

# A Report on the Theory of Pulsating Stars

## Introduction

Pulsating stars are entities that experience a periodic, driven, expansion/contraction cycle in their outer layers – this leads to concurrent variation of the star's luminosity. This paper will focus on the current state of our understanding of the pulsation mechanisms, as well as these stars' applications as test beds for stellar structure/evolution theory and cosmology. Since an exhaustive list of pulsating stars and their governing physics would make a decent textbook, this work will only focus on a *few* of the most well-known and/or useful types of pulsating stars as illustrative examples.

## Stellar Pulsations

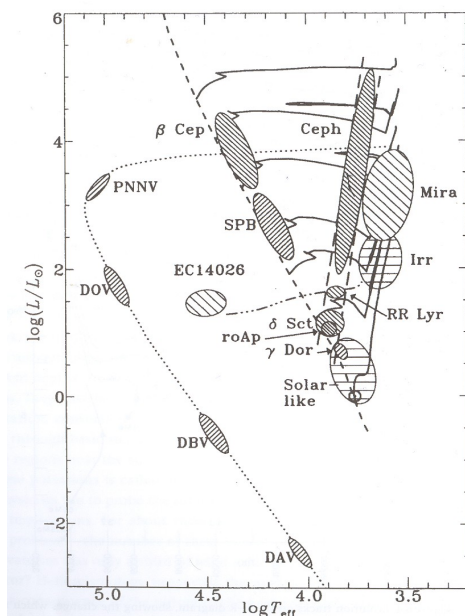
First, it is essential to define what exactly a pulsating star *is*. Pulsating stars (hereafter referred to as **pulsators** for brevity's sake) are observed to be variable in brightness; this variation being the result of changes in the stars' radius, either over the whole or over part of the star's surface (photosphere). These inward-outward motions are described as pulsation *modes*, which occur in *radial* and *non-radial* varieties.

*Radial* modes often produce the largest changes in a star's radius, leading to large changes in luminosity, temperature, and radial velocity. The simplest radial mode is the *fundamental* mode, in which the entire atmosphere of the star expands/contracts in unison. The *first overtone*, the second-simplest pulsation mode, involves the outer zone of the star's envelope expanding whilst the inner zone contracts, and vice versa, while the mass in a thin nodal shell between the two does neither. Higher-order modes have more of these opposed expansion/contraction zones, with the number of nodes being equal to the order of the overtone. Radial modes are dominant in extended envelopes of evolved stars, such as the well-known Cepheids, RR Lyrae, and Mira types (Percy).

*Non-radial* modes are more complicated, as in these modes some parts of the star's surface move inwards while other parts move outwards. Whereas one number, the order of the overtone, is

used to describe radial pulsations, *three* numbers are used to describe non-radial pulsations, *three* numbers are used to describe non-radial pulsations, *three* numbers are used to describe non-radial pulsations. These are the quantum numbers  $n$  (radial), degree  $l$ , and azimuthal number  $m$ . Here the number of nodal lines on the surface of the pulsator is given by  $l$  (DSN). Stars with non-radial pulsations often have very complicated but potentially useful pulsation spectra, a topic that will be expanded upon in a later section. Examples of non-radial pulsators include pulsating white dwarfs, Gamma Doradus stars, Beta Cepheids, and Delta Scuti stars.

To get an idea of just how common pulsating stars are in the universe, the reader's attention is directed to **Figure 1**, at left. Note that not all pulsating stars are giants: several pulsator classes, such as the Beta Cepheids and Delta Scuti



stars, are in fact main sequence, hydrogen burning stars. Several are even more evolved than the Classical Cepheids, such as the pulsating white dwarfs (Percy).

Other terms used to describe non-radial pulsation modes are *p-modes* and *g-modes*. P-modes are pressure (or acoustic) waves, like sound waves that travel through the air. G-modes are gravity waves, and as the name suggests gravity is their restoring force (DSN).

