ACCRETION DISKS AROUND BLACK HOLES

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Abstract. A brief summary of the various types of accretion disks that might occur in astrophysical situations is provided, with emphasis placed on general relativistic effects to be expected in the vicinity of an accreting black hole.

1. Introduction

It is patently impossible to provide a thorough review of a topic as vast as accretion disks around black holes in only 15 relatively small pages, and so this chapter will mention only a few topics, and concentrate on fewer still. In 1996 alone, 250 papers listing accretion or accretion disks as subject headings appeared in *The Astrophysical Journal* and 80 more were published in the *Monthly Notices of the Royal Astronomical Society*, with other leading journals containing much additional research. In that accretion onto white dwarfs, neutron stars, and young stellar objects are also active research areas, only roughly half of the above flood of calculations and data was directly related to accretion onto black holes (hereafter, BHs). Nonetheless, the literature since the early 1970's, when this phenomenon was first considered seriously (e.g., Shakura & Sunyaev 1973; Novikov & Thorne 1973), amounts to several thousand papers. Roughly one international conference per year concentrates on accretion disk phenomena (for a recent one, see Abramowicz et al. 1997a), and entire texts (e.g., Frank et al. 1992), monographs (e.g., Chakrabarti 1990a), and many far more extensive reviews than this current effort (substantial recent ones include Chakrabarti 1996a; Lin & Papaloizou 1996; Papaloizou & Lin 1995), have appeared on this subject. This topic has now become so mainstream that it would be amazing if a modern text on either general relativity theory or basic astronomy did not discuss accretion disks around black holes.

In a volume celebrating the contributions of C.V. Vishveshwara, it seems appropriate to begin by noting some collaborative efforts that began our scientific acquaintance. Vishu had already performed computations of electromagnetic fields and orbits of particles in the vicinity of BHs back in the 1970's (Chitre & Vishveshwara 1975; Prasanna & Vishveshwara 1978). During my first scientific visit to India, Naresh Dadhich and I examined the effect of strong magnetic fields on the efficiency with which matter would be accreted onto non-rotating BHs (Dadhich & Wiita 1982). Using the Ernst (1976) metric, we located the marginally stable and minimum binding energy orbits (r_{ms} and r_{mb}, respectively) for accretion disks around a static BH immersed in a uniform magnetic field. When I met him in Bangalore, Vishu proposed that we generalize this effort to the more difficult case of Kerr BHs, using the effective potential for trajectories worked out by Prasanna and himself (Prasanna & Vishveshwara 1978). We first considered uniform magnetic fields (Wiita et al. 1983) around rotating BHs and later analyzed the situation for dipolar magnetic fields (Iyer et al. 1985). Charged test particles could see the efficiency with which their rest mass is converted to energy rise dramatically in the presence of magnetic fields; however, the increase of accretion efficiency with magnetic field strength is lower for black holes with higher specific angular momenta. Nonetheless, when full-fledged hydrodynamical accretion is considered, the effects of the magnetic field on accretion efficiency are invariably quite small.

Vishu encouraged me to give a series of lectures on accretion disks in Bangalore in the summer of 1981 and then arranged to distribute those lecture notes (Wiita 1981). These talks inspired additional works on black hole accretion disks by Indian astrophysicists, the first of which was an analysis of the maximum luminosity that could emerge from the "funnels" along the rotation axes of optically and geometrically thick accretion disks (§2.2) around black holes (Nityananda & Narayan 1982). This paper was followed by an estimate of the strength of the flow of ejected matter from these funnel regions (Narayan et al. 1983). We shall not further consider the launching of jets from accretion disks here, other than to mention the probable importance of magnetic fields and refer the reader to other reviews (e.g., Wiita 1990, 1996a; Chakrabarti 1996a).

In §2 the major classes of accretion disks (thin and thick) around black holes will be outlined. Over the past few years, models in which much of the energy produced in the disk is advected inwards to the black hole before all the radiation can escape from the surface of the disk have become the focus of intense investigation, and a brief discussion of the properties of these disks will be given in §3; it is worth noting that two of the leading contributors to the study and popularity of such "unified" or "advection dominated" accretion flows are Indians: Sandip Chakrabarti and Ramesh

Narayan. Observable variations produced by fluctuations on accretion disks around black holes will be briefly discussed in §4.

2. Standard Accretion Disk Models

Accretion disks around black holes were first envisaged to occur within a binary stellar system where a very massive primary star had evolved and collapsed into a black hole; the secondary would feed matter onto the BH either through a wind or through Roche lobe overflow. In either case, the accreted matter would clearly possess substantial angular momentum with respect the BH, and would form a ring around it, where centripetal and gravitational forces balance. Viscosity within the fluid, presumably produced by turbulence or magnetic fields, would allow most of the mass to filter inwards, while the bulk of the angular momentum would be transported outwards (e.g. Shakura & Sunyaev 1973). Clear evidence for such accretion disks around white dwarfs in binaries was quickly provided by analysis of cataclysmic variables (e.g., Robinson 1976).

