Energy-dependent polarization variability as a black hole signature

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Abstract

A fully relativistic approach is used to calculate the observed polarization properties of sources orbiting a black hole. Unlike in the Newtonian case, an irregular matter distribution around a black hole exhibits energy-dependent polarization variability; both the observed degree of polarization and the angle of the polarization plane have larger variability amplitudes at higher frequencies. These effects are dominated by the strong bending of light rays by the central mass. Observational confirmation of this predicted effect may serve as a rigorous test of the presence of a black hole.

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Rapid X-ray and UV variability of both Galactic black hole candidates and active galactic nuclei (AGN) with timescales comparable to the time light requires to cross a black hole suggests that we are indeed observing radiation from matter near black holes, though we note that Doppler boosted synchrotron self Compton radiation from relativistic jets probably dominates the variable emission observed from BL Lacertae objects and some other radio loud AGN [1,2]. Nonetheless, despite substantial dynamic evidence [3–5], there is still no *direct* proof that black holes exist in either X-ray binaries or AGN. Since there are only three observable quantities for a black hole, its mass, charge and angular momentum [6], it is highly probable that a direct proof will come from the observation of radiation from matter in orbits close to the event horizon of a black hole.

The inner regions of accretion disks around black holes in X-ray binaries and AGN are probably unstable and therefore clumpy. This scenario is strongly supported by the observed chaotic X-ray and UV/optical light curves [7–9] in both types of objects, as well as by theoretical models [10,11]. Radiation originating close to the black hole, where electron scattering probably provides the dominant opacity, is expected to be linearly polarized [12]. In this *Letter*, we demonstrate that, unlike in the Newtonian case, enhanced regions of emission orbiting around a black hole produce potentially observable rapid, energy-dependent, polarization variability in the X-ray band.

Consider first the polarization variability of a single source in a strong gravitational field, with an orbit around a Schwarzschild black hole characterized by a semimajor axis, a, an eccentricity, e, and an orbital plane inclined to the line of sight by the angle, i. The four-velocity for a photon is

$$\begin{aligned} u^t &= \frac{r}{r-2}, \\ u^r &= \left[1 - \frac{r-2}{r^3}B^2\right]^{1/2}, \\ u^\theta &= r^{-2}\left(B^2 - L^2/\sin^2\theta\right)^{1/2} \\ u^\phi &= r^{-2}L/\sin^2\theta, \end{aligned} \tag{1}$$

where units where $G = c = M_{\rm BH} = 1$ are used. The total impact parameter of the photon is B and L is the impact parameter around the symmetry axis. Since photon motion is planar in Schwarzschild space-time, it can be expressed solely in terms of B.

The velocities of the source measured by a static observer are

$$v^{(j)} = \frac{\vec{u}\cdot e^{(j)}}{u^{(t)}}, \tag{2}$$

where $\vec{u}$ is the four-velocity of the orbiting source and $e^{(j)}$ are components of the basis vector for a local observer. The non-zero components of $v^{(j)}$ are:

$$v_1 = \left[1 - \frac{r-2}{r}\Big(\frac{1}{E_s^2} + \frac{1}{r^2}(\frac{l_s}{E_s})^2\Big)\right]^{1/2}; \tag{3}$$

$$v_2 = \sqrt{1 - \frac{2}{r}}\frac{1}{r}\frac{l_s}{E_s}, \tag{4}$$

where $v_1 \equiv v^{(r)}$, $v_2 \equiv v^{(\phi)}$, and the energy E_s and angular momentum l_s of the orbiting source are given by: $E_s^2 = [(a-2)^2 - a^2e^2]\{a\,[a(1-e^2) - 3 - e^2]\}^{-1}$, $(l_s/E_s)^2 = [a^3(1-e^2)]\,[(a-2)^2 - a^2e^2]^{-1}$.

Here we consider a special case, where the intrinsic polarization direction is taken to be in the orbital plane. We note that the results obtained do not depend on this assumption. The polarization vector in the comoving frame reads,

$$p^{\hat{\mu}} = \frac{1}{\sqrt{u^{\hat{t}2} - u^{\hat{\theta}2}}}(0, \quad -u^{\hat{\phi}}, \quad 0, \quad u^{\hat{r}}). \tag{5}$$

Any vector $\vec{f}$ in a static frame may be obtained through Lorentz transformation

$$f^{(\mu)} = \Lambda^{(\mu)}_{\hat{\nu}} f^{\hat{\nu}}, \tag{6}$$

where

$$\Lambda^{(\mu)}_{\hat{\nu}} = \begin{pmatrix} \gamma & \gamma v_1 & 0 & \gamma v_3 \\ \gamma v_1 & (\gamma-1)\frac{v_1^2}{v^2} + 1 & 0 & (\gamma-1)\frac{v_3 v_1}{v^2} \\ 0 & 0 & 1 & 0 \\ \gamma v_3 & (\gamma-1)\frac{v_3 v_1}{v^2} & 0 & (\gamma-1)\frac{v_3^2}{v^2} + 1 \end{pmatrix} \tag{7}$$

$$\gamma = \frac{1}{\sqrt{1-v^2}}, \qquad v^2 = v_1^2 + v_3^2. \tag{8}$$

Since photon motion is planar, any angle measured in the static frame should be the same as that measured by an observer at infinity, so deriving the polarization angle in the static frame is sufficient.

