-negligible. To eliminate the uncertainty in the physical conditions inside the individual sources for the present approximate study of the role of GR effects, we thus accepted the maximum value of 0.117 given by Chandrasekhar (1960). At the same time, we use the approximation (2.1.6) for the total intensity of the emergent radiation.

2.1.2. Synchrotron radiation

First we consider a single source region. For simplicity, we assume that the magnetic field is along the tube and parallel to the normal of the source orbit; to lowest order, this is reasonable. The relativistic electrons are, as usual, taken to have a power-law distribution in energy: $N(E) = kE^{-s}$, where s is assumed to be a constant. Both the emission coefficient and the absorption coefficient depend on the pitch angles, the angle between the magnetic field, H and the direction of velocity of each electron (e.g., Pacholczyk 1970). If the electrons are highly relativistic, most of the radiation will be confined within a small angle $\sim \gamma^{-1}$ (γ is the Lorentz factor of the electron) around the direction of instantaneous velocity. Obviously, γ^{-1} will be very small for highly relativistic electrons. An observer at infinity can only receive those photons emitted from a specific location if they are emitted at a proper angle $\hat{\theta}$ (see the following section). Thus, to a first approximation, the synchrotron radiation with pitch angle equal to $\hat{\theta}$ will dominate what is received by the observer.

In the comoving frame of a magnetic tube, the total emission coefficient for synchrotron radiation coming from a stationary region with such a power-law distribution of electrons reads (Pacholczyk 1970)

$$\epsilon_\nu \propto (H \sin\hat{\theta})^{(s+1)/2} \nu^{(1-s)/2}, \tag{2.1.7}$$

and the average absorption coefficient is

$$K_\nu \propto (H \sin\hat{\theta})^{(s+2)/2} \nu^{-(s+4)/2}. \tag{2.1.8}$$

Unfortunately, the form of such a modified magneto-hydrodynamic flow is not well understood, and it is fair to say that no convincing models for flux tube sizes and strengths or for particle distributions exist. Still, a reasonable approach is given by Schwartzman (1971) who suggested that a disk will allow reconnection of field lines between adjacent "cells", and there should be a rough equipartition of the gravitational potential energy between magnetic field, kinetic infall of gas, turbulent energy and thermal kinetic energy of particles. Thus a magnetic event at a distance r to the central object can be taken to have

$$H \propto [kT(r)]^{1/2},$$

and then we may write the intensity of radiation measured in the local comoving frame as

$$I_\nu \propto f(\hat{\theta}) \sin\hat{\theta}^{(s+1)/2} \nu^{(1-s)/2} [T(r)]^{(s+1)/4}, \tag{2.1.9}$$

for an optically thin homogeneous source and $\nu > \nu_c$, and

$$I_\nu \propto f(\hat{\theta}) \sin\hat{\theta}^{-1/2} \nu^{5/2} [T(r)]^{-1/2}, \tag{2.1.10}$$

for an optically thick source and $\nu < \nu_c$. Here $f(\hat{\theta})$ is an angle dependent function, defined in the following section, and ν_c is the critical frequency at which synchrotron self-absorption starts.

Formulae (2.1.9) and (2.1.10) ignore the probable increase of the number of supra-thermal electrons available for acceleration to relativistic speeds as well as the possibility that the high energy cutoff for the emitting electrons also increases as T increases. Therefore these relations probably are only an underestimate of the radial dependence of synchrotron emission. Also, the distribution of $T(r)$ of a flux tube as a function or r is not clear. We plausibly take $T(r)$ to track that of a standard relativistic accretion disk model (Novikov & Thorne 1973). Clearly, the formulae we give here for are simplified, and may not be terribly realistic; however, they should capture the basic trend of additional emission arising in the inner portions of the disk.

The ellipticity of polarization of the synchrotron radiation from an ensemble of electrons will be zero, so we expect only linear polarization. If the Faraday rotation is small the degree of the polarization from a single source does not depend on any local quantity except the distribution of the electrons (Pacholczyk 1970), and thus it does not change when the source orbits the black hole. The fractional polarization for such a distribution can be expressed as

$$\delta_e = \frac{s+1}{s+7/3}$$

for the optically thin case, and

$$\delta_e = \frac{3}{6s+13},$$

for the optically thick case.

The polarization vector will be parallel to the normal to the orbital plane for the optically thin case and perpendicular to the normal for the optically thick case.

2.2 Polarization variability of a single source

Throughout this calculation for electron scattering opacity we consider that the photons scattered within geometrically thin disk regions cross only a small geometric distance before their final scattering. Therefore, general relativistic effects during the radiative transfer process are small and can be neglected. Then we can use the Newtonian result of Chandrasekhar (1960) to describe the local polarization properties.