The tremendous powers required to energize quasars, Seyfert galaxies, BL Lacertae objects, and other active galactic nuclei (hereafter AGN) demanded an engine that could very efficiently convert matter into electromagnetic radiation. The rapid variability in luminosity arising from AGN also implied very compact central engines. Accretion from a disk around a Schwarzschild (non-rotating) BH produces efficiencies of $\eta \simeq 0.057$, where the total power liberated is related to the mass accretion rate by $L = \eta \dot{M} c^2$, roughly an order of magnitude above that attainable through nuclear fusion. Accretion onto a maximally rotating Kerr BH is even more efficient, yielding $\eta \simeq 0.42$, though most models provide limits of $\eta \simeq 0.32$ for a Kerr BH with $a/M \simeq 0.998$ (Thorne 1974). These computations are based upon the binding energy of a test particle at the last stable circular orbit around the BH; $r_{ms} = 6GM/c^2 \equiv 6r_g \equiv 3R_s$ for a Schwarzschild BH. In that the bulk of this luminosity is found to emerge from distances of $< 20R_s$ for accretion disks around black holes, both the efficiency and compactness desiderata are beautifully satisfied, and this picture, with $M_{BH} \sim 10^6$–$10^9 M_\odot$, quickly became the dominant model for the AGN powerhouses (e.g., Rees 1984). The more powerful quasars still required substantial mass influxes ($\sim 10 M_\odot \mathrm{yr}^{-1}$) which are difficult, but not impossible, to find from mass loss processes in dense stellar clusters. The uncertainty in the net angular momentum of the gas falling into a central mass implies that the disk only might form at relatively small radii, where rotation finally becomes important enough to balance gravity.

2.1. THIN ACCRETION DISKS

The original accretion disk models (Shakura & Sunyaev 1973; Novikov & Thorne 1973) assumed that the mass accretion rate was substantial, but that $L \ll L_E$, where $L_E = 4\pi c G M_{BH} m_p/\sigma_T = 1.3\times10^{38}\mathrm{erg\,s^{-1}}(M_{BH}/M_\odot)$ is the Eddington luminosity, or the maximum luminosity that can escape from a spherical system and still allow matter to be accreted inwards (i.e., when radiation pressure on electrons is just balanced by gravitational attraction on protons). The other key assumptions were that the disk had a Keplerian angular momentum distribution so that $v_\phi \propto r^{-1/2}$ (in the Newtonian approximation), and that the radial velocity of infall, $v_r \ll v_\phi$. Under these circumstances, the radial equations of the steady state disk structure decouple from the vertical ones, and complete solutions can be obtained based on the relativistic conservation laws for mass, energy and angular momentum. Define: the surface mass density as $\Sigma(r) = \int_{-h}^{+h} \rho(r,z)dz$, with ρ the mass density and $h(r)$ the disk half thickness; $F(r)$ is the flux radiated from the upper disk surface; and $W(r) \equiv \int_{-h}^{+h}\langle T_{\phi r}\rangle dz$ is the integrated stress between two disk annuli. Then, in terms of non-dimensional radii, $r_* \equiv r/M$, and angular momentum parameter of the BH, $a_* \equiv a/M$, one can find the equations of radial structure to be (Novikov & Thorne 1973):

$$\dot{M} = -2\pi r \Sigma v_r (1 - 2/r_* + a_*^2/r_*^2)^{1/2}, \tag{1}$$

$$W = \frac{\dot{M}}{2\pi}\Big(\frac{M}{r^3}\Big)^{1/2} \frac{(1 - 3/r_* + 2a_*/r_*^{3/2})^{1/2} Q}{r^2(1 - 2/r_* + a_*^2/r_*^2)(1 + a_*/r_*^{3/2})}, \tag{2}$$

$$F = \frac{3\dot{M}}{8\pi r^2}\Big(\frac{M}{r}\Big)\frac{Q}{(1 + a_*/r_*^{3/2})(1 - 3/r_* + 2a_*/r_*^{3/2})^{1/2}}, \tag{3}$$

where Q is an extremely messy integral, which, however, $\approx 1 - (r_{ms}/r_*)^{1/2}$ for small a_*.

The Newtonian approximation shows that the total luminosity emerging from both surfaces of the disk, $L_{disk} = GM\dot{M}/2r_{in}$ and this is one-half of the maximum total luminosity contained in the infalling matter. Therefore this matter retains as kinetic energy half the potential energy it lost while in-spiralling, implying that an equivalent amount of additional energy could be radiated in a boundary layer, where the disk "rubs up against" the star (e.g. Frank et al. 1992). This phenomenon is possible for central objects with hard surfaces, such as white dwarfs and neutron stars (though strong magnetic fields may prevent the disk from reaching the stellar surface); however, BHs do not possess hard surfaces, and once inside the marginally stable orbit the infalling matter is trapped and very little additional radiation will escape; instead it will be swallowed by the BH, along with the accreted gas.

Standard thin disk models approximate the shear as proportional to the pressure in the disk, $T_{\phi r} = \alpha P$, with $\alpha < 1$. With this simplification, values of F, Σ, h, and T are found as algebraic functions of $r, M, \dot{M}, a$ and α (Novikov & Thorne 1973). Small corrections to some of these results, particularly for h, were recently demonstrated (Riffert & Herold 1995). Such disks are usually considered to be divided into three regions: at large radii (typically $r_* > 100$) the 'outer region' has gas pressure much greater than radiation pressure and the opacity is mostly free-free; a 'middle region', from which most of the luminosity arises, still has gas pressure dominating, but the opacity is mostly due to electron scattering; the 'inner region' (typically $r_* < 4$), is dominated by radiation pressure and electron scattering. While the flux is essentially independent of the viscosity (shear) prescription, the disk thickness, surface density and central temperatures all do depend on α. Much effort has gone into producing self-consistent models for the viscosity and this may finally be bearing fruit (cf. §3.3), with the possibility that $\alpha \sim 0.01$–0.3 is generally appropriate.