Note: "Understanding Variable Stars" by John R. Percy has proven to be an invaluable, comprehensive resource for this topic. Thus, that work will be frequently cited in these pages.

### **Pulsation Mechanisms & the Instability Strip**

We now begin a discussion as to what exactly *causes* these pulsations. Stars are a great balancing-act: the outward pressure generated by the core is constantly opposed to the ever-present force of gravity, attempting to crush the star to a point. The result is a body in hydrostatic equilibrium (mostly), which is neither shrinking nor expanding. Pulsations in stars, then, basically just amount to an oscillation about this equilibrium point. The question is, then, what causes these oscillations?

For many pulsators, the culprit is the well-known  $\kappa$  mechanism. As one might surmise from the title, the  $\kappa$  mechanism is an opacity-driven phenomenon. As one moves inward from the surface through the atmosphere of a star, one passes through regions that are progressively hotter and more ionized. Below the photosphere (normally) one first encounters the hydrogen ionization zone, and below that the first helium ionization zone. It is in the helium ionization zone where many stars gain the ability to become pulsators. The theory goes like this: begin with a non-pulsating star, with a sufficient surface temperature such that the He ionization zone contains a significant portion of the envelope mass. The helium is only singly ionized, meaning it still has one electron bound to the nucleus. Suppose a small perturbation causes the star to contract: this causes the temperature of the star to rise via elementary thermodynamics, since we know that reducing the volume that a gas occupies increases its temperature. The extra thermal energy associated with this causes the helium to become fully ionized – this increases the opacity of the entire layer in a manner known as an *opacity bump*. This increased opacity is more effective at capturing radiation streaming out of the core of the star, which causes the helium layer to expand outward: this naturally leads to an expansion of the star's total radius. But as the star expands, and thus cools, the electrons can recombine with their helium nuclei, which lowers the opacity, reduces the layer's ability to absorb radiation, and causes the layer to contract, reducing the radius of the star. This contraction triggers the next expansion via the manner described above, and thus the star pulsates in radius (Percy).

All of this, of course, depends on whether or not the  $\kappa$  mechanism is able to lift the stellar material in and above the helium ionization layer: i.e. whether it acts as a restoring or damping force, and is able to overcome other damping effects such as non-adiabatic heat transfer between different stellar layers and the inertia of the stellar material itself. Consider the equation for a luminosity perturbation  $\lambda$ , calculated for a quasi-adiabatic approximation:

$$\lambda = \frac{P_0}{\nabla_{ad} P_0} \nabla_{ad} P'_ad + 4 \nabla_{ad} P_{ad} - 4 \nabla_{ad} P_{ad} - (\kappa_P + \nabla_{ad} \kappa_T) P_{ad}$$

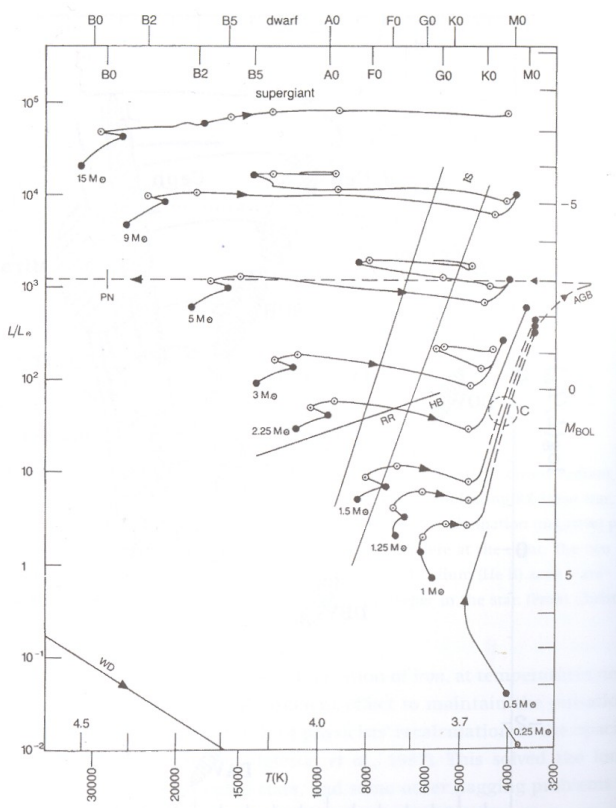
where  $P_0$  is the initial pressure,  $\kappa_p$  is the derivative of  $\ln(\kappa)$  of the opacity with respect to  $\ln(P)$  at constant  $T$ ,  $\kappa_T$  is the derivative of same with respect to  $\ln(T)$  at constant  $P$ ,  $\nabla_{ad}$  is the adiabatic gradient,  $p_{ad}$  is an adiabatic eigenfunction, and primed values represent perturbations from the original. The condition for convective stability in this situation is given by the sign of the right-hand side of the following term:

