Probing the Upper Mass Limit of the Main Sequence Observationally

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There is a large gap in our knowledge of the upper part of the Main Sequence. A significant piece of this is the question of how massive the most massive star actually is. Is there an upper cut-off to stellar mass? Astronomers have been struggling with this question for some time. It seems logical that there would be some sort of physical limitation on the mass of a star, but this upper mass limit has not been clearly detected with observations. There are multitudes of papers detailing the upper mass limit from a theoretical point of view, but here I only discuss the search for an upper mass limit through observation.

It is very difficult to observe extremely high mass stars. They do not live very long and there just are not that many of them around. A good place to look for stars pushing the upper mass limit is in a large, relatively nearby young star cluster.

The more massive the cluster as a whole, the more likely it is that there will be extremely high mass stars. The cluster needs to be young because otherwise, due to the short lifetime of such high mass stars, the most massive stars will have evolved off of the main sequence, through the super giant phase, gone supernova, and may no longer be visible. However, ideally the cluster would be old enough such that the molecular cloud from which the cluster formed has mostly vanished. If the cluster is still very young, then the molecular cloud is still rather thick and it will simply be impossible to detect the extremely massive stars. By the time the cluster is just a bit older, almost all of the gases and dust of the molecular cloud will have gone into forming stars, so the cluster is mostly free of this gas and dust and it no longer obscures the stars.

The cluster should be relatively nearby simply because you need to be able to clearly resolve all of the stars in your observations. Also, if the cluster is nearby, then it is likely that the distance to the cluster is well established, enabling more accurate mass estimates.

There are two main methods to observationally measuring stellar masses: a dynamical approach and an empirical approach. The dynamical, or direct, method involves observing the orbital motion of binary stars and calculating the masses using Kepler's Third Law. The empirical, or indirect, approach is more theoretical in that it involves applying different types of models to observed data in order to derive a mass. For instance, you can observe the luminosity of the star and then use a mass-luminosity relation to estimate the mass of the star based on its observed luminosity. The empirical approach gives only indirect estimates of the masses and those mass values have large uncertainties. The dynamical approach is "the only direct way of measuring accurate masses of distant stars" and is accomplished by observing "double-lined spectroscopic binary systems with eclipses present in their light curves" (Bonanos et al. 2004). However, it should be noted that the indirect approach, i.e. using stellar structure theory to estimate masses based on observed luminosities, yields much higher mass values ($\sim$100-140 M_{sun}) than the largest masses calculated directly from observing binary stars

($\sim$70 M$_{sun}$ for O-type stars) (Antokhina 2000, Weidner & Kroupa 2006, Orosz et al. 2007).

Dynamical Mass Measurements

As previously mentioned, observations of binary or multiple star systems can directly yield the masses of the stars through the observed Keplarian motion.

For observations of a double-line spectroscopic binary, you can measure the Msin^{3}i value for each star from the radial velocity curve. Since the orbital inclination angle i ranges from 0° to 90°, then the sin^{3}i value ranges from 0 to 1. This means that the measured Msin^{3}i values are the minimum values for the mass of each star.

If the double-line spectroscopic binary is also eclipsing, then you can measure the inclination angle i from the observed light curve. With the value of i, you can then calculate the absolute, or actual, individual masses by using the Msin^{3}i values. This is the most straightforward method to dynamically determine stellar masses and it is the method used for many of the results discussed below.

Williams et al. (2008) provides a list of some of the largest dynamically determined stellar masses (see Figure 1). I added an additional dynamically determined stellar masses at the bottom. Below, I will discuss many of these observational results in chronological order.

Dynamically Determined Stellar Masses

Star Name	Mass ($M_\odot$)	Spectral Type	Reference
WR 20a (Primary)	82.7 ± 5.5	WN6ha	Rauw et al. (2005); Bonanos et al. (2004)
WR 20a (Secondary)	81.9 ± 5.5	WN6ha	Rauw et al. (2005); Bonanos et al. (2004)
M33 X–7 (Secondary)	70.0 ± 6.9	O7-8III	Orosz et al. (2007)
WR 47 (O-star)	~60	O5:V	Lamontagne et al. (1996)
R136-38 (Primary)	56.9 ± 0.6	O3 V	Massey et al. (2002)
WR 22 (Primary)	55.3 ± 7.3	WNv+	Schweickhardt et al. (1999)
[L72] LH 54-425 (Primary)	47 ± 2	O3 V	This work
HD 93205 (Primary)	45 (Probable)	O3 V	Antokhina et al. (2000)

Figure 1: Some of the largest dynamically determined stellar masses to date. (Williams et al. 2008)

WR 22

Several different astronomers have published results for WR 22, a binary system composed of a Wolf-Rayet star and an O-type star. A Wolf-Rayet (WR) star is a very massive star with strong stellar winds. These strong winds cause rapid mass loss from the star. WR 22 is one of the brightest Wolf-Rayet stars and "all analyses agree in that WR 22 is the most massive Wolf-Rayet star ever weighed" (Schweickhardt et al. 1999).