We start with the study of polarization variation due to a single orbiting source. The knowledge of this will facilitate understanding the collective behavior of distributed sources simultaneously emitting around the central black hole. We consider a general case in which a polarized source lies on an eccentric orbit around a Schwarzschild black hole with an eccentricity of e, a semi-major axis a, and an inclination i (angle between the normal of the orbital plane and the line of the sight).

The four-basis vectors in a static frame can be easily written as

$$\begin{aligned} e^{(t)} &= \xi^{1/2} e^t; \\ e^{(r)} &= \xi^{-1/2} e^r; \\ e^{(\theta)} &= r e^\theta; \\ e^{(\phi)} &= r\sin\theta e^\phi. \end{aligned} \tag{2.2.1}$$

where $\xi = 1 - 2/r$ and units of $G = c = M = 1$ are used.

The four-velocity for both the source and the photon is

$$\begin{aligned} u^t &= \xi^{-1} E, \\ u^r &= \left[E^2 - \xi(\eta + r^{-2} b^2)\right]^{1/2}, \\ u^\theta &= r^{-2}\left(b^2 - l^2/\sin^2\theta\right)^{1/2}, \\ u^\phi &= r^{-2} l/\sin^2\theta, \end{aligned} \tag{2.2.2}$$

where $\eta = 0$ or 1 correspond, respectively, to the photon and the emitting matter, while E, b and l are the standard constants of motion. Especially for a photon ($\eta = 0$), we have $B = b/E$ is the total impact parameter of photon, and $L = l/E$ is the impact parameter around the symmetry axis. Since photon motion is planar, it can be written as $L = -B \sin i \sin \psi'$, where ψ' is the observed position angle of the orbiting source given by (Bao 1992)

$$\sin \psi' = \frac{\sin \phi_s}{\sqrt{1 - \cos^2 \phi_s \sin^2 i}}, \tag{2.2.3}$$

and ϕ_s is the local phase of source. As one can see from Eq. (2.2.2), photon motion in Schwarzschild space time is solely described by the impact parameter B. The velocities of the source measured by a static observer are

$$v^{(j)} = \frac{\vec{u} \cdot e^{(j)}}{u^{(t)}}, \tag{2.2.4}$$

and its non-zero components are

$$v_1 = \left[1 - \xi \Big(\frac{1}{E_s^2} + \frac{1}{r^2} (\frac{l_s}{E_s})^2 \Big)\right]^{1/2}, \tag{2.2.5}$$

$$v_3 = \xi^{1/2} \frac{1}{r} \frac{l_s}{E_s}, \tag{2.2.6}$$

where $v_1 \equiv v^{(r)}$ and $v_3 \equiv v^{(\phi)}$ in units of c, the speed of light. Here l_s and E_s are the energy and the angular momentum of the orbiting source, and $E_s^2 = [(a-2)^2 - a^2 e^2][a(a(1-e^2) - 3 - e^2)]^{-1}$, while $(l_s/E_s)^2 = [a^2(1-e^2)^2][(a-2)^2 - a^2 e^2]^{-1}$; note there was a typographical error in the equivalent formula in Bao et al. (1996).

Consider two special cases for the polarization angle that bracket all possibilities, i.e., the intrinsic polarization direction is either perpendicular to the orbital plane or parallel to it. For a polarization vector perpendicular to the source orbital plane in the comoving frame we find that

$$p^{\hat{\mu}} = \frac{1}{u^{\hat{t}} \sqrt{(u^{\hat{t}})^2 - (u^{\hat{\theta}})^2}} (0,\ -u^{\hat{\theta}} u^{\hat{r}}, (u^{\hat{t}})^2 - (u^{\hat{\theta}})^2,\ -u^{\hat{\theta}} u^{\hat{\phi}}), \tag{2.2.7}$$

where $u^{\hat{\mu}} (\mu = t,\ r,\ \theta,\ \phi)$ is the four-velocity of the photon measured in the comoving frame. For a polarization vector parallel to the source orbital plane we use the result of Bao et al. (1996),

$$p^{\hat{\mu}} = \frac{1}{\sqrt{(u^{\hat{t}})^2 - (u^{\hat{\theta}})^2}} (0, \quad -u^{\hat{\phi}}, \quad 0, \quad u^{\hat{r}}). \tag{2.2.8}$$