For the disk to remain thin ($h/r \ll 1$), it is easily seen that the local Keplerian velocity is highly supersonic, $c_s \ll v_\phi$, in that vertical pressure balance implies that $h \approx c_s(r/GM)^{1/2}r$. In this case, the circular matter velocity will be very close to the Keplerian value, but this will not be true very close to r_{ms}. However, the radial velocity must be substantially subsonic in the bulk of the disk, since $v_r \approx \alpha c_s(h/r) \ll c_s$.

2.2. THICK RADIATION SUPPORTED ACCRETION DISKS

Of course the assumptions implying accretion disks are always thin can break down, and they almost certainly do in the innermost regions when careful consideration of the effects of general relativity show that the flow must go supersonically through a cusp (e.g., Abramowicz et al. 1978). If the accretion rate is very high and/or the viscosity is very low then the matter will pile up in the vicinity of the central BH. The accreting material forms a thick, radiation pressure supported, accretion disk, which is often called an accretion torus. Lynden-Bell (1978) first showed the basic shape such flows should have, and pointed out that there would be funnels along the BHs rotation axis in which high radiation pressures could drive matter outwards, possibly producing jets. Paczyński & Wiita (1980) produced the first thick disk models that connected smoothly to more standard thin accretion disks at larger radii, and pointed out several important features of these configurations. These computations approximated the effects of the Schwarzschild metric in a very simple way, by using a pseudo-Newtonian potential of the form, $\Phi_{PW} = -GM/(r - 2r_g)$ in lieu of the usual $\Phi_N = -GM/r$. This potential gives the correct values of $r_{ms} = 6r_g$ and $r_{mb} = 4r_g$, and closely approximates the efficiency, with $\eta(r_{ms}) = 0.0625$. A recent study (Artimova et al. 1996) has shown that this potential remains better in nearly every regard than others that have been employed.

These disks are characterized by accretion rates so high that more than the Eddington luminosity should be produced, i.e., $\dot{M} > \dot{M}_E$. Under these circumstances the radiation produced in the accreting matter exerts large pressures on the gas and forces significant changes in the disk structure, so that $h(r) \sim r$, at least for the inner 10–100r_g. When the pressure gradient term is important (basically when $GM/r \approx P/\rho \approx c_s^2$), it modifies the angular momentum distribution. In the inner portion (out to a few r_{in}) the specific angular momentum will be above the Keplerian value (typically close to constant), while outside of that distance (corresponding to the densest portion of the disk on the midplane) the angular momentum distribution will be below the Keplerian value, though it becomes very close to it at large r. The inner edge of the disk is pushed closer to the BH than r_{ms}, and in the limit of extremely high $\dot{M}$, it approaches r_{mb} (Paczyński & Wiita 1980); this also means that the structure in the inner region is nearly independent of the viscosity. This inward motion of the disk's inner edge implies that the efficiency of accretion drops dramatically, so that, while in principle, super-Eddington luminosities can be produced, the growth of the luminosity is far below that of the accretion rate. Further, these early models involved only barytropic equations of state, and did not properly compute how much of the generated luminosity would be advected with the matter and swallowed by the BH.

More sophisticated models of radiation supported thick disks included limits imposed by the disk's self-gravity and nuclear fusion (Wiita 1982). Later work was able to include self-gravity in a reasonable approximation and make full general relativistic computations of the angular momentum distribution (Chakrabarti 1988; Lanza 1992). The spectra emitted from thick disks are similar to those of thin disks in the optical, but thick disks produce much more radiation at high frequencies (EUV and soft X-ray), if viewed essentially face-on; however, the spectra are much weaker and softer if viewed at high inclination angles (so that the hot, radiation drenched funnels are blocked from view by the rest of the disk) (Madau 1988). Computation of fusion reactions indicate that nucleosynthesis is probably unimportant for thick disks around supermassive BHs in AGN, but that it might be of real importance around solar mass BHs, and the ejected mass might have significantly enhanced metallicity (Chakrabarti et al. 1987).

Despite being able to generate jets (albeit, probably not highly relativistic ones, cf. Melia & Königl 1989), super-Eddington luminosities, sometimes better fitting spectra than thin disks, and nucleosynthesis, the study of thick disks fell from favor in the late 1980's. This was mainly because of the demonstration that thin tori were dynamically and globally unstable to non-axisymmetric perturbations (Papaloizou & Pringle 1984; Goldreich et al. 1986; Hawley & Blaes 1987). However, other calculations showed that this instability grows much more slowly for more realistic thick disks when compared to slender tori (Kojima 1986) and that actual accretion reduces the reflection from the inner boundary that allows the modes to grow quickly (e.g., Blaes 1987).

Therefore, this radiation supported thick disk picture may remain viable, if high accretion rates involving both low angular momentum and low radial infall velocities can be achieved. Accomplishing both simultaneously is difficult in the simplest astrophysical situations, but could well occur through the production of shocks in initially thin disks (e.g., Chakrabarti 1990a, 1996a). Inside of that shock zone, probably at 10–20 r_g, the flow is likely to puff up and resemble the thick disk models we have just summarized.