The unit normal vector $\vec{n}$ of the plane in which the photon moves reads,

$$n^{(\nu)} = \frac{1}{\sqrt{u^{(t)^2} - u^{(r)^2}}}(0,\ 0,\ -u^{(\phi)},\ u^{(\theta)}). \tag{9}$$

Thus, the angle between the polarization vector and the normal of the photon-motion plane is

$$\psi = \cos^{-1}[n^{(\theta)}p_{(\theta)} + n^{(\phi)}p_{(\phi)}]. \tag{10}$$

Finally, the observed angle of the polarization plane is expressed as

$$\psi_p = \psi' + \psi \pm \frac{\pi}{2}. \tag{11}$$

where ψ' is the observed phase of the orbiting source, and "$\pm$" means that ψ_p describes the orientation of the polarization plane. As $n^{(\nu)}$ and $p^{(\nu)}$ are expressed in terms of the four-velocity of the photon, they are solely determined by the total impact parameter of the photon B. Since B is a function of $i,\ \phi_s,\ e$, and r, i.e., $B = B(i, \phi_s, e, r)$, we have

$$\psi_p = \psi_p(i, \phi_, e, r). \tag{12}$$

There is no analytic expression for B, which must be solved through numerical calculation [13,14], but it is a strong function of r.

As a simple, but important, example, we assume that the polarization is caused by electron scattering [6]. In this case the original electric vector of the polarization lies in the disk plane, and the degree of polarization, δ, is a function of the angle, θ, between the disk normal and the direction in which the photon is emitted [15]. The value of δ changes from 0.117 when the disk is viewed edge-on to 0 when the disk is face-on. This angle, measured in the comoving frame, is

$$\cos\hat{\theta} = \frac{p^{\hat{\theta}}}{p^{\hat{t}}} = \frac{\sqrt{(\frac{B}{r})^2 - (\frac{L}{r})^2}}{\gamma\{(1-\frac{2}{r})^{-1/2} - \frac{v_1^2}{v}\sqrt{(1-\frac{2}{r})^{-1} - (\frac{B}{r})^2} - \frac{v_3^2}{v}\frac{L}{r}\}}, \tag{13}$$

where $\gamma = (1 - v^2)^{-1/2}$ with $v^2 = v_1^2 + v_3^2$. This demonstrates how gravitational bending of light changes the angle at which rays leave the surface of the source from the Newtonian value

i. Since the fractional polarization depends on $\hat{\theta}$, ultimately, $\delta = \delta(i, \phi_s, e, r)$. Although δ is a Lorentz invariant (it does not change along a given ray), it does change during the orbital period in which different beams of photon are emitted. Both the observed degree of linear polarization and the observed angle of the plane of the polarization change rapidly within the orbital period; this is illustrated for different values of r and i in Fig. 1. Relativistic effects (gravitational bending and special relativistic aberration) play vital roles in these changes. The amplitude of the changes depend on the parameters r, i, and e of the source orbit. These features distinguish this picture from the Newtonian case, where no significant changes in either the degree or the plane of polarization are likely to occur, as we now show.

In special relativity we consider, for simplicity, only a circular orbit ($v_1 = 0,\ v_3 = v = r^{-1/2}$). In flat spacetime, photons go along straight lines, and we have an analytic expression for the total impact parameter, which allows one to derive easily the expression for the polarization angle. Ignoring general relativistic (GR) effects, the angle between the polarization vector and the plane of the photon is

$$\psi = \cos^{-1}\{\cos\psi' \sin i \gamma[\gamma^2(1 + v\sin i \sin\phi_s)^2 - \cos^2 i]^{-1/2}\}, \tag{14}$$

where in this approximation, $\gamma = [r/(r-1)]^{1/2}$. For the slow motion Newtonian case, ($v \ll 1,\ \ \gamma \approx 1$), the physical solution of the polarization angle is $\psi_p \approx \pm\frac{\pi}{2}$, implying that the polarization remains in the disk plane and does not rotate during the orbital motion of the source; (ψ_p is measured from the projected symmetry axis in the observer's sky). Similarly, we find that

$$\hat{\theta} = \cos^{-1}\{\cos i[\gamma(1 + \frac{v}{r}\sin i \sin\phi_s)]^{-1}\}. \tag{15}$$

When $r \gg 1$ we get, unsurprisingly, $\hat{\theta} = i$. We thus conclude that for the same initial polarization, the Newtonian approximations to the plane of polarization and to the degree of the polarization, measured at infinity, are nearly the same for each point of origin on the disk, i.e., r-independent.