Any vector $\vec{f}$ in a static frame may be obtained through a Lorentz transformation

$$f^{(\mu)} = \Lambda^{(\mu)}_{\hat{\nu}} f^{\hat{\nu}}, \tag{2.2.9}$$

where

$$\Lambda^{(\mu)}_{\hat{\nu}} = \begin{pmatrix} \gamma & \gamma v_1 & 0 & \gamma v_3 \\ \gamma v_1 & (\gamma - 1)\frac{v_1^2}{v^2} + 1 & 0 & (\gamma - 1)\frac{v_3 v_1}{v^2} \\ 0 & 0 & 1 & 0 \\ \gamma v_3 & (\gamma - 1)\frac{v_3 v_1}{v^2} & 0 & (\gamma - 1)\frac{v_3^2}{v^2} + 1 \end{pmatrix},$$

$$\gamma = \frac{1}{\sqrt{1 - v^2}}, \qquad v^2 = v_1^2 + v_3^2.$$

Because photon motion is planar, any angle measured in the static frame should be the same as that measured by an observer at infinity. Therefore, it is only necessary to derive the polarization angle in the static frame. One can get the polarization vector measured in the static frame through Eq. (2.2.9), i.e., $p^{(\mu)} = \Lambda^{(\mu)}_{\hat{\nu}} p^{\hat{\nu}}$.

The unit normal vector $\vec{n}$ of the plane in which photon moves reads (Bao et al. 1996),

$$n^{(\nu)} = \frac{1}{\sqrt{(u^{(t)})^2 - (u^{(r)})^2}} \; (0,\ 0,\ -u^{(\phi)},\ u^{(\theta)}). \tag{2.2.10}$$

Thus, the angle between the polarization vector and the normal of the photon-motion plane is

$$\psi = \cos^{-1}[n^{(\theta)} p_{(\theta)} + n^{(\phi)} p_{(\phi)}]. \tag{2.2.11}$$

Therefore, the observed angle of the polarization plane is expressed as

$$\psi_p = \psi' + \psi \pm \frac{\pi}{2}, \tag{2.2.12}$$

with ψ' the observed phase of the orbiting source and the notation "$\pm$" allows ψ_p to describe the orientation of the plane of polarization. As we know, both the normal vector of the photon plane $n^{(\nu)}$ and the polarization vector $p^{(\nu)}$ are expressed in terms of the four-velocity of the photon, which is solely determined by the total impact parameter of the photon, B. Since B is a function of i, ϕ_s, e, and r, i.e., $B = B(i, \phi_s, e, r)$, we have

$$\psi_p = \psi_p(i, \phi_s, e, r). \tag{2.2.13}$$

Unfortunately, there is no analytic expression for B, which must be solved through numerical calculation (Bao 1992; Bao, Hadrava & Østgaard 1994); however, we note that B is a strong function of r.

Assuming that the polarization arises from electron scattering, the polarization vector lies in the disk plane and the degree of polarization δ is a sensitive function of the angle between the normal of the disk and the direction in which photon is emitted (Chandrasekhar 1960; §2.1). The value of δ changes from 11.7% when the disk is edge-on to 0 when disk is face-on. The angle measured in the comoving frame reads

$$\begin{aligned} \cos\hat{\theta} &= \frac{p^{\hat{\theta}}}{p^{\hat{t}}} \\ &= \frac{\sqrt{(\frac{B}{r})^2 - (\frac{L}{r})^2}}{\gamma(\xi^{-1/2} - v_1\sqrt{\xi^{-1} - (\frac{B}{r})^2} - v_3\frac{L}{r})}, \end{aligned} \tag{2.2.14}$$

where p^{ν} and $p^{\hat{\mu}}$ are the four-momenta of the photon. (Note that this differs slightly from Eq. (13) in Bao et al. (1996), which is in error; however, because all results given there assumed $v_1 = 0$, so $v = v_3$, this error did not affect any conclusions in that paper.) Again $\hat{\theta} = \hat{\theta}(i, \phi_s, e, r)$ and thus the degree of polarization

$$\delta = \delta(i, \phi_s, e, r). \tag{2.2.15}$$

Although the degree of polarization, δ, is a Lorentz invariant (i.e., it does not change along a given ray), it varies during the orbital period in which different beams of photon are emitted.

3. The polarization variability of distributed sources

It is likely that the innermost part of an accretion disk is full of clumpy sources at different radii. In the following study, we assume that the sources are distributed randomly in their orbital phases but have a power law distribution in their radii. The polarization of radiation from sources can be described by the normalized Stokes parameters,

$$u_\nu = U_\nu/I_\nu, \quad q_\nu = Q_\nu/I_\nu, \tag{3.1}$$

where I_ν is the intensity at frequency ν. If the polarization is induced by electron scattering, there is no circular polarization for initially unpolarized radiation; similarly, if synchrotron emission dominates the polarized emission, the polarization will also be essentially linear. The degree of polarization is

$$\delta_\nu = \sqrt{u_\nu^2 + q_\nu^2}, \tag{3.2}$$

and the angle of the plane of polarization is

$$\psi_p = \frac{1}{2}\tan(\frac{u_\nu}{q_\nu}). \tag{3.3}$$

The Stokes parameters (u_ν, q_ν) and intensities of the sources are combined through the following formulae

$$I_\nu = \sum_{i=1}^{n} \int g_i^3 I_\nu^i f(\hat{\theta}) d\Pi_i, \tag{3.4}$$

$$u_\nu = \frac{1}{I_\nu}\sum_{i=1}^{n} \int g_i^3 \delta_i \sin 2\psi_p^i I_\nu^i f(\hat{\theta}) d\Pi_i, \tag{3.5}$$

and

$$q_\nu = \frac{1}{I_\nu}\sum_{i=1}^{n} \int g_i^3 \delta_i \cos 2\psi_p^i I_\nu^i f(\hat{\theta}) d\Pi_i. \tag{3.6}$$

