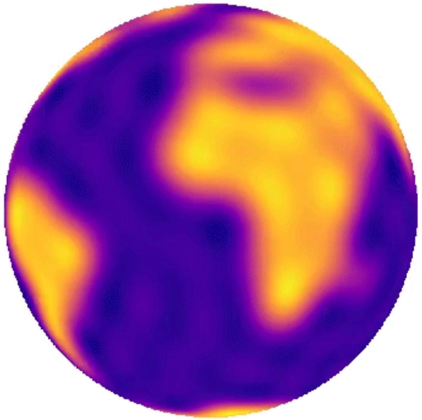
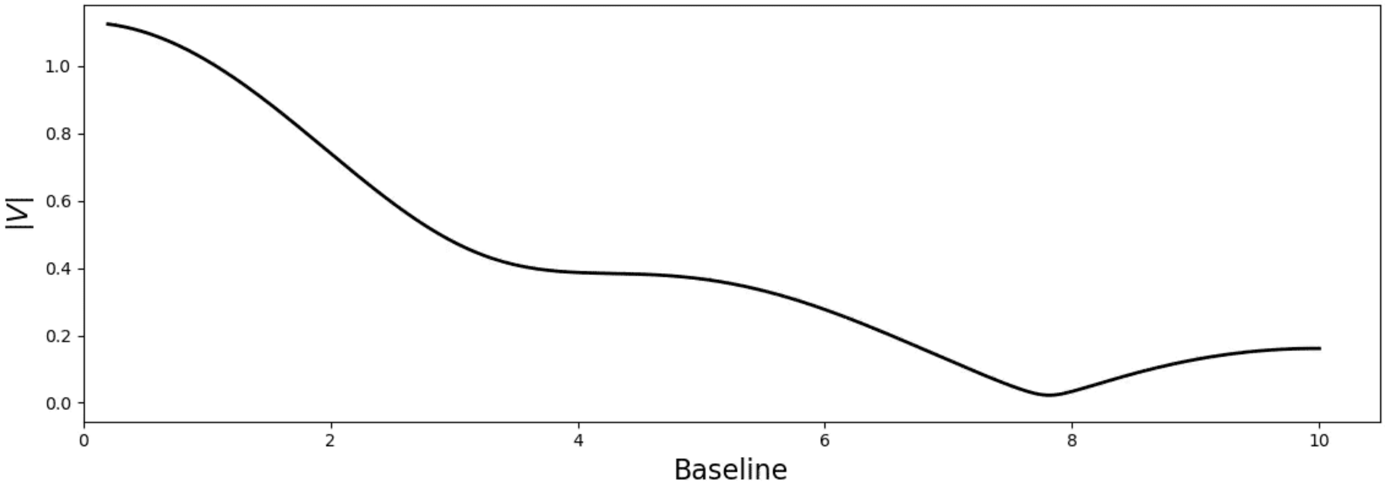
Harmonix: Analytic models of rotating stars





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Problem statement

- Understanding stellar surfaces can help solve many enigmas in stellar physics (magnetism, interiors, chemistry)
- Stellar surfaces represent a noise floor for other fields of astrophysics (exoplanet detection, mass measurements & atmospheres)
- Mapping stars interferometrically is *hard*
    - Need very long baselines to image spots
    - Computationally complex to interpret data
    - Difficult to scale to most stars (e.g. main-sequence stars)
    - Need to widen in scope to tackle the above science problems

Approaches to interferometric stellar mapping

Forwards model:

1. See if there exists an analytic model for the star you're working with (limb darkened disk)
2. If not, discretize star into a tessellation on a sphere
3. Rotate, project & take numerical Fourier transform

Many tried-and-tested models! (`SIMTOI`[1], `rotir.jl`[2], `SURFING`[3])

Fit these non-analytic forwards models to data (the inverse problem)