Before Schweickhardt et al. (1999), Rauw et al. (1996) calculated a minimum mass of 72 M$_{sun}$ for the primary star. However, the mass determined by Schweickhardt et al. (1999) "revises down considerably the last published value for the mass." The O-type companion star is very faint in comparison to the WR star, so it is extremely difficult to

find the velocity curve of the companion. This is the cause of the discrepancies in the dynamically measured masses.

Schweickhardt et al. (1999) collected a "large data set" of new spectra of WR 22 and combined that with previously published data. This gave them a 26 year time span, from which they derive a mean orbital period of 80.336 +/- 0.0013 days. The then calculated the minimum mass of the WR star ($M\sin^3 i$) to be 55.3 +/- 7.3 M_{sun}, indeed the most massive Wolf-Rayet star yet detected, even though the mass is much smaller than the 72 M_{sun} value published previously. The physical parameters determined are shown in Figure 2.

Physical parameters of WR 22

P [days]	80.336	±0.0013
e	0.598	±0.010
γ [km s^{-1}]	0	±15
T_0 [JD 2 400 000.5+]	50 126.97	±0.14
ω_{WR} [deg]	268.2	±1.6
$\omega_O = \omega_{WR} - 180°$ [deg]	88.2	
K_{WR} [km s^{-1}]	70.6	±0.8
K_O [km s^{-1}]	190	±10
mass ratio $q = M_{WR}/M_O$	2.69	±0.14
$a_{WR} \sin i$ [10^6 km]	62.5	±0.9
$a_O \sin i$ [10^6 km]	168.2	±9.0
$(M_{WR} + M_O) \sin^3 i$ [$M_\odot$]	75.9	±9.0
$M_O \sin^3 i$ [$M_\odot$]	20.6	±1.7
$M_{WR} \sin^3 i$ [$M_\odot$]	55.3	±7.3

Figure 2: Results for physical parameters. Image from Schweickhardt et al. (1999).

HD 93205

HD 93205 is a binary system containing an O3 V and an O8 V star "in the very young open cluster Trumpler 16, within the Great Carina Nebula" (Antokhina et al. 2000). Antokhina et al. (2000) were interested in studying it more thoroughly because previous observations of the binary system indicated that one of the stars is potentially very massive. They wanted to determine the orbital inclination of the system, and thus calculate the mass of the O3 star, by gathering more precise photometric observations than previously collected.

The data gives $M_1\sin^3 i = 29.0$ M_{sun} and $M_2\sin^3 i = 12.9$ M_{sun}. The best fit inclination angle (i) to the light curve is between 35° and 77°, with i = 60° being optimal. This gives mass ranges for the O3V star from 31 to 154 M_{sun}, with the best fit (i = 60°) mass being 45 M_{sun}. The best fit mass for the O8V star is 20 M_{sun}, with a range between 14 and 68 M_{sun}. The probable 45 M_{sun} mass of the O3V star was one of the highest masses directly observed via Keplerian motion at the time the work was published.

R136-38

The R136 cluster, a "super star cluster," is part of the 30 Doradus nebula in the Large Magellanic Cloud. It is large and has many young stars, but is also the nearest cluster to us of its type, allowing "its stellar population [to] be studied directly" (Massey & Hunter 1998). Massey & Hunter (1998) used the spectrograph on the Hubble Space Telescope to collect data on many stars in R136.

Using temperature models, they estimated the temperature of the star based on its spectral type and they used "evolutionary models to determine ages and masses." They saw right away "that the R136 cluster contains a large number of extremely massive stars." The evolutionary models do not go past 120 M_{sun}, so Massey & Hunter extrapolated the values for the larger masses "using the mass-luminosity relation derived from the models." They estimate that these masses go up to 140-150 M_{sun}. Massey & Hunter (1998) conclude by saying that more detailed observation is needed on the hottest stars, but that R136 "contains a large number of the most luminous and massive stars ever identified."