In the above equations the index i refers to the i^{th} source, so ψ_p^i and δ_i are the polarization angle and the degree of the polarization of the i^{th} source, and Π_i is the solid angle that source subtends at the observer. The function $f(\hat{\theta})$ describes the angular dependence of the radiation, which may be expressed by

$$f(\hat{\theta}) = (1 + a\cos\hat{\theta})\cos^b\hat{\theta}, \tag{3.7}$$

where a and b are constants, depending on the local physics. For example, in an optically thick electron scattering atmosphere, we have $a = 2.06$, $b = 0$ (Chandrasekhar 1960). The g_i is the red-shift factor of the i^{th} source, which is defined as the ratio of photon energy measured at infinity to the energy measured in the comoving frame of the i^{th} source. It reads

$$\begin{aligned} g &= \frac{p^t(\infty)}{p^{\hat{t}}(r)} \\ &= \frac{1}{\gamma(\xi^{-1/2} - v_1\sqrt{\xi^{-1} - (\frac{B}{r})^2} - v_3\frac{L}{r})}. \end{aligned} \tag{3.8}$$

At present, no theory is available to describe the temperature distribution of discrete sources on the disk. Our assumption is that it may follow the disk temperature distribution under many conditions. However, it need not always do so: for example, if the sources are gas-pressure dominated while the disk is radiation pressure dominated; or if the sources are not embedded in the disk, but are on its surface where the temperature and density are different from the vertically averaged values. For simplicity, we adopt the (vertically averaged) temperature of standard relativistic accretion disks (Novikov & Thorne 1973).

4. Polarization in the Special Relativistic and Newtonian Approximations

In flat space time a photon goes along a straight line. So, we have an analytic expression for the total impact parameter B (see Fig. 1 in Bao 1992),

$$B = r\sqrt{1 - \sin^2 i \cos^2 \phi_s}. \tag{4.1}$$

Thus the four-velocity of the observed photon at (ϕ_s, r_s) reads

$$u^\nu = \left(1, \quad \sin i \cos \phi_s, \quad \frac{\cos i}{r}, \quad -\frac{\sin i \sin \phi_s}{r}\right). \tag{4.2}$$

For simplicity, we consider only a circular orbit ($v_1 = 0,\ v_3 = v = 1/\sqrt{r}$) and first consider the case where the polarization vector lies in the disk plane. By using Eqns. (4.1) and (4.2), after some algebra, we find that the angle between polarization vector and the plane of photon is

$$\cos\psi = \cos\psi' \frac{\gamma \sin i}{\sqrt{\gamma^2(1 + v\sin i \sin\phi_s)^2 - \cos^2 i}}, \tag{4.3}$$

in the SR approximation where $\gamma = \frac{1}{\sqrt{1-v^2}} = \sqrt{\frac{r}{r-1}}$. From Eqns. (3.3) and (2.2.12) we see that ψ_p is a weak function of r, and is only changed by relativistic aberration. For large r or for slow motion of the source ($v \sim 0, \quad \gamma \sim 1$), formula (3.3) attains the Newtonian limit of

$$\cos\psi = \cos\psi'. \tag{4.4}$$

In this limit, the physical solution of the polarization angle is

$$\psi_p = \pm\frac{\pi}{2}, \tag{4.5}$$

implying that the polarization lies in the disk plane independent both of r and of ϕ_s of the emitting matter (ψ_p is measured from the projected symmetry axes in the observer's sky). Formula (4.3) is then only a direct function of i and ϕ_s, and hardly depends on the radial location, r, of the source. It is interesting that for very low inclination ($i \sim 0$) from Eq. (4.2) we have

$$\psi_p = \phi_s, \tag{4.6}$$

i.e., ψ_p changes linearly with the orbital motion of the source. This is also true even if we include gravity, as gravitational bending plays little role in such a case.