Requires optimizing over >1000 parameters

Intractable to find uncertainties and degeneracies in the maps

Need to understand all possible maps consistent with data

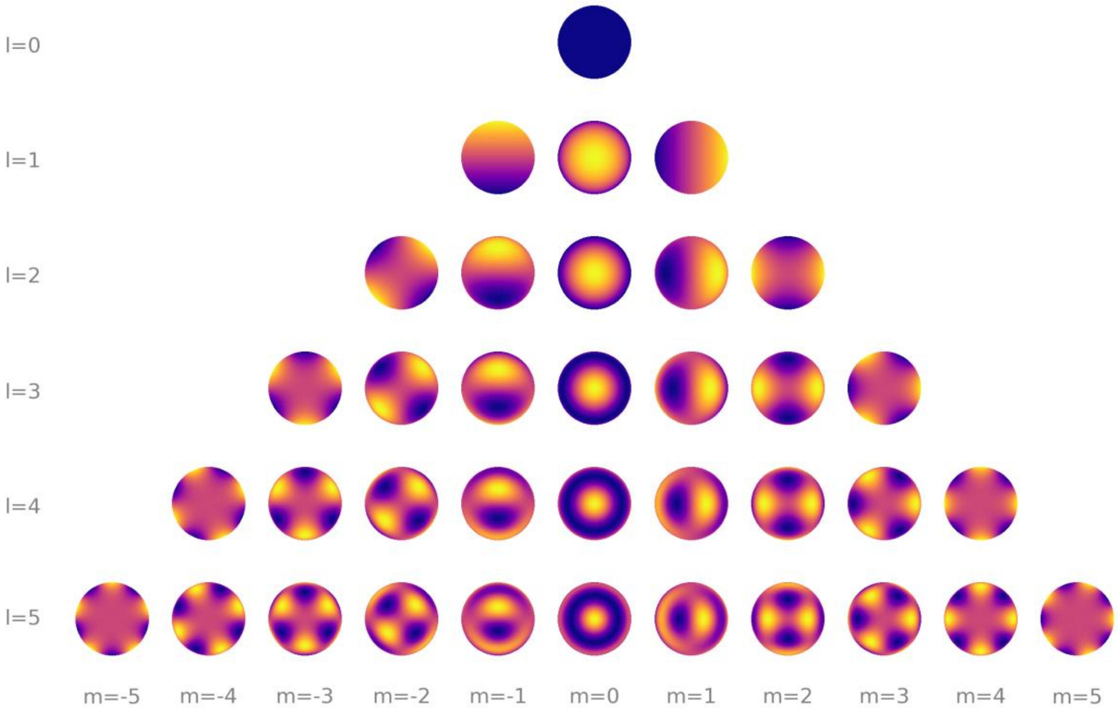
[1]Kloppenborg & Baron 2012, [2]Baron & Martinez 2018, [3]Roettenbacher et al. 2016)

Spherical Harmonics

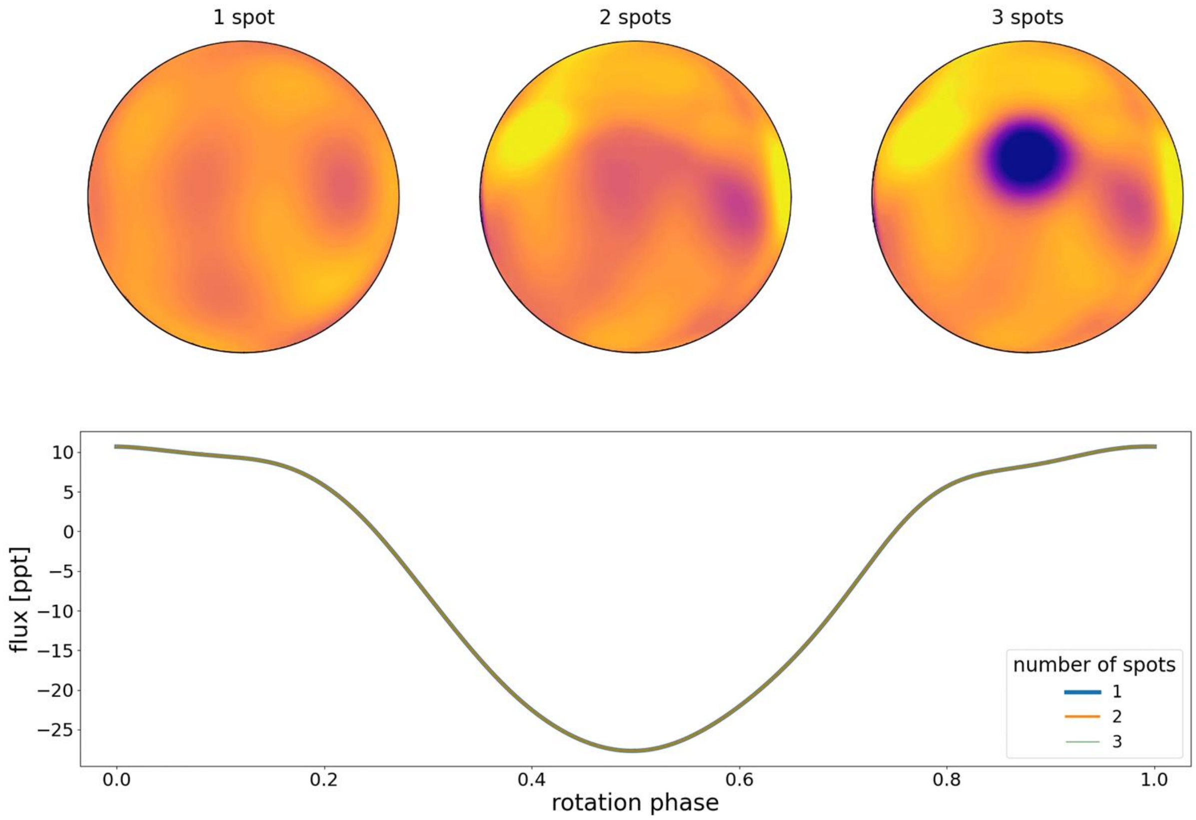
Provides a way of understanding the information theory of the mapping problem