3. Composite Disk Models

Over the past few years, a great deal of attention has been focused on "advection dominated accretion flows" (or ADAFs, as they are often abbreviated). These more complex models now appear to be necessary to match the spectral and variability properties of observed black hole binaries, and they may also be relevant to AGN powerhouses. The possibilities that electrons and protons do not necessarily equilibrate before being accreted and that disks can form with separate temperatures for each were pointed out by Lightman & Eardley (1974) and Shapiro et al. (1976), but they emphasized the thermal instability of such disks. Ichimaru (1977) studied two-temperature disks where the proton cooling time exceeded the accretion time and he showed that these could be thermally stable; this work probably qualifies as the first discussion of ADAFs (Svensson 1997). The idea that geometrically-thick, but optically thin, two-temperature disks (with $T_{ion} \sim T_{virial} \sim 10^{11} K \gg T_e$) could be the natural state for very low accretion rates was stressed by Rees et al. (1982). Such ion-supported accretion disks (or tori) would radiate very little directly, but could tap the rotational energy of the central BH through magnetic fields, and thereby rather easily produce relativistic jets with small mass fluxes. Rees et al. argued that such central engines were good models for radio galaxies as opposed to quasars, where the tremendous optical brightness probably originates from thin or radiation supported thick disks. Optically thick "slim accretion disk" models allowing for advection were produced (e.g., Abramowicz et al. 1988), but may be unstable.

Two basic approaches to a detailed analysis of the more complex accretion disk models have been undertaken. One approach to these unified models stresses the topologies of the solutions to steady-state models for the radial structure, and can naturally include shock transitions; however, to solve these equations, one must integrate over the vertical structure and must make approximations concerning the heating and cooling rates (e.g., Matsumoto et al. 1984; Abramowicz & Chakrabarti 1990; Chakrabarti 1990a,b 1996a,b; Chakrabarti & Titarchuk 1995). The other route to studying these flows includes a more explicit treatment of vertical structure and heating and cooling processes, but makes other simplifications, such as self-similarity, which preclude recognition of shocks in the flows (e.g., Narayan & Yi 1994, 1995; Abramowicz et al. 1995; Chen et al. 1995; Esin et al. 1996; Lasota et al. 1996; Narayan 1996). We now turn to brief descriptions of some key results obtained from these studies to date.

3.1. FLOW TOPOLOGY AND UNIFIED DISK MODELS

An easy way to think about the topology of rotating accretion flows is to first consider the specific energy of an inviscid, thin, isothermal flow, which can be shown to be (e.g., Chakrabarti 1996a)

$$e = \frac{1}{2}v_r^2 + c_s^2 \log(\Sigma) + \frac{1}{2}\frac{\lambda^2}{x^2} - \frac{1}{2(x-1)}, \tag{4}$$

where $x = r/2r_g$, λ is the assumed constant specific angular momentum, c_s is the constant sound speed, and the pseudo-Newtonian potential is employed for simplicity. Close to the BH ($x \sim 1$), the potential energy term dominates the rotational kinetic energy term, and $e \approx 0.5[v_r^2 - (x-1)^{-1}]$. The negative sign connecting the two terms indicates that contours of constant energy in the (v_r, x) plane are hyperbolic and the asymptotes cross to form a saddle type ('X') inner sonic point. Similarly, at great distances from the BH, since the rotational energy term falls as $\propto x^{-2}$ while the potential falls as $\propto x^{-1}$, the contours of constant e also have a hyperbolic character and there is an outer saddle type sonic point. But, for intermediate distances, where $x \sim \lambda$, the rotational energy term dominates the potential term, and $e \approx 0.5[v_r^2 + (\lambda/x)^2]$; the positive sign indicates elliptical contours for e, implying the presence of an 'O' type sonic point (i.e., as for a harmonic oscillator). In the presence of viscosity, this 'O' type point changes to a spiral, or 'S', type, as for a damped harmonic oscillator (Chakrabarti 1996a). A general relativistic treatment for the Kerr metric obviously changes many details but not the fundamental topologies and number of sonic points (Chakrabarti 1990a), so this type of analysis is generally useful.

Now the actual presence of all three possible sonic points is determined by the angular momentum of the flow and the heating and cooling processes that yield the internal energy of the fluid, which was neglected in the above simple approximation. For no angular momentum in the accreting matter, the three critical points merge into a single transonic radius, as for simple spherical accretion (Bondi 1952). Up to two stable shocks can occur in these accretion flows for higher λ, but even the outer one typically occurs at $x < 30$. The inner sonic point exists only for black hole accretion (Chakrabarti 1990a), while the outer two can exist around neutron stars. Therefore the flow onto the BH is supersonic in going through the horizon, while the fluid should strike a neutron star's surface subsonically. This difference in flow topologies can have observable consequences. If the accretion rate is high enough, the flow onto a BH through an inner shock will be sufficiently convergent to Comptonize soft photons into a power-law spectrum with fairly steep spectral index (e.g., Chakrabarti & Titarchuk 1995), while this spectral component should be absent from flows onto neutron stars. Some spectral characteristics of AGN can be well fit by flows including shocks (e.g., Chakrabarti & Wiita 1992).

When viscosity is included in the analysis, not only does the central 'O' type sonic point become an 'S' type but the outer stable shock also becomes weaker (Chakrabarti 1990b; Chakrabarti and Molteni 1995). Once the viscosity rises above a critical value, α_{cr}, the outer shock disappears, and the topology changes to allow two smooth solutions. The one with higher dissipation passes through the inner sonic point, while the one with lower dissipation passes through the outer sonic point. The necessary viscosity depends on the disk temperature, sonic point location and angular momentum at that point, but typically corresponds to $\alpha_{cr} \sim 0.02$.