$$4 \nabla_{ad} P_{ad} - (\kappa_P + \nabla_{ad} \kappa_T) P_{ad}$$

If the side of the right-hand side is  $> 0$ , then the star is stable against pulsation, and if it is  $< 0$  the star will be prone to pulsations (Kippenhahn).

Of course, the  $\kappa$  mechanism cannot account for all observed pulsation behavior. For instance, it only works for stars of a moderate temperature, of say around 6,500 K: this temperature defines the blue edge of the **Cepheid instability strip** (Kippenhahn). This is a band on the Hertzsprung-Russel Diagram (HRD) that contains several variables: Cepheids, RR Lyrae, Gamma Doradus, and Delta Scuti variable stars, all of which pulsate due to a helium opacity bump. Stars with greater surface temperatures do not pulsate due to the helium-based  $\kappa$  mechanism because helium in such stars is almost completely ionized up to the photosphere.

However, the  $\kappa$  mechanism can still produce pulsations in stars such as the Beta Cepheids, which rely on partially ionized *iron* at deep layers instead of helium. The red edge (low temperature)



of the strip is due to the rising prevalence of convection in lower-temperature stars, which serves to damp the pulsations of Cepheids, since the  $\kappa$  mechanism works by trapping energy transported via *radiation*, and not *convection*. Convection, though, does act as a driving force for pulsations in some stars, notably the pulsating red giants farther to the right on the HRD, and in stars like our own Sun (Xiong). Note also that the Cepheid instability strip is not a vertical band, but one that slopes toward lower temperatures as one goes to higher luminosities on the HRD. The reason for this lies in the physical phenomenon that gives rise to the pulsations in the first place: helium ionization. The higher-luminosity stars are progressively less dense in their outer regions, and since helium ionizes more easily at lower densities for a constant temperature, the temperature needed

to ionize helium decreases as one moves up the HRD, and hence the Strip shifts toward lower temperatures (Percy).

One interesting fact about the Cepheid instability strip is the vast variety of stars, in terms of the range of initial mass, which will, at some point in their evolution, call the strip home. **Figure 2**, above, shows an HRD which displays the partial evolution track of several stars of various masses, along with the Cepheid instability strip, denoted by two parallel lines on the right side of the diagram. Notice how stars not born on the Strip eventually move there at some point in their post-MS evolution, sometimes at several different evolutionary phases (the stars actually born in the instability strip are the Delta Scuti stars) (Percy).

Yet another potential source for pulsations is known as the *Mechanism*. This mechanism is discussed on pp 412 & 417 of Kippenhahn, and basically shows how the same small oscillations that give rise to pulsations via the kappa mechanism can have the same effect by modulating the nuclear energy generation rate in an oscillatory way. Hence, this method can only apply to parts of the star in the core, where energy is produced. This mechanism is important in stars where radiation pressure vastly dominates as the energy transport mechanism, such as in the high-mass end of the hydrogen main sequence. Numerical models indicate stars above about  $60 M_{\text{sol}}$  should be unstable to pulsations. These stars are not observed because the pulsations set up by this mechanism have the effect of ejecting material from the surface of the star, until a total stellar mass of  $60 M_{\text{sol}}$  is reached and pulsations are unable to continue (Appenzeller). This mechanism is, however, important for some evolved stars, particularly the long-period red giants and Mira pulsators.