Special set of polynomials on a unit sphere

Any surface map can be expressed as a linear combination of spherical harmonics



The rotational light curve problem



Looking at a star's brightness over time gives you some information about its surface—a disk integrated flux at each unit time

But there are significant degeneracies in mapping stars with just photometry

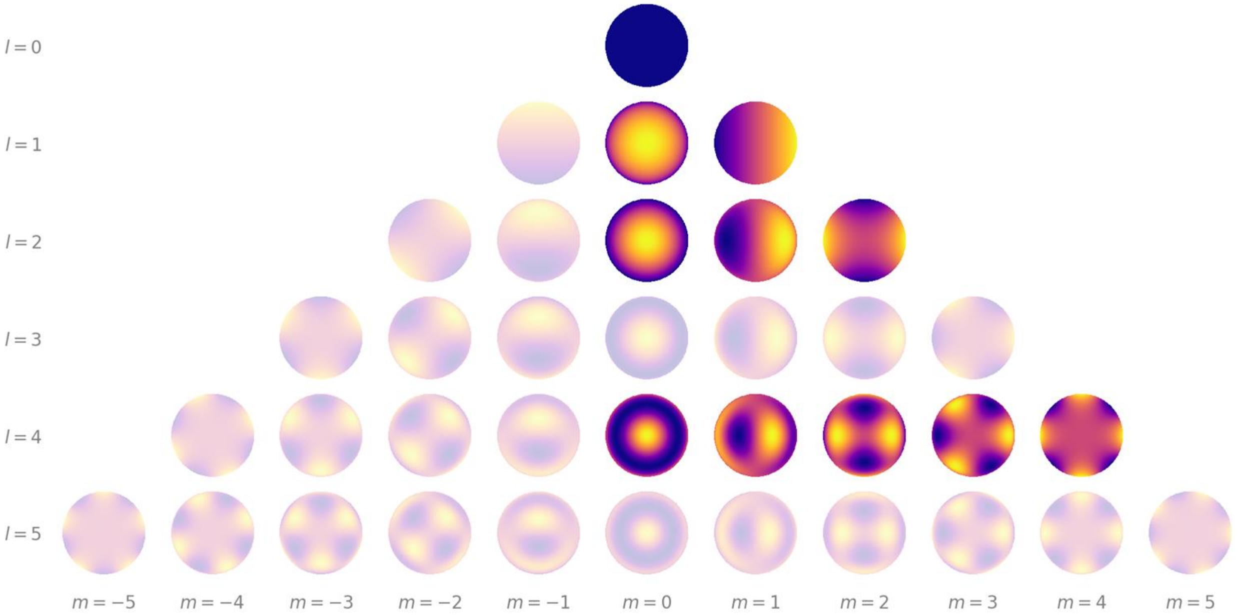
The null space

$$F = \mathbf{A}\,\mathbf{y}$$

Null space is the set of harmonics that do not contribute to the light curve

Most of the information in the map is destroyed when transforming to a light curve

Contains all of the degeneracies of the stellar map



Spherical Harmonics: Modeling interferometry

$$I(x,y) = \underbrace{\tilde{\mathbf{y}}(x,y)}_{\text{basis}} \; \underbrace{\mathbf{R}}_{\text{rotation}} \; \underbrace{\mathbf{y}}_{\text{map}}$$