When the polarization vector is perpendicular to the disk plane, we have

$$\cos\psi = \frac{\sin^3 i \sin\phi_s + \gamma^2\cos^2 i(v + \sin i \sin\phi_s)}{\gamma(1 + v\sin i\sin\phi_s)\sqrt{1 - \sin^2 i\cos^2\phi_s}\sqrt{\gamma^2(1 + v\sin i\sin\phi_s)^2 - \cos^2 i}}, \tag{4.7}$$

and for large r or for slow motion of the source ($v \sim 0, \quad \gamma \sim 1$) we get

$$\cos\psi = \frac{\sin\phi_s}{\sqrt{1 - \sin^2 i\cos^2\phi_s}}. \tag{4.8}$$

From formulae (2.2.3) and (4.8) we see $\cos\psi = \sin\psi'$, so the angles differ by $\frac{\pi}{2}$ and from formula (2.2.12) we obtain

$$\psi_p = 0 \quad (\text{or} \quad \pi),$$

implying that the polarization vector is perpendicular to the disk plane everywhere in the disk, independent both of r and of the ϕ_s of the emitting matter.

By using (4.1) we find that the SR limit is

$$\cos\hat{\theta} = \cos i \frac{1}{\gamma(1 + v\sin i \sin\phi_s)}. \tag{4.9}$$

When $r \gg 1$, we get, unsurprisingly,

$$\cos\hat{\theta} = \cos i. \tag{4.10}$$

This implies that the degree of polarization does not depend on r either. We thus reiterate the conclusion of Bao et al. (1996) that for the same initial polarization, the Newtonian approximation to the plane of polarization as well as the degree of polarization measured at infinity are nearly the same for each region on the disk, or effectively r-independent, regardless of the initial direction of polarization.

5. Results and Discussions

In this section, we use quasi-analytical methods (Bao 1992; Bao et al. 1994) to trace every photon trajectory from the emitting source to the observer at infinity. This is needed for the calculation of total impact parameters of photons emitted at each point of the orbit. The degree of polarization and the angle of plane of polarization are then calculated by using the formulae presented in §3.

Fig. 1 presents the variation in the degree and the angle of the plane of polarization with orbiting phase for inclination $i = 80°$ and for different eccentricities, i.e., $e = 0,\ 0.3,\ 0.6$ with semi-major axis $a = 20m$ (where $m = GM/c^2$). The source starts from the closest point (at phase zero) to the observer (phase π is the point behind the black hole relative to the observer) and then returns to 2π. It is clearly seen that both the degree and the plane of polarization vary with phase and eccentricity. We note that a strong variation occurs when the emitting source is at phase π. For the degree of polarization, a dip appears, and for the angle of the polarization plane, a peak shows up. Both are attributable to the strong gravitational bending effect as photon trajectories connecting the source at phase π and the observer at infinity have to pass very close to the black hole. This variation as a function of eccentricity is easy to understand, as different orbits have different minimal distances to the central black hole and are therefore subject to different strengths of the bending effect by the central hole. Apart from the the eccentricities, the orientation of the orbit also affects the variations. However, for simplicity, here we only present the case for longitudinal phase of periastron $= -\frac{\pi}{2}$.

Our main focus in this paper is to examine the influences of GR effects on polarization variations arising from sources close to the central black hole and to compare them with the Newtonian and SR approximations. Fig. 2 shows how the variation in the degree of polarization depends on the radius of the circular orbit over one period, for $r = 8m, 20m$, and $100m$. We also present the cases for different inclinations of the orbits, i.e., $i = 50°$ (left), $i = 70°$ (middle) and $i = 85°$ (right). Let us first examine the degree of polarization for the case of $i = 50°$ where the changes in the degree of polarization are almost like a sinusoidal signal, and are due to the bending and relativistic aberration. As the inclination increases, the bending effect begins to dominate the changes, which is indicated by the dips at phase π when the source is behind the black hole. All variations are seen to be strong functions of the radius. The general trend is that the smaller the r of the source, the larger the changes (see the solid line in the upper row of the figure). At the same time we must pay attention to the time scale of the variation. Apart from the larger amplitude of variation, the time scale for smaller r is shorter than for the larger r; for example, T(100m) is 44 times longer than T(8m). This means that larger variation of polarization with a very short time scale occurs only in those sources with their orbits at small radii. Similar to the variation in the degree of polarization, the angle of the plane of polarization depends very much on r and on the inclination of the orbit. The variation amplitude at a fixed time scale decreases rapidly. In addition, we notice that the major changes in the angle occur near phase π, even when $i = 50°$; the variation in fractional polarization dips, while the usually more dramatic variation in polarization angle spikes.

To compare the effects of the relativistic aberration with those of the gravitational bending, Fig. 3 presents the variation in the degree and the angle of polarization plane with orbital phase for the case where photon goes along a straight line (the SR case). The changes in the polarization are produced entirely by relativistic aberration. The figure includes different radial orbits, i.e., $r = 8m$, $20m$ and $100m$ respectively. In order to clearly distinguish GR from SR effects, we chose the inclination $i = 85°$. First, we explicitly see that the variation in polarization is not a strong function of the location of the source r for the SR case. Second, there is no special feature at phase π (since lensing is absent). But in the case when GR is included (see Fig. 2 for $i = 85°$), variations are strongly r-dependent and peaks and dips appear at the phase π. Moreover, the variation amplitude is much larger than that for the SR approximation.