By allowing for different assumptions about heating and cooling mechanisms, these topological results permit the development of a 'unified model' for accretion disks (Chakrabarti 1989, 1996a,b). As long as α viscosity provides the heating, $Q_+ \propto \alpha P x d\Omega/dx$. For an optically thick flow, the cooling is reasonably approximated by the diffusive case, so $Q_- = 4acT^4/3k\Sigma$; but if the flow is optically thin, then Q_- must be examined more carefully, with terms for bremsstrahlung, Comptonization, and synchrotron losses included. The idea of this unification scheme is that while α_{cr} $(0 < \alpha_{cr} < 1)$ depends on the relative importance of Q_- to Q_+ (or how much energy is advected inwards), the location of the inner sonic point, and the specific angular momentum at the inner edge of the disk (Chakrabarti 1996a), the fundamental possible solution topologies do not change. The key point is that if $\alpha < \alpha_{cr}$ the flow can sustain a shock, but if $\alpha > \alpha_{cr}$ a shock will not form and the fluid can accrete smoothly through the inner sonic point.

3.2. ADAFS AND CRITICAL ACCRETION RATES

The other approach to multi-phase disk models has stressed the change in structure that can be expected as the accretion rate varies. Most models for BH powerhouses in AGN were concerned with the need to generate high luminosities and therefore emphasized models with high efficiencies and relatively high accretion rates; the appropriate models were therefore the standard thin and radiation supported thick disks summarized in §2. But the possibility of starved accretion flows, with $\dot{m} \ll 1$ ($\dot{m} \equiv \dot{M}/\dot{M}_E$) must be considered, particularly in binary systems with the secondary underfilling its Roche lobe, or in ex-(or future) AGN where there is little gas available near the central massive BH.

The basic idea can be summarized as follows (e.g., Ichimaru 1977; Svensson 1997): there are two stable branches in the as $\dot{m}$ vs. Σ plane. The high value of Σ branch corresponds to having $t_{cool} < t_{heat}$ at the outer edge of the disk (r_{out}), and yields the Shakura–Sunyaev disks with gas pressure support; as $\dot{m}$ rises the radiation pressure grows, leading towards the more unstable thick disks. The low value of Σ branch is selected when $t_{cool} > t_{heat}$ at r_{out} and corresponds to advection dominated flows; there exists a maximum value of $\dot{m}$ (and Σ) for which such flows can exist. An actual accretion flow can also be a mixture of these two physical branches; typically the outer portion would correspond to a standard thin disk, but the inner portion could go over to an ADAF.

In these ADAF models, the key consideration is the nature of the heating and cooling processes, for $Q_+ = Q_- + Q_{adv}$ must be considered separately for protons and electrons. While the electrons can almost always radiate efficiently, in the innermost portions of accretion flows with low $\dot{m}$, the protons will not, as long as Coulomb processes are the only thing that share energy between electrons and protons. This produces a two-temperature disk, with $T_i \sim 10^{12}\mathrm{K}/x$ and $T_e \sim 10^9$–10^{10}K (Shapiro et al. 1976; Rees et al. 1982). At these temperatures the electrons are mildly relativistic, electron scattering dominates the opacity, and the models appear to be both thermally and viscously stable. Depending upon the angular momentum, this inner portion could resemble either the ion-supported torus (Rees et al. 1982) or a nearly spherical shape (Narayan & Yi 1995). It is important to realize that here 'advection' does not mean that photons are trapped in an optically thick fluid and dragged into the BH; rather, not many photons are produced in the first place, and matter is swallowed without emitting much radiation.

The critical accretion rate below which these two-temperature models become important is given by $\dot{m}_{cr} \sim 0.3\alpha^2$ (e.g., Narayan & Yi 1994, 1995; Abramowicz et al. 1995; Chen et al. 1995). These underluminous models have $\eta \sim 0.1(\dot{m}/\dot{m}_{cr})$ so that the total power is only $L \approx (\dot{m}/\dot{m}_{cr})^2 L_E$. The spectra of these disks are very hard, and inverse Compton scattering of synchrotron emission (based upon reasonable approximations to magnetic field strength) allows quite remarkable fits to spectra of various objects over about 15 decades of frequency. In particular, fits to soft X-ray transients such as A0620−00 and V404 Cygni in their quiescent states (Narayan et al. 1996, 1997), Sagittarius A* at the center of the Milky Way (Narayan et al. 1995) and the Seyfert AGN NGC 4258 (Lasota et al. 1996) have all been quite good, and require setting very few parameters; these are M_{BH} (which can often be independently estimated), $\dot{m}$, α, P_{mag}/P_{gas}, and r_{tr}, the radius at which the standard disk converts to the two-temperature flow. The possibility that ADAF models could also work for luminous accreting black holes has been explored by Narayan (1996) who found that good spectral fits were possible, but that an unlikely high value of α (≈ 1) was necessary to obtain such fits. Advection dominated one-temperature models have also been considered (Esin et al. 1996). When the effects of non-local radiation on the heating and cooling of such accretion flows is included then they are apparently only possible for extremely low accretion rates, $\dot{m}_{cr} < 0.003\alpha^2$ (Esin 1997), and have not been able to fit the spectral of any particular astrophysical object (Narayan 1997).