Representing intensity in the spherical harmonic basis (read from right to left):

$$V(u,v) = \iint_{S(x,y)} I(x,y) e^{-i(ux+vy)} dx\, dy$$

van-Cittert Zernike theorem: Fourier transform of the intensity on sky is the complex visibility

$$= \iint_{S(x,y)} \tilde{\mathbf{y}}(x,y)\, e^{-i(ux+vy)} dx\, dy\, \mathbf{R}\, \mathbf{y}$$

Because of linearity of the Fourier transform, we can remove the dependence of the rotation operation and the $\mathbf{y}$ inside the integral

$$= \mathfrak{F}[\,\tilde{\mathbf{y}}(x,y)]\, \mathbf{R}\;\; \mathbf{y}$$

We only need to solve for the FT of the basis itself

Spherical Harmonic: Analytic solution

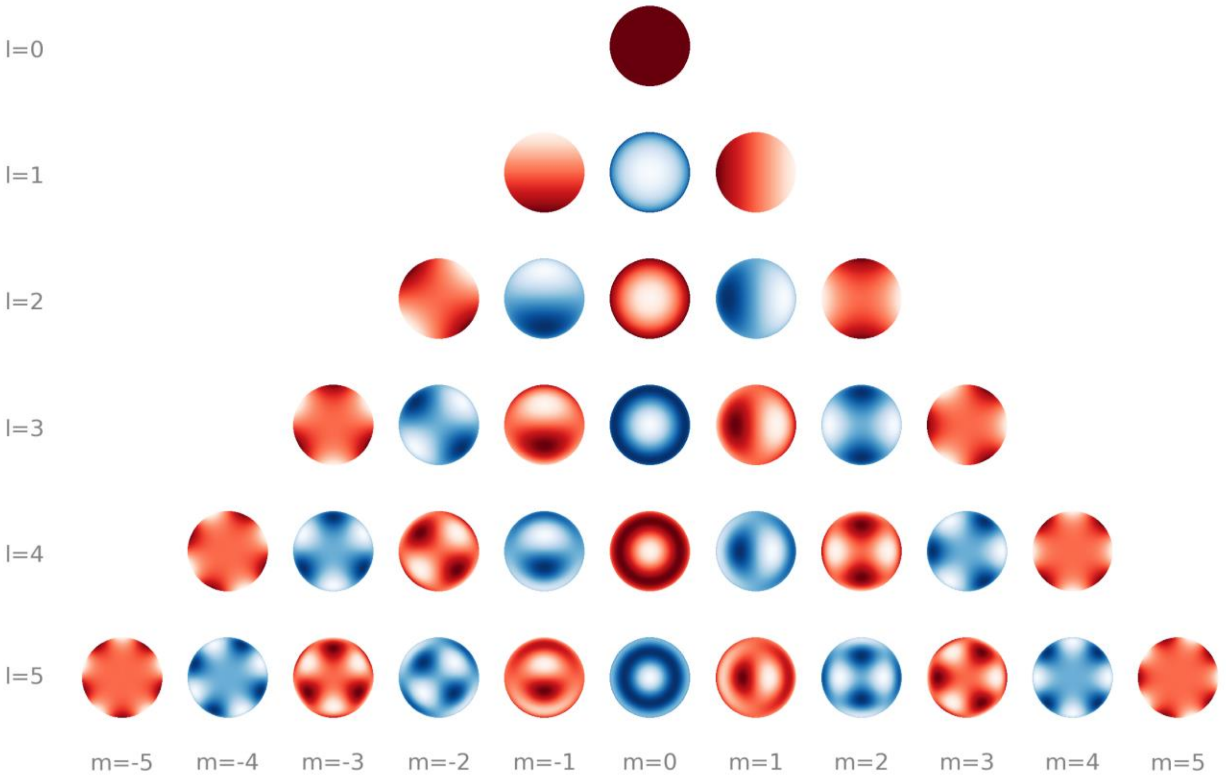
Two families of solutions:

Hemispheric harmonics

- Solved by changing basis to Zernike polynomials
- Solution in terms of Bessel functions