### **Building Models of Stellar Pulsations**

Building a theoretical model that accurately describes the observed behavior of a particular pulsating star is, not surprisingly, extremely difficult. A complete model must take into account several variables, including: the nuclear energy generation rate in the core, the chemical composition of each layer, its initial mass and composition, temperature gradient, the importance of convection, and how all of these and *more* parameters change with time. Fortunately, perfection need not have been achieved on the first try, as there have been several methods tried successively and successfully that have gradually increased our understanding of these objects. In order to not get bogged down in gory, nigh-incomprehensible detail pertaining to codes used by individual groups, I will describe, in general, the three distinct types of models that have thus far been used.

The first (and simplest) modeling method is the *linear, adiabatic theory* (LAT), first proposed by Arthur Eddington in the early twentieth century. The main assumptions for this theory are 1) small-amplitude oscillation about a static equilibrium point (Christy), and 2) no energy transfer between zones in the star (hence adiabatic). While this method, due to its simplicity, is unable to accurately reproduce the pulsation behavior in pulsators, it is able to illuminate the relationships between some pulsation behavior and certain stellar parameters, especially for the simpler radial modes. For instance, the method shows that “*the period of any mode is approximately inversely proportional to the square root of the mean density of the star*” (Percy). This is what gives rise to the famous period-luminosity relation among some classes of pulsators (notably Cepheids). This should make sense: stars inside the

instability strip are of roughly equal temperature, and pulsation period & luminosity are related to the radius of the star, so period and luminosity must be related to each other. LAT also predicts that pulsation amplitude should be inversely related to density, so that the outer layers of a star, where density is lowest, should experience greater expansion/contraction.

The next-most complex modeling method comes in the form of *linear, non-adiabatic theory* (LNAT). As could be deduced from the name, this method takes into account energy transfer between different parts of the star. This allows for the possibility for certain modes to grow in amplitude at the expense of other modes, depending of the physical properties of a particular pulsator. In other words, the method allows theorists to be able to delineate between modes which would be stable or unstable in a given pulsator. However, this method is still unable to predict what the final amplitude will be for a given mode. This method is most useful when looking at multi-mode pulsators. The ratio of periods of the observed pulsator can be compared with theoretical period ratios to gain information about the properties of the star.

Finally, we come to the most complex class of methods yet proposed: *non-linear, non-adiabatic theory*. This basically entails building a detailed hydrodynamic and time-dependent model of a star, and taking into account the motions of and forces acting on each layer of the star. The model is subjected to many slight perturbations and allowed to run through several cycles, until it finally settles on a stable amplitude in one or a group of modes. It is important to note that even with these sophisticated models, pulsation periods and amplitudes can only found for stars that exhibit only one or two radial modes, and cannot predict amplitudes of multi-mode and/or non-radial pulsators.

### **Helioseismology & Asteroseismology**

Helioseismology and asteroseismology, like their terrestrial counterpart, aim to use the vibrations (pulsations) manifest in the surfaces of stars to deduce information about their physical parameters and internal structure. Many of the more famous pulsators, like the Cepheids, pulsate radially in the fundamental mode or one of the lower (1<sup>st</sup> or 2<sup>nd</sup>) overtones. This allows them to keep their spherical symmetry during the course of the pulsation. But there are many pulsators (indeed, some would likely say most of them) that do not behave in this way: they pulsate in non-radial modes, higher-order radial modes, a mix of radial & non-radial modes, a mix of *p*- and *g*-modes, or in combinations of all these. All of these different modes acting together in the same star have the effect of adding more observed pulsation periods to the star.