Up to now, despite much evidence of chaotic variability and occasional quasi-periodic oscillations, no periodic X-ray flux variability of either AGN or Galactic black hole candidates has been observed, which suggests that no single, dominant bright source is likely to exist in the innermost part of the engine for a relatively long time. However, it is reasonable to hypothesize the coexistence of multiple "equally bright" sources in the region, which produce the observed chaotic flux variability (Abramowicz et al. 1991; Wiita et al. 1991). Since the temperature distribution of spots is a rapidly varying function of their distances from the central black hole, the polarization variability of a collection of sources should be frequency-dependent or energy-dependent. For a quantitative estimate of this, we finally present studies of many simultaneously present polarized sources located in the inner part of a relativistic accretion disk (Novikov & Thorne 1973).

Fig. 4 and Fig. 5 exhibit the polarization variability amplitude with photon energy for 100 simultaneously radiating sources. The methods and formulae used have already been described in §3. Since a uniform distribution of the sources (i.e., equal brightness, equal life time, number of the sources per unit ring are constant) may produce flux variability consistent with the observation (Lawrence & Papadakis 1993), here we simply assume that the polarization sources are also uniformly distributed on the disk and take electron scattering as the origin of the polarization. We calculate first the time series of both the degree and the angle of polarization plane for each randomly distributed source, and then its variability power spectrum. The contribution from all sources is obtained through summation of the power at a given frequency (or timescale). The variability amplitude, defined as the square root of the variability power at a temporal frequency of 2×10^{-3} Hz (or timescale of 500s), is plotted for a black hole mass of $10^6 M_\odot$. The solid line refers to the case of $i = 80°$, the dashed line refers to $i = 50°$.

First of all, we can see clearly that the variability amplitude is energy dependent, that is, the higher the photon energy, the larger its polarization variability amplitude (variation in the degree of polarization, *and* in the angle of plane of polarization). We also performed the calculation for photon energy from 0.0001 keV to 0.1 keV, and found that the variability amplitude is nearly a constant through the IR/optical and UV energy bands, at least for black hole masses expected for most Seyfert galaxies. As shown in Fig. 2, the variation in both the degree of polarization and the angle of plane of polarization decays rapidly as one goes to larger radii (considering also the timescale). According to the standard theory of accretion disks, the innermost region is dominated by high energy photons, and here the gravitational field is strongest. Given that polarization variation is created by strong gravitational fields through bending, it should not be surprising that only X-ray photons manifest the energy-dependent polarization variation. It should also be natural that this variability depends on the inclination of accretion disk. Higher inclinations imply that more of the the photon trajectories must be bent more strongly in order to reach the observer than is the case for disks viewed at low inclination. The variability amplitudes are significantly smaller for low inclinations. The difference gets even bigger when one goes from high energy to low energy photons.

The energy dependent trend in polarization variability does not depend strongly on the local physics assumed, e.g., the emission law or the polarization mechanism, as long as there exist polarized sources. We have done the calculations for various situations. Here we just show a few results. Fig. 6 presents the case for different dependences of the local emission angle; all other conditions are the same as in Fig. 4 and Fig. 5. There is a common trend of the variability increasing with energy, and there are no special features in the variability pattern to indicate differences between emission laws.

We have also calculated many cases where synchrotron radiation is the source of the polarization; the electron distribution index was taken as $s = 2$. The energy dependent variability trend is still seen, but the dependence is not as strong as in the case of electron scattering, and this dependence occurs at somewhat lower range of energies. Both the absolute amplitudes and the difference of variability amplitudes at different frequencies are small; this is because in synchrotron radiation the percentage of polarization of a single source is invariant if s is held constant, which dramatically reduces the variability amplitude. Since the angle of polarization plane is varying during an orbital period, the percentage of polarization of many sources varies with time. A plot of this is not very interesting, instead, we just give several specific examples: in the optically thick case, when $i = 80°$, the normalized variability amplitudes are 1, 1, 0.23, and 0.001 for the percentage of polarization and 1, 1, 0.96, and 0.001 for the polarization angle at energies of 10, 1, 0.1, and 0.01 keV, respectively; in the optically thin case, the normalized variability amplitudes are 1, 0.989, 0.93, and 0.0004 for percentage of polarization, and 1, 1, 0.95, and 0.0009 for the polarization angle at energies of 10, 1, 0.1, 0.01 keV. However, the average actual variability amplitudes for both are roughly two decades smaller for synchrotron induced polarziation than they are for electron scattering induced polarziation, even when all other conditions are the same. It is likely that the emission from the inner region of an accretion disk will arise from both synchrotron radiation and thermal radiation simultaneously; therefore, we expect that polarization variability amplitude from a realistic source should generally be larger than the situation where only synchrotron radiation contributes.