Complementary hemispheric harmonics

- Solution in terms of a single spherical Bessel function



Harmonix

Analytic forwards models of interferometry in Python

Open source and easy to use

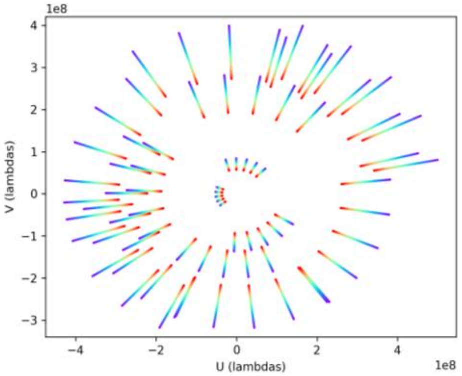
`Jax`: fast and automatically differentiable:
- Gradient-based Monte Carlo for high-dimensional models

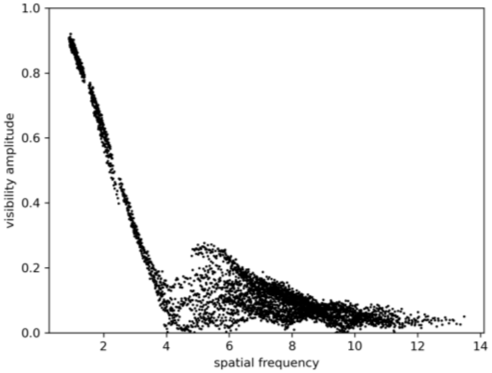
Works with other codes in `jax` (`jaxoplanet`) for joint fits to multiple types of observations (interferometry, photometry)

Gives you uncertainties and unlocks information theory

Information theory of mapping

Simulation of a spotted, resolved star with CHARA/SPICA





Gradients allow computing the Fisher information matrix

- How many bits of information does a dataset permit us to obtain about a star's map?

Properties of Fisher information:

- Multiple datasets can be combined by simply adding their information

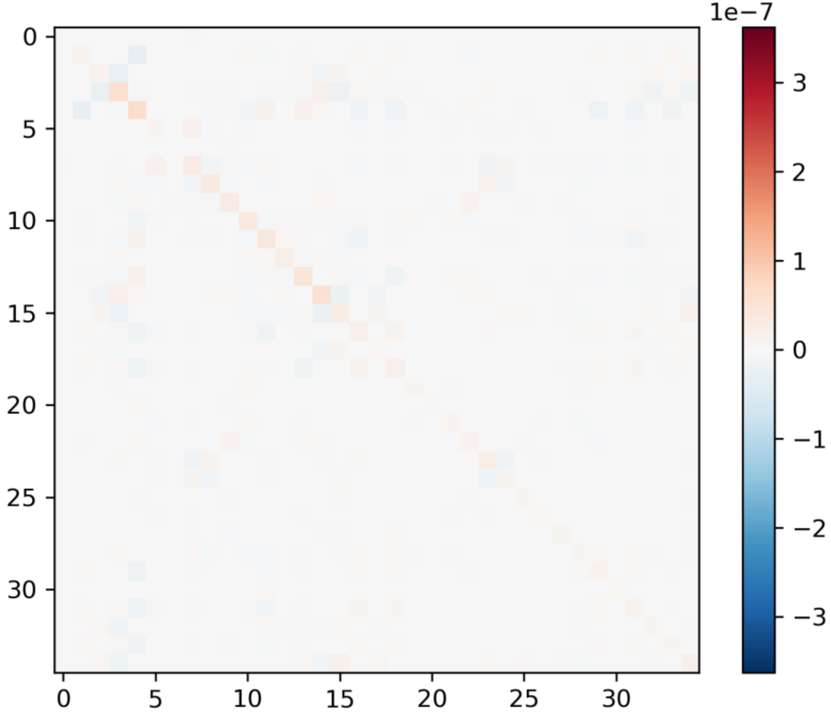
- Inverse of the Fisher information matrix at the best-fit parameters is the covariance matrix

Interferometry + space-based photometry

CHARA does great at high resolution information but doesn’t provide coverage at the shortest baselines

Photometry buys back the low-res details

By combining both together, you can get an order of magnitude improvement in stellar surface measurements



Covariance matrix of map coefficients
How to read: Diagonal is variance on each map coefficient
White=better constraint

Future work

- Using `harmonix` to fit observations of rotating, spotted stars
- CHARA observations of Eps UMa upcoming (March 22-27th), simultaneously with TESS
- Chaining `harmonix` stars together into binaries (and multiples), solving for orbital parameters & map coefficients simultaneously
- Rewriting starry Doppler imaging into `jax` to allow modelling interferometry, photometry & Doppler imaging simultaneously