What may look like nightmarish problem, however, might actually be a blessing in disguise. As discussed previously, measurement of a single pulsation period can give an estimate of the average density. But in the case of, say, a double-mode Cepheid, the two expressed periods can yield measurements of the star's mass and radius. Each period can thus be thought of as an individual measurement of the stellar structure, so the more periods being expressed by a star, the more information about the star one has.

Often, multi-mode pulsators have not just two periods expressed, but *several*. Our Sun, the most accessible multi-mode pulsator, possesses a staggering  $10^6$  pulsation modes! These are *p*-modes, driven by pressure, and have periods of about 5 minutes (compared to ~1 hour for the fundamental

radial mode). This abundance of information, combined with detailed pulsation models, has produced extremely detailed information about the interior structure of the Sun, for example the depth of the convective zone. The science of using surface pulsations to study the Sun's interior is called **Helioseismology** (Christensen-Dalsgaard).

The extension of this method of determining the internal structure of other stars is called **Asteroseismology**. Asteroseismology theorists attempt to construct realistic pulsation spectra by constructing a model pulsating star and subject it to perturbations until it settles on one or more p- or g-modes, described by the previously-mentioned  $n$ ,  $l$ , and  $m$  quantum numbers. The p-modes can be thought of as sound waves, and contain information about the atmosphere of the star, since they cannot penetrate the core, whilst g-modes are oscillations driven by buoyancy forces (gravity) and can penetrate the core of the star. The synthetic pulsation spectra produced by these models are then compared with pulsation spectra of real stars to test the accuracy of the models (DSN). Asteroseismology studies focus on classes of pulsators that are prone to have members with rich pulsation spectra, such as Delta Scuti and white dwarf stars, as opposed to the typically mono-periodic Cepheids.

## Cepheids

Our discussion of various types of pulsators begins with perhaps the most well-known class of pulsating stars, the Classical Cepheids (hereafter simply referred to as Cepheids). These stars exhibit single-mode, radial pulsations of one to over a hundred days, with absolute magnitudes between -2.6 to -5.3 and pulsation amplitudes between 0.4 and 1.4 magnitudes. As has already been described, the mechanism responsible for the pulsations is an opacity bump due to the ionization of helium in a sub-photosphere layer. These stars have spectral types between F and K and lie above the main sequence on the Hertzsprung-Russell diagram, making them evolved (non-hydrogen-burning) stars (Percy). By definition, they also lie on the Cepheid instability strip, which is a zone a few hundred Kelvin wide, going nearly vertically up the HR diagram.

Looking back at **Figure 2**, one can see that stars of a wide range of zero-age main sequence masses can all be called “Cepheids” at one or more points in their post-main sequence evolution. For example, notice the  $5 M_{\text{sol}}$  star in the figure. It crosses the strip no less than three times during its post-main sequence life: the first during the crossing of the Hertzsprung Gap after the exhaustion of core hydrogen (though this is a very rapid phase of evolution and not likely to be observed), and two more times during various phases of shell burning.

It is prudent to note here that Cepheids are organized into two groups: Type I and Type II. The Type I Cepheids are the true “Classical” Cepheids. These are evolved Population I stars, with masses greater than the Sun, which allows many of them to have evolved off the main sequence by the current epoch.

The primary reason for interest in these stars is their utility as “standard candles” in cosmological distance measurements. The pulsation period of a Cepheid is tightly correlated with its mean intrinsic luminosity, such that longer-period Cepheids are intrinsically brighter than short-period ones. They follow the correlation between average density of the pulsator and the pulsation period

given by:

$$\Pi\sqrt{\rho_{avg}} = Q$$

where  $Q$  is known as the *pulsation constant* (Kippenhahn). Recall that this relation was derived above in the description of LAT models. These stars derive their importance from several properties, including tight period-luminosity relation, fairly bright absolute magnitude (they can be seen out to the Virgo Cluster), and the relative frequency with which they appear in the local universe. This makes them one of the strongest rungs on the cosmic distance ladder, and an indispensable part of the establishment of an extragalactic distance scale that is not dependent on the cosmic microwave background radiation (Bono).