6. Astrophysical Implications

We have investigated the polarization variability due to orbiting sources around a black hole for the situation where each source is assumed to have an intrinsic linear polarization. In a single source, if the polarization is due to electron scattering, both the observed degree of linear polarization and the angle of the plane of the polarization change rapidly within the orbital period. If the polarization source is due to synchrotron radiation the percentage of polarization may not change despite the changes in total intensity (see Eqns. (2.3.5) and (2.3.6)); however, the new polarization of a collection of many sources does change. Relativistic effects (both general relativistic gravitational bending and special relativistic aberration) are seen to underlie these changes. The change in polarization depends on the eccentricity, the radius, and the inclination of the source orbit. These features distinguish this picture from the Newtonian case, where no significant changes in either the degree or plane of polarization are likely to take place. We demonstrate that large r-dependent polarization variability can be detected only if GR effects are considered, i.e., this phenomenon is related to the presence of a strong gravitational field. The most important feature of the polarization variability due to distributed sources is its energy dependent behavior, i.e., the higher the photon energy, the larger its variability amplitude. Such a property is related to a well-known character of the black hole, i.e., gravitational bending. In the Newtonian approximation, the degree of the polarization measured at infinity is the same for each point of origin on the disk (for the same initial polarization), resulting in its variability amplitude being constant for all energies of photons emerging from the accretion disk.

Fundamentally, polarized radiation arises from the presence of hot X-ray emitting matter near the black hole. Under this condition, electron scattering opacity is important, and thus any initially unpolarized radiation undergoing electron scattering becomes partially polarized with its polarization vector lying in the disk plane. If the X-ray emission region is dominated by events involving strong magnetic fields, synchrotron radiation may also contribute to (and even dominate) the polarization. Although, in general, Faraday rotation may reduce the percentage of polarization (Agol & Blaes 1996), this depolarization mainly occurs in optical and ultraviolet bands. The X–ray polarization feature may offer important information on the accretion disk structure, a theoretical possibility first noted by Rees (1975) and by Lightman & Shapiro (1975). Stark & Connors (1977) and Pineault (1977) further pointed out that, unlike the overall disk spectrum, the polarization feature can be strongly affected by GR effects.

One might argue that since little is known yet about the inner region close to a black hole, the actual situation may be far more complicated. Also, since confident predictions on the polarization properties pertaining to an accretion disk are still hardly possible in the absence of gravity (e.g., Lamb 1991), it is impossible to determine the precise changes in the polarization produced by gravity. While admitting the truth in these arguments, we maintain that as long as the inner part of accretion disk is unstable, which is almost certainly the case, and as long as the black hole exists there, the *energy-dependent variability* trend, studied here, should always remain, irrespective of any local physics. Also, as the emission from rotating matter around a central object may yield rapid flux variability because of the Doppler effect, but not necessarily produce polarization variability if the central object is not gravitationally strong, as the variability is only related to the light bending effect of the black hole. Therefore, we argue that polarization variability measurements provide, in principle, a more sensitive test for black hole effects than can variations in the total flux.

Moreover, observed polarization variability can provide additional information on the mass of the central black hole, the viewing angle of the observer, and the physics of the accretion disk. First, the shortest timescale of the polarization variability due to the orbital motion is $T = 4.6 \times 10^4 m_8$ seconds (Bao & Østgaard 1994), where m_8 is the mass of the black hole in units of $10^8 M_\odot$; thus the observed shortest timescale of polarization variability can yield information on the black hole mass. Second, as was shown in §5 (see Fig. 4), by comparing the polarization variability amplitude of an observed source with a standard object whose inclination is well defined through other reliable measurements (such as the width of optical emission lines, or the orientation of nuclear radio jets), one may set a limit on the inclination for the observed source. Third, instabilities of an accretion disk may exist in a limited region of the accretion disk, outside of which we should not observe polarization variability even when that stable part is still emitting polarized light. The border of this region would correspond to the lowest energy that still displays polarization variability, after which the polarization amplitude will remain a constant through all lower energies. Knowledge of this kind may shed light on the nature of instabilities to which accretion disks are subject. Last, but not least, since the variability amplitude also depends on the temperature distribution of the clumps, it may provide clue to the local physics of the accretion disk.

It is well established that the orbiting matter around black holes may create rapid variability in fluxes (Abramowicz et al. 1989; Mangalam & Wiita 1993) and is subject to both general relativistic effects and Doppler effects. Note that both of these effects are r-dependent. The feature of energy-dependent (or r-dependent) polarization which we are studying in this work is purely produced by general relativistic effects (not other non-gravitational ones), and is apparently unique to the clumpy disks around black holes. Thus, comparing the flux and polarization changes in the observed signal should therefore be very helpful in separating disk physics effects from pure relativistic ones.