One particularly interesting application of these ADAF models is to the spectral changes that should occur as the accretion rate changes. Esin et al. (1997) have argued that the differing spectra seen for of Nova Muscae in the X-ray can be explained in terms of differing accretion rates. The idea is that if $\dot{m} \approx 0.01$ the transition radius is very large, corresponding to the quiescent state and a hard spectrum; as the accretion rate rises r_{tr} decreases. For $\dot{m} \approx 0.05$ a low state with a flatter spectrum is achieved and for $\dot{m} \approx 0.08$ a transition to an intermediate state occurs, with a peak in the hard X-rays. For slightly larger $\dot{m}$, the ADAF portion of the flow disappears and the spectrum is the soft one of a standard thin disk. Narayan (1997) has argued that the excellent fit of ADAF spectra to X-ray transient sources that appear to have massive primaries is strong evidence that one is directly detecting a BH in those cases. This is because these solutions are only available for central objects possessing event horizons, and the X-ray spectra and L/L_E ratios of neutron star candidates differ systematically from those of BH candidates.

3.3. CAN α BE DETERMINED?

The hope that the viscosity parameter, α, could be determined from either *ab initio* calculations or observations has long been held by astrophysicists modeling accretion systems. Values of $\alpha \sim 0.1$ seem to give good fits to dwarf nova outbursts (e.g., Cannizzo et al. 1988), and nearly everyone accepted the arguments of Shakura and Sunyaev (1973) that $\alpha < 1$. Further constraints from observations relevant to BHs or convincing theoretical models have been lacking until very recently. The ADAF models discussed in §3.2 fit the observed spectra best if $\alpha \approx 0.2$–0.3 (Narayan 1997). The critical value of α in the unified scheme is about 0.02 (§3.1).

Three-dimensional numerical simulations involving magnetic fields in sheared flows have strongly indicated that MHD turbulence does indeed transport angular momentum and that the magnetic fields are actually amplified (Brandenberg et al. 1995; Stone et al. 1996; Hawley et al. 1996). The values of α deduced from these simulations are typically quite low, ranging from 0.001–0.01. However, these results depend both on the resolution of the grid used in the simulation and at what values the magnetic fields saturate. Stronger angular momentum transport is found for higher resolution simulations (Brandenberg 1997). Therefore the results from the simulations to date are probably lower limits that could be raised somewhat when bigger computers allow for smaller grids; nonetheless, it will be important to see if direct simulations can actually produce the substantially higher values of α apparently favored by spectral modeling.

4. Variability from Accretion Disks

In view of space constraints, only a couple of topics can be mentioned here; for a more extensive reviews, see Chakrabarti (1996a) and Wiita (1996b). Although most strong variability, particularly from radio loud AGN, is attributed to shocks in relativistic jets, there is no question that instabilities of various types exist on accretion disks. Observations of rapid X-ray variability in Seyfert galaxies (e.g., Lawrence & Papadakis 1993), and intra-night optical variability in radio quiet QSOs (e.g., Sagar et al. 1996) are explicable in terms of bright spots on accretion disks (e.g., Abramowicz et al. 1991; Mangalam & Wiita 1993).

The key idea of these models is that any intrinsic variability produced in or just above an accretion disk will be strongly modulated by the orbital motion of the radiating flare. Both Doppler effects and purely general relativistic light bending effects come into play, and are particularly important for fluctuations produced in the inner portions of accretion disks. Phenomenological bright spot models were first applied to X-ray time series and were shown to produce excellent fits to generic light curves and their power spectral densities (Abramowicz et al. 1991). They were also shown to be viable models for the smaller optical fluctuations in BL Lacertae objects and other AGN (Mangalam & Wiita 1993). The anti-correlation between X-ray luminosity and rapid variability for Seyfert galaxies (Lawrence &

Papadakis 1993) is at least partially explained by these models (Bao & Abramowicz 1996). The high temporal coherence between different X-ray bands recently seen for galactic black hole candidates (Vaughn & Nowak 1997) is difficult to explain in most models, but can be fit by reasonable bright spot scenarios (Abramowicz et al. 1997b).

Although various physical causes for such fluctuations have been considered, none has been worked out to the extent to provide fully convincing pictures. Vortices forming within a disk (Abramowicz et al. 1992) could be relevant, as could plasma dominated events just above the disk surface (e.g., Krishan & Wiita 1994). Spiral shocks produced in the accretion disk around a BH by passing massive stars, molecular clouds or companion BHs could also yield suitable luminosity fluctuations from the disks (e.g., Chakrabarti & Wiita 1993). The possibility that self-organized criticality plays a role in producing many flares of different strengths has also been seriously considered (e.g., Mineshige et al. 1994; Wiita & Xiong 1997).

An interesting feature of these models involves the variations in polarization that should be produced. The X-rays originating from the hot inner region are expected to be partially linearly polarized as electron scattering dominates the opacity. If strong magnetic fields are present on or above the disk then synchrotron radiation from electrons may also contribute to the polarization. Rapid polarization variability arises from these orbiting regions and the variability of both the net degree of polarization and the net angle of the polarization plane are energy-dependent, i.e., the polarization variability amplitudes are larger at higher energy (Bao et al. 1996). This trend depends only weakly on the local physics such as the specific polarization mechanism or optical depth of the sources (Bao et al. 1997). Energy-dependent polarization variability is a direct result of near field bending of light rays by the central BH and it is unique to black hole systems involving accretion disks. Since accretion disks around BHs are expected in both active galactic nuclei and X-ray binaries, future X-ray polarimetry missions could confirm the prediction of this phenomenon.

5. Conclusions

Tremendous effort continues on models for accretion disks around black holes. These models have progressed to the point that detailed comparisons with the spectra and variability characteristics of many astrophysical systems are being made. Effects arising from the strong gravitational fields in the immediate environs of black holes are possibly being observed, and convincing evidence for such effects should be available in the near future.