Massey et al. (2002) again used the Space Telescope Imaging Spectrograph on the Hubble Space Telescope to observe some targets in R136 that previous observations showed were potentially massive "spectroscopic binaries caught at favorable phases." They hoped their observations would help resolve the discrepancy between masses estimated with evolutionary tracks and masses estimated with stellar atmosphere models by leading to clear orbital solutions with definite masses. (Massey et al. 2002) They took both photometric and spectroscopic observations of each target.

The most striking results from Massey et al. (2002) are for R136-38, detailed in Figure 3. They used "light-curve synthesis code" and found a "well-determined orbital inclination." They derived the orbital parameters "from the spectroscopic solution" and used spectral classifications to estimate the physical parameters of the stars.

At the time of this work, the 57 M_{sun} mass of the primary star was arguably the largest mass measured in a binary system (Massey et al. 2002).

ORBIT SOLUTIONS AND PHYSICAL PARAMETERS: R136-38

Parameter	System	Primary	Secondary
Orbital:			
P (days)	3.39 (adopted)	...	...
e	0.00 (adopted)	...	...
T primary conjunction (MJD)	51,621.40 $\pm$ 0.03	...	...
γ (km s^{-1})	...	278.2 $\pm$ 0.4	272.3 $\pm$ 0.3
K (km s^{-1})	...	174.7 $\pm$ 0.5	424.8 $\pm$ 0.2
m_p/m_s	0.41	...	...
R1 (km s^{-1})	...	2.3	20.3
$a \sin i$ ($R_\odot$)	40.2	11.7	28.5
$m \sin^3 i$ ($M_\odot$)	...	53.8 $\pm$ 0.2	22.1 $\pm$ 0.1
Spectral and photometric:			
Spectral type	...	O3 V	O6 V
T_{eff} (K)	...	48,500 $\pm$ 4850[a]	42,200 $\pm$ 4220[a]
Δm	1.0 $\pm$ 0.2	...	...
M_V	-5.3	-4.9 ± 0.2[b]	-3.9 ± 0.2[b]
Bolometric correction	...	-4.4 ± 0.3[a]	-4.0 ± 0.3[a]
M_{bol}	...	-9.3 ± 0.4[a,b]	-7.9 ± 0.4[a,b]
Radius ($R_\odot$)	...	9.3 $\pm$ 1.0[a,b]	6.4 $\pm$ 0.7
v_{sync}	...	110 $\pm$ 12	76 $\pm$ 9
$v \sin i$	...	130 $\pm$ 20	90 $\pm$ 20
Eclipse depths	...	0.23	0.20
Inclination: geometry (deg)	79.0 $\pm$ 1.0	...	...
Inclination: GENSYN model (deg)	79.0 $\pm$ 1.0	...	...
Masses:			
m, orbit($M_\odot$)	...	56.9 $\pm$ 0.6	23.4 $\pm$ 0.2
m, evolutionary tracks ($M_\odot$)	...	53 $\pm$ 5[c]	29 $\pm$ 2[c]

[a] Adopting a 10% uncertainity in the spectral type–T_{eff} relationship.
[b] Errors (anti)correlated.
[c] Errors on the masses from the evolutionary tracks are based solely on the errors in M_V.

Figure 3: Orbital and physical results of R136-38. Image from Massey et al. (2002).

WR 20a

Currently, the two largest dynamically determined stellar masses belong to the two stars in the WR 20a binary system. WR 20a "is believed to be a member of the young open cluster Westerlund 2" (Rauw et al. 2004).

Rauw et al. (2004) "performed a snapshot spectroscopic survey" of 30 objects that previous observations had shown were potentially binary systems containing a WR star and an OB star. Their observations confirmed that WR 20a is a binary system and they found that the two stars had nearly identical spectra, meaning they have the same spectral type. The fact that the spectral types are identical causes some confusion when analyzing the Keplarian motion of the stars. When they plot the radial velocities and fit a Gaussians, it is very difficult to measure the period of the orbit because the stars are almost indistinguishable. Rauw et al. (2004) estimated the period by tracking the absolute value of the difference in radial velocities and looking for the peak. The result is three possible orbital periods: 3.675 +/- 0.030 days (preferred value), 4.419 +/- 0.044 days, and 1.293 days (least likely). The different periods give very different minimum masses (meaning Msin^{3}i) for stars, which are detailed in Figure 4. It is important to note

that the minimum mass "values strongly depend on the adopted orbital period" (Rauw et al. 2004). Rauw et al. (2004) called for more photometric monitoring of WR 20a in order to more accurately determine the orbital period and inclination.