Several particular types of polarization variations in AGN in both the radio and optical portions of the spectrum have been observed. These include both temporal and frequency changes in fractional polarization and in polarization angle (e.g., Ballard et al. 1990; Hughes, Aller, & Aller 1985, 1991; Impey, Lawrence, & Tapia 1991; Jones et al. 1985; Quirrenbach et al. 1991, 1992; Saikia & Salter 1988; Valtaoja et al. 1990, 1991; Wagner et al. 1990). The modeling of polarization variations has concentrated on radio emission and shock-in-jet models (e.g., Hughes et al. 1985, 1991; Gopal-Krishna & Wiita 1992), and with few exceptions (e.g., Krishan & Wiita 1996), polarization variation at higher frequencies has also been addressed within shock-in-jet models (e.g., Marscher, Gear & Travis 1992). The preliminary work of Bao et al. (1996) and this present study have revealed an alternative component of AGN polarization variability, thus contributing to a more comprehensive understanding of the phenomenon.

Realistic accretion disks will probably have various opacities dominating in different portions of the disk and the optical depth will vary from place to place. This will modify the intrinsic polarization we used. For example, if the optical depth, $\tau \sim 1$, then δ will be quite different from Chandrasekhar's value. Also, for very low optical depth the angle of the polarization plane will no longer lie in the disk, but will be perpendicular to it. Moreover, the geometry of the accretion disk will also affect the intrinsic polarization. For example, a thick accretion disk may have a bigger intrinsic percentage polarization than would a thin disk for a given inclination (e.g., a face-on disk may have a non-zero degree of polarization). All of these generalizations could be important for some specific situations and will be studied in the future.

Though the model presented here has not considered the detailed physical mechanism of the source, our calculation elucidates the kind of effects possible from more detailed models. Our conclusions are grounded on calculations using a non-rotating black hole; however, they also should basically apply to Kerr black holes. This is because the difference between Kerr and Schwarzschild black holes is marginal in terms of their first-order images (Cunningham & Bardeen 1973; Bao, Hadrava, & Østgaard 1994). While our calculation considers the emitting clumps to be geometrically thin, a more precise model would have to include the geometric size of the clumps, in which case the local spectrum as well as the polarization properties may be quite different from the Newtonian value and vulnerable to relativistic effects. This implies that relativistic radiative transfer within the matter should really be considered, however, the incorporation of such complex considerations should not greatly affect the basic results obtained here.

This work is supported by NASA grant NAG 5-5098, and by Chancellor's Initiative Funds at Georgia State University.

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Figure captions

Fig. 1. Variation of linear polarization (δ) and variation of plane of polarization (φ_p in degrees) with orbital phase for different eccentricities ($e = 0.0$, solid line; $e = 0.3$, dashed line; $e = 0.6$, long dashed line) with inclination of $i = 80°$ and semimajor axis $a = 20m$.

Fig. 2. Variation of linear polarization (δ) and variation of plane of polarization (φ_p in degrees) with orbital phase for different inclinations: $i = 50°$ (left), $70°$ (middle), $85°$ (right), and different radii: $r = 8m$ (solid line), $20m$ (dashed line), $100m$ (long dashed line). The source is on a circular orbit.

Fig. 3. Variation of linear polarization (δ) and variation of plane of polarization (φ_p in degree) with orbital phase in special relativity for inclination $i = 85°$ at different radii, i.e., $r = 8m$ (solid line), $20m$ (dashed line), and $100m$ (long dashed line). The source is on a circular orbit.

Fig. 4. The variability amplitude of δ_ν (left) and variability amplitude of φ_p (right) at 2×10^{-3} Hz plotted as a function of hard X-ray energies. One hundred sources were uniformly distributed on the surface of a Novikov-Thorne disk around a $10^6 M_\odot$ black hole from $r_{in} = 4r_g$ to $r_{out} = 60r_g$. All sources have equal life-time taken as $4p(r_{out})$. The dashed line is for inclination $i = 50°$; the solid line for $i = 80°$.

Fig. 5. The variability amplitude of δ_ν (left) and variability amplitude of φ_p (right) at 2×10^{-3} Hz plotted as a function of photon energies. The inclination of the accretion disk is $i = 80°$; otherwise as in Fig. 4.

Fig. 6. The variability amplitude of δ_ν (left) and variability amplitude of φ_p (right) at 2×10^{-3} Hz plotted as a function of hard X-ray energies. The inclination of the disk is $i = 50°$. All other conditions are the same as in Fig. 4. Solid line for $a = 1,\ b = 0$, dashed line for $a = 0,\ b = -1$.