However, Cepheids have been studied for almost a century, which has naturally led to that classic problem in the sciences where we have been looking at a simple problem so long that it has, in fact, ceased being simple.

One of the problems with Cepheids was discovered by comparing the populations of the stars in our Galaxy, the LMC, and the SMC. One of the cornerstone assumptions of using Cepheids as standard candles is that all Cepheids in the universe have the same Period-Luminosity-Color (PLC) relation: color being important for making corrections due to reddening by dust in the interstellar medium. Recall also that color is a function of temperature, which affects the observed luminosity. Consideration of the Stefan-Boltzmann law and the fact that radius fluctuations are small in Cepheids leads to the following relation between the minimum and maximum temperatures during a pulsation cycle and the minimum and maximum V-band magnitudes:

$$\log T_{\max} - \log T_{\min} = \frac{1}{10}(V_{\min} - V_{\max})$$

It has been observed that discrepancies exist for the Period-Color (PC) relations for the aforementioned populations, namely that during maximum light, Cepheids with periods  $> 10$  days the PC relation flattened out, giving these Cepheids an almost constant spectral type (temperature/color). The source of this discrepancy is explained by the interaction between the photosphere and the hydrogen ionization layer (a.k.a. Hydrogen Ionization Front, or HIF), and by considering the effect of differing metallicities on this interaction. During a pulsation, the photosphere and HIF do not move in unison, and that the higher metallicities present in Galactic Cepheids cause the HIF to extend into the photosphere during maximum light for these longer-period stars. In the types of low-density environs found in the atmospheres of these stars, embedding the photosphere in the HIF has the effect of making the photosphere appear to be the same temperature as the HIF, namely at the temperature at which hydrogen ionizes. As has been mentioned, ionization temperatures for Cepheid atmospheres vary only slightly with increasing period, which explains the flattening-out of the PC relation (Kanbur).

Another problem with Cepheids is a discrepancy between the masses derived from an analysis of their pulsation periods, versus masses calculated via stellar evolution models, and consideration of

their HRD positions. The magnitude of this discrepancy is non-uniform, but a +25% discrepancy for Polaris, the nearest Cepheid, is representative (Bono). Possible explanations for this phenomenon include considering stronger rolls for rotation and radiative opacity, enhanced mass loss rates, and the possibility of convective overshoot into the core during the star's hydrogen burning phase (Keller). Currently, this is still considered an outstanding problem for Cepheid theory..

**W Virginis** stars, also known as **Type II Cepheids** are pulsating stars that occupy the same part of the instability strip as Classical Cepheids and have similar pulsation characteristics. The difference between the two is that the Classical Cepheids are younger, Population I stars, with higher metallicities, and that W Virginis stars are older, metal-poor Population II stars ( $Z=0.00001$  for Extreme Pop. II). Being older means that W Virginis stars must have lower masses than the Sun to still be around ( $M > 0.8 M_{\text{sol}}$ ). Having fewer metals means they also have greater intrinsic luminosities than Pop. I stars of the same mass, which means that W Virginis stars must have a Period-Luminosity relation that differs from that of the Classical Cepheids (Percy).

### RR Lyrae Stars

These stars are variable on timescale of a tenth to one full day, and in amplitude by up to 1.5 magnitudes. They are of spectral type A5 to F5, have masses of around half that of the Sun, and possess typically low metallicities that can vary anywhere from  $Z = 0.00001$  (extreme Population II) to  $Z = 0.01$  (half the solar value). They are divided into subclasses a, b, and c based on the dimensions of their light curves. On the HRD, they lie on the Horizontal Branch, which means that they are all at about the same mean intrinsic luminosity (about +0.6 magnitudes in V), and are currently undergoing core-helium fusion. Their common average luminosity makes it possible to get distance estimates, but due to their lack of brilliance compared to Cepheids, they are essentially restricted to measuring distances in the local group. Stars on this branch with effective temperatures ranging from 6000 K to 7500 K are unstable against pulsation, the pulsation mechanism being the same as for the Cepheids: the  $\kappa$  mechanism caused by a helium opacity bump (Percy).