As it is simultaneously subjected to many different instabilities and varying modes thereof, the innermost part of an accretion disk is likely to be home to many orbiting sources of varying amplitude rather than being dominated by a single bright region [16,17]. The distribution of the temperatures of these source regions is expected to be a rapidly varying function of their distances from the central black hole [18]. Therefore the polarization variability of a collection of sources should be frequency- (or energy-) dependent. The polarization of radiation from many sources can be described by the normalized Stokes parameters, $u_\nu = U_\nu / I_\nu$ and $q_\nu = Q_\nu / I_\nu$. If the polarization is induced by electron scattering, there is no circular polarization for initially unpolarized radiation. The degree of polarization and the angle of the plane are, respectively

$$\delta_\nu = \sqrt{u_\nu^2 + q_\nu^2}, \qquad \psi_p = \frac{1}{2} \tan\Big(\frac{u_\nu}{q_\nu}\Big). \tag{16}$$

The Stokes parameters are calculated through

$$u_\nu = \frac{\sum_{i=1}^{n} \int g_i^3 \delta_i \sin(2\psi_p^i) I_\nu^i d\Omega_i}{\sum_{i=1}^{n} \int g_i^3 I_\nu^i d\Omega_i} \quad \text{and} \tag{17}$$

$$q_\nu = \frac{\sum_{i=1}^{n} \int g_i^3 \delta_i \cos(2\psi_p^i) I_\nu^i d\Omega_i}{\sum_{i=1}^{n} \int g_i^3 I_\nu^i d\Omega_i}, \tag{18}$$

where ψ_p^i and δ_i are the polarization angle and the fractional polarization of the i^{th} source, Ω_i is the solid angle that source subtends at an observer, and

$$g_i = \frac{p^t(\infty)}{p^{\hat{t}}(r)} = \frac{1}{\gamma\{(1-\frac{2}{r})^{-1/2} - \frac{v_1^2}{v}\sqrt{(1-\frac{2}{r})^{-1} - (\frac{B}{r})^2} - \frac{v_3^2}{v}\frac{L}{r}\}} \tag{19}$$

is the redshift factor for that source [14]. We have considered cases in which sources are distributed randomly in their phases but have power-law distributions in their radii. Fig. 2 presents the variability amplitude as a function of energy from a Novikov –Thorne accretion disk [18]. The amplitude shown is the square root of the variability power density at a frequency of 2×10^{-3} Hz for $M_{\rm BH} = 10^6 M_\odot$, and is a strong function of photon energy. Both the degree and the angle of the plane of polarization vary more at higher energies in the X-ray band. Unfortunately, no recent X-ray satellites have had polarimetry capability, though such an instrument was planned for the future AXAF mission, and could be capable of detecting the degree of variability predicted here.

The radiation arises from the hot X-ray emitting matter near the black hole. Electron scattering opacity is important for these temperatures, and thus any initially unpolarized radiation undergoing electron scattering becomes partially polarized with its polarization vector lying in the disk plane. It was realized long ago that X–ray polarization features could offer information on the accretion disk structure [12] and that unlike the spectrum, polarization is strongly affected by GR light bending [19,20]. Unlike the Newtonian case, where the degree and the angle of polarization of photons emerging from an accretion disk are both essentially constant through hard X-ray energies, in the GR case, both the observed degree of polarization and the angle of the polarization plane are strong functions of photon energy.

It is well established that excess emission from patches of the orbiting matter around black holes can create the rapid fluctuations in brightness seen in radio quiet AGN from the optical [17] through X-ray [16] bands. The observed emission is significantly modified by both GR as well as Doppler effects, both of which are r-dependent. The type of energy-dependent (or r-dependent) polarization variability described here should be a unique signature of black holes fed by accretion disks.

One might argue that too little is yet known about the inner disk region close to a black hole, so that the actual situation may be far more complicated. Also, since confident predictions of the polarization properties of an accretion disk are not available even in the absence of general relativistic effects, it will certainly be difficult to determine the changes in the polarization produced by strong gravity. While admitting the truth of these points, we maintain that as long as the inner part of an accretion disk is unstable and hotter, and as long as the black hole exists there, the energy-dependent variability trend should always remain, as it barely depends on local physics. In other words, polarization variability measurements provide a sensitive test for the effects of a black hole.

While our calculations were made for a non-rotating black hole, the fundamental aspects will also apply to rotating black holes, in that the differences between the first-order images of Kerr and Schwarzschild black holes are marginal [21,13]. Although the simple model presented here has not considered the detailed physical mechanisms involved in the polarized emission or the temperature variations of the sources, our calculation should give a good approximation to the main effects to be expected in a more detailed model.

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FIGURES

FIG. 1. Variation of the observed fractional linear polarization (δ, top) and variation of the observed plane of polarization (ψ_p in degrees, bottom) with orbital phase (in degrees), for three different radii and two different inclination angles. The emitting source is always assumed to be on a circular orbit.

FIG. 2. The variability amplitude of δ_ν (top) and variability amplitude of ψ_p (bottom) at a temporal frequency of 2×10^{-3}Hz plotted as a function of observed photon energy. One hundred sources of luminosity were uniformly distributed in area from $r_{in} = 8GM/c^2$ to $r_{out} = 120GM/c^2$ on the surface of a relativistic accretion disk, which prescribes their temperatures, once a value of the viscosity parameter, α, is chosen. All sources have circular orbits and equal life-times taken as 4 times the orbital period at r_{out}.