Orbital Period (days)	3.675 +/- 0.030	4.419 +/- 0.044	1.293
$M_1\sin^3 i$ (M_{sun})	70.7 +/- 4.0	84.0 +/- 5.3	25.9 +/- 1.4
$M_2\sin^3 i$ (M_{sun})	68.8 +/- 3.8	80.8 +/- 5.1	24.9 +/- 1.4

Figure 4: Some orbital parameter results of Rauw et al. (2004).

Bonanos et al. (2004) quickly answered their call and collected more photometry for WR 20a. The orbital period measured from their radial velocity curve basically agreed with Rauw et al. (2004), but refined the value to P = 3.686 +/- 0.01 days. From their "well-sampled light curve," Bonanos et al. (2004) easily derived a best-fit inclination angle for the orbit, i = 74.5° +/- 2.0°. Their velocity curve yielded slightly larger masses, as shown in Figure 5.

LIGHT-CURVE PARAMETERS FOR WR 20a

Parameter	Value
Period, P	3.686 ± 0.01 days
Primary eclipse, T_{prim}	2453124.569
Inclination, i	$74°5 \pm 2°0$
Eccentricity, e	0 (fixed)
Effective temperature, T_{eff1}	42,000 K (fixed)
Effective temperature, T_{eff2}	$40,300 \pm 1000$ K
Surface potential ($\Omega_1 = \Omega_2$)	3.92 ± 0.03
Radius, r_{pole}	$18.7 \pm 0.3\ R_\odot$
Radius, r_{point}	$22.0 \pm 0.3\ R_\odot$
Radius, r_{side}	$19.3 \pm 0.3\ R_\odot$
Radius, r_{back}	$20.4 \pm 0.3\ R_\odot$

FOR WR 20a

Parameter	Value
Systemic velocity, γ	8.0 ± 5.0 km s^{-1}
Semiamplitude, K_1	362.2 ± 8.0 km s^{-1}
Semiamplitude, K_2	366.4 ± 8.0 km s^{-1}
Semimajor axis, $a \sin i$	$53.0 \pm 1.0\ R_\odot$
Mass ratio, $q = m_2/m_1$	0.99 ± 0.03
Mass, $m_1 \sin^3 i$	$74.3 \pm 4.0\ M_\odot$
Mass, $m_2 \sin^3 i$	$73.4 \pm 4.0\ M_\odot$

Figure 5: (a) Orbital parameters derived from light-curve. (b) Orbital parameters derived from radial velocity curve. Both images from Bonanos et al. (2004).

Using their derived inclination, Bonanos et al. (2004) calculated the individual masses of the two stars to be 83.0 +/- 5.0 M_{sun} and 82.0 +/- 5.0 M_{sun}. They note that the errors in the masses "include the error in the minimum mass and the error in the inclination" (Bonanos et al. 2004).

Using the orbital period and inclination values derived by Bonanos et al. (2004), Rauw et al. (2005) reexamined the data from their 2004 paper and recalculated some orbital elements of WR 20a. The Rauw et al. (2004) data with the Bonanos et al. (2004) inclination and period give absolute masses for the two stars as 82.7 +/- 5.5 M_{sun} and 81.9 +/- 5.5 M_{sun}.

Both of these results make the primary and secondary stars in WR 20a the two most massive stars with well-determined masses to date.

M33 X-7

        Orosz et al. (2007) was interested in observing black holes in close binary systems. M33 X-7 is an X-ray source located in the spiral Triangulum Galaxy (M33). The system contains "the only known black hole that is in an eclipsing binary" (Orosz et al. 2007). Spectroscopic observations yielded an almost sinusoidal radial velocity curve and an orbital period of 3.453 days and indicated that the companion is an O-type star. Using a "light-curve synthesis code," they found a best-fit model for the light curve, giving an inclination angle $i = 74.6° +/- 1.0°$. The dynamical measurements give the mass of the O-star as $70.0 +/- 6.9$ M_{sun} and the mass of the black hole as $15.65 +/- 1.45$ M_{sun}. These and other results are included in Figure 6.