The evolution of these stars is of particular interest. The small mass of these stars requires them to be quite old in order to have exhausted their core hydrogen and migrated up into the horizontal branch. This explains their low average metal content: at the time of their formation, had not been enriched in elements heavier than helium, since such nuclei are synthesized by stars over several stellar generations.

A peculiar phenomenon observed in RR Lyrae stars worth noting here is the *Blazhko Effect*, named for its discoverer. Since these stars have been studied for about a century now, modulations in their pulsation amplitudes and phases that act over timescales of decades have been observed. The amplitude modulation amounts to about 20-30% for RR Lyrae stars that pulsate in fundamental overtone (a & b subtypes, excepting the long-period pulsators which do not display this effect) and about 5% for stars that pulsate in the first overtone (subtype c). Unfortunately, there is still no widely-accepted theory as to the cause of this phenomenon. One hypothesis proposes a resonance between the radial fundamental period and a non-radial period (or possibly more than one). Like two sinusoidal waves that have different periods, the two modes slowly move in-and-out of phase over time,



alternately enhancing and damping the expressed pulsation amplitudes in a phenomenon known as “beating”. Another possible mechanism put forward to explain this behavior invokes the presence of a strong magnetic field, on the order of about 1 kG, that deforms, or even splits, the radial mode of the star. Unfortunately for this hypothesis, no RR Lyrae star displaying the Blazhko Effect has yet been observed with such a strong magnetic field (Blazhko Project).

As a group, RR Lyrae pulsators are very amenable to analysis via theoretical models. This is partly due to their populous nature – thousands are known, so the sample size is large and they tend to congregate in clusters, ensuring common ages, distances, reddening, and metallicities for co-cluster members. The other advantages are a consequence of their mostly homogeneous physical properties: they all have similar masses, ages, mean intrinsic luminosities, are undergoing core-helium fusion, and are evolving from hotter to lower temperatures. This allows grids of stellar models to be made for these stars that cover the complete range of their observed parameters.

One interesting cosmological result that came from modeling the pulsations of these stars has been that the blue edge of the instability strip is a function of  $Y$ , the helium abundance in a star. Since RR Lyrae stars are older stars, mapping a change in the position of this blue edge would map out how the abundance of helium has changed with time. The result is, surprisingly (to the author, anyway) a null one: nucleosynthesis of heavy elements in stars has certainly increased their abundance in the universe since the Big Bang, but that process has not had much of an effect on the abundance of helium (Percy).

### **Delta Scuti Stars**

Delta Scuti stars, sometimes referred to as “dwarf Cepheids”, are of spectral type A5 to F2, have pulsation periods ranging from 30 minutes to over 8 hours, and often support several modes of pulsation in a single star (giving them rich, complex pulsation spectra), though they also often have small pulsation amplitudes: down into the milli-magnitude range. They lie on the lower part of the instability strip, where it crosses the main sequence. This means two things; 1) these are main-sequence pulsators, burning hydrogen in their cores, and 2) there *a lot* of examples of this class out there, since this is a very dense part of the HRD (in fact, only pulsating white dwarfs poses higher membership amongst all the pulsating stars). Stars in this part of these spectral types on the main sequence are too massive to be as old as RR Lyrae pulsators, and possession of higher metallicities means that they are Population I objects.

From a theorist’s standpoint, the really interesting aspect about these stars is the large number of pulsation periods displayed by individual members of this group. As discussed above, stars with an abundance of expressed pulsation periods are prime targets for asteroseismological studies, since each period is a measure of the stellar interior. Delta Scuti stars can have dozens of periods that arise from several  $g$  modes and/or  $p$  modes (DSN).