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References

Abramowicz, M.A., Bao, G., Lanza, A. and Zhang, X.-H. (1991) *Astron. Astrophys.*, **245**, 454.
Abramowicz, M.A., Bao, G., Larsson, S. and Wiita, P.J. (1997b) *Astrophys. J.*, 10 November, in press.
Abramowicz, M.A., Björnsson, G., and Pringle, J.E., eds., (1997a) *Theory of Black Hole Accretion Disks*, Cambridge University Press, Cambridge, in press.
Abramowicz, M.A. and Chakrabarti, S.K. (1990) *Astrophys. J.*, **350**, 281.
Abramowicz, M.A., Chen, X., Kato, S., Lasota, J.-P., and Regev, O. (1995) *Astrophys. J.*, **438**, L37.
Abramowicz, M.A., Jarosyński, M. and Sikora, M. (1978) *Astron. Astrophys.*, **63**, 221.
Abramowicz, M. A., Lanza, A., Spiegel, E. A., Szuszkiewicz, E. (1992) *Nature*, **356**, 41.
Abramowicz, M.A., Czerny, B., Lasota, J.P. and Szuszkiewicz, E. (1988) *Astrophys. J.*, **332**, 646.
Artimova, I.V., Björnsson, G., and Novikov, I.D. (1996) *Astrophys. J.*, **461**, 565.
Bao, G., and Abramowicz, M. (1996) *Astrophys. J.*, **465**, 646.
Bao, G., Hadrava, P., Wiita, P.J., and Xiong, Y. (1997) *Astrophys. J.*, 20 Sept., in press.
Bao, G., Wiita, P. J., and Hadrava, P. (1996), *Phys. Rev. Lett.*, **77**, 12.
Blaes, O.M. (1987) *Mon. Not. Roy. Astr. Soc.*, **227**, 975.
Bondi, H. (1952) *Mon. Not. Roy. Astr. Soc.*, **112**, 195.
Brandenberg, A. (1997) in *Theory of Black Hole Accretion Disks*, Abramowicz, M.A. et al. eds., Cambridge University Press, Cambridge, in press.
Brandenberg, A., Nordlund Å., Stein, R.F., Torkelson, U. (1995) *Astrophys. J.*, **446**, 741.
Cannizzo, J.K., Shafter, A.W., and Wheeler, J.C. (1988) *Astrophys. J.*, **333**, 227.
Chakrabarti, S.K. (1988) *J. Astrophys. Astron.*, **9**, 49.
Chakrabarti, S.K. (1989) *Astrophys. J.*, **347**, 365.
Chakrabarti, S.K. (1990a) *Theory of Transonic Astrophysical Flows*, World Scientific, Singapore.
Chakrabarti, S.K. (1990b) *Mon. Not. Roy. Astr. Soc.*, **243**, 610.
Chakrabarti, S.K. (1996a) *Phys. Reports*, **266**, 229.
Chakrabarti, S.K. (1996b) *Astrophys. J.*, **464**, 664.
Chakrabarti, S.K., Jin, L. and Arnett, W.D. (1987) *Astrophys. J.*, **313**, 674.
Chakrabarti, S.K. and Molteni, D. (1995) *Mon. Not. Roy. Astr. Soc.*, **272**, 80.
Chakrabarti, S.K. and Titarchuk, L.G. (1995) *Astrophys. J.*, **455**, 623.
Chakrabarti, S.K. and Wiita, P.J. (1992) *Astrophys. J.*, **387**, L21.
Chakrabarti, S.K. and Wiita, P.J. (1993) *Astrophys. J.*, **411**, 602.
Chen, X., Abramowicz, M.A., Lasota, J.-P., Narayan, R., and Yi, I. (1995) *Astrophys. J.*, **443**, L61.
Chitre, D.M. and Vishveshwara, C.V. (1975) *Phys. Rev.*, **D12**, 1538.
Dadhich, N. and Wiita, P.J. (1982) *J. Phys. A.: Math. Gen.*, **15**, 2645.
Ernst, F.J. (1976) *J. Math. Phys.*, **17**, 54.
Esin, A.A. (1997) *Astrophys. J.*, **482**, 400.
Esin, A.A., McClintock, J.E. and Narayan, R. (1997) *Astrophys. J.*, in press.
Esin, A.A., Narayan, R., Ostriker, E., and Yi, I. (1996) *Astrophys. J.*, **465**, 312.
Frank, J., King, A. and Raine, D. (1992) *Accretion Power in Astrophysics*, Cambridge University Press, Cambridge.
Goldreich, P., Goodman, J. and Narayan, R. (1986) *Mon. Not. Roy. Astr. Soc.*, **221**, 339.
Hawley, J.F. and Blaes, O.M. (1987) *Mon. Not. Roy. Astr. Soc.*, **225**, 677.
Hawley, J.F., Gammie, C.F., and Balbus, S.A. (1996) *Astrophys. J.*, **464**, 690.
Ichimaru, S. (1977) *Astrophys. J.*, **214**, 840.
Iyer, B.R., Vishveshwara, C.V., Wiita, P.J. and Goldstein, J.J. (1985) *Pramāna*, **25**, 135.
Kojima, Y. (1986) *Prog. Theor. Phys.*, **75**, 251.
Krishan, V. and Wiita, P.J. (1994) *Astrophys. J.*, **423**, 172.
Lanza, A. (1992) *Astrophys. J.*, **389**, 141.