Selected parameters for M 33 X-7			
Parameter	**Value**	**Parameter**	**Value**
Θ (degrees)	46 ± 1	M_2 ($M_\odot$)	70.0 ± 6.9
T_{eff} (K)	34,000–36,000	r_d	0.45 ± 0.03
$V_{rot}\sin i$ (km s^{-1})	250 ± 7	e	0.0185 ± 0.0077
R_2 ($R_\odot$)	19.6 ± 0.9	ω (degrees)	140 ± 27
$\log L_2$ ($L_\odot$)	5.72 ± 0.07	Ω	0.903 ± 0.037
$\Delta\phi$	0.0045 ± 0.0014	f_2	0.777 ± 0.017
i (degrees)	74.6 ± 1.0	a ($R_\odot$)	42.4 ± 1.5
K_2 (km s^{-1})	108.9 ± 5.7	M ($M_\odot$)	15.65 ± 1.45

Figure 6: Selected parameter results of the binary system. Image from Orosz et al. (2007).

        Both of these masses are striking for various reasons. The mass of the O-star "puts it among the most massive stars whose masses are well-determined" and the black hole mass makes it "the most massive stellar black hole known" (Orosz et al. 2007). Also, the mass values and the small seperation between the two stars (~ 42 R_{sun}) make the system "very difficult to explain using stellar evolutionary models" (Orosz et al. 2007). It is unclear what the initial masses of the secondary (O-type) star or the progenitor star (the star that formed the black hole) could have been because their mass loss rate undoubtedly changed over time. Currently, no stellar evolution models accurately predict this situation.

        Could the initial mass of either of the stars have been more than the most massive star observed to date? We do not yet know. However, the O-type companion star is probably between 2 and 3 million years old (Orosz et al. 2007) and its current mass ranks it as the third highest dynamically determined stellar mass.

Empirical Mass Measurements

        Previously, I mentioned you could indirectly estimate the mass of a star by using a theoretical mass-luminosity relation. You could also use a spectral type-mass relation, to name another way. However, there are many uncertainties in stellar structure theory

for massive, luminous stars and these uncertainties "affect how the observed luminosity is converted into mass" (Antokhina et al. 2000).

The Arches Cluster

The work of Figer 2005 in the Arches cluster is an example of an empirical measurement of the upper mass limit. The Arches is the densest cluster in the Milky Way and near the Galactic Center. As Figer (2005) points out, this cluster is a well-suited place to try and find extremely high mass stars for many of the reasons I stated earlier. It is very massive, young, but not too young, and nearby.

Using the Hubble Space Telescope's Near-Infrared Camera and Multi-Object Spectrometer, Figer collected photometry on the most massive stars in the Arches cluster. He then applied some corrections to these observations and used the Geneva stellar evolution mass-flux relation models to get initial masses from the observed fluxes. Though the models end at 120 M_{sun}, he had some data points extending beyond that. For these three brighter stars, he "assigned masses through a linear extrapolation of the mass-flux relation from points immediately below" the 120 M_{sun} value. The extrapolation gives mass estimates that do not exceed 130 M_{sun}. Though theoretical mass function models predict maximum stellar masses of at least 500 M_{sun} for the Arches cluster, stars of that mass were definitely not found. There is a clear cutoff at 130 M_{sun}. Figer estimates a 10% error in his calculations, so he conservatively gives 150 M_{sun} as the upper mass limit for the Arches cluster.

After further analysis of the significant deficit in massive stars, including a Monte Carlo simulation, Figer concludes that "stars with masses about ~150 Msun should still be visible if they were formed, given [his] estimate of the age for the Arches cluster."

Figer goes on to discuss the error in such calculations. The models do not accurately take into account everything going on in the star. For example, fairly significant errors occur because the "stellar wind/atmosphere models…do not model the effects of increased opacity produced by metals in stellar winds (that is, line-blanketing)." Other errors in the masses come from "uncertainties in distance, reddening and photometry." However, in this case, there is hardly any uncertainty in the distance because the Arches cluster is so close to the Galactic Center, which is a well-known distance from us.

Conclusion

Despite lots of hard work, astronomers still do not know the upper mass limit of the main sequence. We still do not know the mass of the most massive star possible. Two of the approaches to finding this limit lead astronomers in somewhat different directions. The empirical method indicates a much higher mass limit than the dynamic method. Which method is more accurate? The indirect approach depends on stellar models that have many uncertainties, so there is more uncertainty in the masses generated this way. However, the indirect approach does a better job of incorporating data about similar and surrounding stars. The direct approach has fewer uncertainties, but it does not take into account as much about other stars. I think dynamical mass measurements say a lot about that star itself, but they do not say as much about other stars that might be similar.

To me, it does not seem likely that one could confidently say that a dynamically measured mass is definitely the upper limit. How would you know that you are not going to turn around and observe another large young cluster containing a binary system with a larger primary component? I think it is more likely that empirical mass estimates will end up leading to a proposed or possible upper mass limit. Then the direct approach would be used to test that limit.

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