Even with all this information, though, there are still many outstanding issues concerning Delta Scuti stars. For instance, many Delta Scutis exhibit variability in pulsation amplitude and phase (though not in the same manner as in the Blazhko Effect). Another outstanding issue concerns the inability of pulsation models to predict pulsation *amplitudes* for these stars, as well as period. There is

some evidence along these lines, though: higher amplitudes tend to be associated with more rapidly rotating stars, as well as with fewer pulsation modes being expressed in the star overall, indicating that there may be some sort of energy-sharing rule amongst pulsation modes (Percy).

### **Pulsating Red Giants & Mira Variables**

We now leave the instability strip to look upper-right of the HRD. Here reside the pulsating red giants. These stars occupy nearly the whole region of the HRD between the red edge of the Cepheid instability strip and the Hayashi Line. Indeed, these stars sort of define the absolute edge of the strip, since the convection that dominates in their atmospheres disallows the operation of the  $\kappa$  mechanism, which is driven by radiative energy transport.

The stars in this region have diverse and complicated pulsation characteristics, but general properties can still be identified. For instance, all the stars in this region with spectral types between K2 to M0 are variable in brightness due to pulsation (Xiong), pulsation amplitude increases with stellar radius and with decreasing temperature, and pulsations are likely due to radial modes in the fundamental or first overtone. The pulsators in this region are also bounded by an instability strip. The red edge of this strip is bounded by the Hayashi line. Utilizing models that relied on a non-local, non-adiabatic method, Xiong & Deng found that the location of the blue edge of this instability strip depended of metal abundance: it moved red-ward with increasing  $Z$  values.

The excitation mechanism for these stars is known, but not well-understood; it is *convection*, which is perhaps the least-understood process in all of astrophysics, and which becomes ever-more important as one approaches the Hayashi limit. There does not yet exist an adequate time-dependant theory of convection that theorists can use to describe the connections between convection and pulsation in these stars.

An important subset of the pulsating red giants is the **Mira** type pulsators. These are simply the coolest, largest-radius members of the red giants. They possess mostly stable periods of 100 to 1000 days, and vary in visual amplitude by greater than 2.5 magnitudes by definition (with up to a 5% variation). They near that of the Sun ( $0.6\text{-}3.0 M_{\text{sol}}$ ) and are  $10^4$  times as luminous, having extended envelopes out to several hundred times the diameter of Sol. From their position of the HRD we can tell that they have dense carbon cores, and are undergoing hydrogen- and helium-shell burning. These stars are a very short-lived phenomena, however. The pulsations that they are experiencing in the outer atmosphere (driven by the *hydrogen* ionization zone) are working to expel the envelope of the star at a rate of  $\sim 3 \times 10^{-7} M_{\text{sol}} \text{ yr}^{-1}$  (Wood). This will cause the deplete the envelope of mass in only a few million years, eventually leaving behind a naked core and planetary nebula. That core will eventually run out of fusible material, and might just become a...

### **Pulsating White Dwarfs**

Finally, let us take a look at the pulsating white dwarfs. These are objects are composed of matter being supported against gravity solely by electron degeneracy pressure, and represent natural laboratories for studying matter under normally unobtainable temperatures and pressures. These are difficult to study, for a number of reasons. One is their small size – about the same radius as Earth –

which gives them low intrinsic luminosity, even though they are extremely hot (up to 100,000 K). Additionally, their pulsations are small, producing changes in brightness on the order of milli-magnitudes, and they have rapid periods on the order of minutes.

However, they offer great potential rewards if they can be studied correctly, and do possess properties that work in the asteroseismologist's favor: they lack convection, are not generating energy in their cores, and are often highly differentiated as a result of various shell-burning processes that took place during the star's post-MS life.

## **Conclusion**

In this work I have attempted to present a broad, comprehensive overview of the theory of pulsating stars. I have described the terminology used in the field, the various physical mechanisms that are responsible for making these stars change their radius/luminosity, and I have described several (but by no means all) of the more important/interesting classes of pulsating stars in terms of their physical properties and their broader application to our understanding of astrophysics and cosmology.

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