Lasota, J.-P., Abramowicz, M.A., Chen, X., Krolik, J., Narayan, R. and Yi, I. (1996) **462**, 142.
Lawrence, A. and Papadakis, I. (1993) *Astrophys. J.*, **414**, L85.
Lightman, A.P. and Eardley, D.E. (1974) *Astrophys. J.*, **187**, L1.
Lin, D.N.C. and Papaloizou, J.C.B. (1996) *Ann. Rev. Astr. Astrophys.*, **34**, 703.
Lynden-Bell, D. (1978) *Physica Scripta*, **17**, 185.
Madau, P. (1988) *Astrophys. J.*, **327**, 116.
Mangalam, A.V. and Wiita, P.J. (1993) *Astrophys. J.*, **406**, 420.
Matsumoto, R., Kato, S., Fukue, S., and Okazaki, A.T. (1984) *Pub. Astron. Soc. Japan*, **36**, 71.
Melia, F. and Königl, A. (1989) *Astrophys. J.*, **340**, 162.
Mineshige, S., Ouchi, N.B., and Nishimori, H. (1994) *Pub. Astr. Soc. Japan*, **46**, 97.
Narayan, R. (1996) *Astrophys. J.*, **462**, 136.
Narayan, R. (1997) in *Theory of Black Hole Accretion Disks*, Abramowicz, M.A. et al. eds., Cambridge University Press, Cambridge, in press.
Narayan, R., Barret, D., and McClintock, J.E. (1997) *Astrophys. J.*, **482**, 448.
Narayan, R., McClintock, J.E. and Yi, I. (1996) *Astrophys. J.*, **457**, 821.
Narayan, R., Nityananda, R., Wiita, P.J. (1983) *Mon. Not. Roy. Astr. Soc.*, **205**, 1103.
Narayan, R. and Yi, I. (1994) *Astrophys. J.*, **428**, L13.
Narayan, R. and Yi, I. (1995) *Astrophys. J.*, **444**, 231.
Narayan, R., Yi, I., and Mahadevan, R. (1995) *Nature*, **374**, 623.
Nityananda, R. and Narayan, R. (1982) *Mon. Not. Roy. Astr. Soc.*, **201**, 697
Novikov, I. and Thorne, K.S. (1973) in *Black Holes*, eds. C. DeWitt and B. DeWitt, Gordon and Breach, New York, p. 343.
Paczyński, B. and Wiita, P.J. (1980) *Astron. Astrophys.*, **88**, 23.
Papaloizou, J.C.B. and Lin, D.N.C. (1995) *Ann. Rev. Astr. Astrophys.*, **33**, 505.
Papaloizou, J.C.B. and Pringle, J.E. (1984) *Mon. Not. Roy. Astr. Soc.*, **208**, 721.
Prasanna, A.R. and Vishveshwara, C.V. (1978) *Pramāna*, **11**, 359.
Rees, M.J. (1984) *Ann. Rev. Astr. Astrophys.*, **22**, 471.
Rees, M.J., Begelman, M.C., Blandford, R.D., and Phinney, E.S. (1982) *Nature*, **295**, 17.
Riffert, H. and Herold. H. (1995) *Astrophys. J.*, **450**, 508.
Robinson, E.L. (1976) *Ann. Rev. Astr. Astrophys.*, **14**, 119.
Sagar, R., Gopal-Krishna, and Wiita, P.J. (1996) *Mon. Not. Royal Astr. Soc.*, **281**, 1267.
Shakura, N.I. and Sunyaev, R.A. (1973) *Astron. Astrophys.*, **24**, 337.
Shapiro, S.L., Lightman, A.P. and Eardley, D.M. (1976) *Astrophys. J.*, **204**, 187.
Stone, J.M., Hawley, J.F., Gammie, C.F., Balbus, S.A. (1996) *Astrophys. J.*, **463**, 656.
Svensson, R. (1997) in *Theory of Black Hole Accretion Disks*, Abramowicz, M.A. et al. eds., Cambridge University Press, Cambridge, in press.
Thorne, K.S. (1974) *Astrophys. J.*, **191**, 507.
Vaughn, B.A. and Nowak, M.A. (1997) *Astrophys. J.*, **474**, L43.
Wiita, P.J. (1981) *Accretion Disks in Astrophysics*, Raman Research Institute, Bangalore.
Wiita, P.J. (1982) *Astrophys. J.*, **256**, 666.
Wiita, P.J. (1990) in *Beams and Jets in Astrophysics*, ed. P.A. Hughes, Cambridge University Press, Cambridge, p. 379.
Wiita, P.J. (1996a) in *Energy Transport in Radio Galaxies and Quasars*, P. Hardee, A. Bridle, and A. Zensus, eds. **ASP Conf. Series, Vol. 100**, Astron. Soc. Pacific, San Francisco, p. 395.
Wiita, P.J. (1996b) in *Blazar Continuum Variability*, H.R. Miller, J.R. Webb, and J.C. Noble, eds. **ASP Conf. Series, Vol. 110**, Astron. Soc. Pacific, San Francisco, p. 42.
Wiita, P.J., Vishveshwara, C.V., Siah, M.J. and Iyer, B.R. (1983) *J. Phys. A.: Math. Gen.*, **16**, 2077.
Wiita, P.J. and Xiong, Y. (1997) in *Theory of Black Hole Accretion Disks*, Abramowicz, M.A. et al. eds., Cambridge University Press, Cambridge, in press.