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Mount Design Issues for the CHARA Array

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1. OVERVIEW

The CHARA optical train must transmit light to the central laboratories without significant degradation of wavefront quality. Of particular concern in this report will be the undesired motions of the telescope structure which may be induced by wind or other vibrations, and result in output wavefront tilt and optical path difference change. This report discusses general considerations for the telescope mount structural design, sketches some scaling laws which point to strategies for minimizing disturbances due to mechanical resonances and finite drive bandwidth, derives analytical approximations for estimating the optical disturbances arising from wind, and derives the expected reduction in fringe visibility for angular vibration of some mounts in particular circumstances.

2. GENERAL STRUCTURAL CONSIDERATIONS

The mount is required to track accurately and smoothly. The major criteria are the positional stability and the optical path stability. Of special concern are fluctuations in these induced by drive noise and by wind buffeting.

2.1. Resonances

Disturbances of all types will cause the largest amplitude effects at resonant frequencies. The impact of resonances is normally controlled by raising them to the highest possible frequency where less disturbance power is expected to found, by limiting the disturbance power, and by passive and active damping.

2.1.1. Mechanical Resonances

The characteristic frequency of a simple structure will be given by

$$f \approx \sqrt{\frac{k}{m}} \quad , \tag{1}$$

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where k is the spring constant and m is the mass. In general, it is desirable to keep structural resonances as high as possible. In this context, it is normally easier to increase the spring constant than it is to decrease the mass. For example, consider a tube of variable diameter d but constant wall thickness. Since $k \propto d^4$ and $m \propto d$, the characteristic frequency will be

$$f \propto d^{3/2} \quad , \tag{2}$$

because increasing stiffness wins over increasing mass. This is a generic argument for concentrating on building a stiff structure rather than a light-weight structure. It is not a strictly quantitative argument since engineered structures need not follow the proportionality which applies to a single truss member.

We can hope to devise a drive which does not introduce significant noise into the mechanical structure. We can try to protect the structure from wind, but some exposure is unavoidable, and even desirable from the point of view of thermal uniformity. The wind power spectrum drops rapidly with frequency. One way to limit the resonant response to wind is to move resonances to high frequency where the wind has little power. This is an argument for keeping resonances at high frequency, to the extent that this is possible. This is best accomplished by increasing the stiffness. The modest dependence of the frequency on mass ($f \propto m^{-2}$) severely limits the changes that can be achieved with mass changes — even gaining a reduction of 25% in frequency by reducing mass can be extremely difficult. It is true, however, that reducing mass also reduces thermal capacity, which is desirable both in keeping the thermal response time short and in limiting the amount of heat which must be liberated from the mount when temperatures drop.

Another strategy for limiting the response to wind buffeting is to introduce extra mass into the structure. The extra mass will provide inertial resistance to wind buffeting. As an undesired side effect, it will also lower the resonant frequencies to where the wind has more power. However, the tilt and optical path variations introduced by the atmosphere also increase at low frequency, and consequently the atmosphere limit is more generous at low frequency than at high frequency. At very low frequencies, the telescope servo systems come into play, preserving pointing in spite of low frequency disturbances. Thus it is not immediately obvious what strategy will best minimize the interferometric impact of resonances.

2.1.2. Drive Response

The response of the drives will be determined by a combination of mechanical and electronic properties. The stiffness will be limited by drive motor power and mechanical characteristics of the drive train. We can expect the drive response to be resonance free, and to be limited in response to frequencies well below mechanical resonances. Therefore resonances in the drives should not be a problem. However, the limited stiffness of the drives will allow the telescope to respond to wind buffeting with angular motion about the driven axes. This response must be considered.

2.2. Static and Dynamic Approximations

The static approximation is sometimes useful. In the case of a telescope mount bending against a stiff mechanical restoring force, the wind-induced motion will very quickly reach equilibrium with the restoring force. This happens so quickly that the deflections can be accurately estimated from a static calculation — that is, assuming that the instantaneous wind force acts steadily and long enough for the mount to reach an equilibrium position.

In some situations the static approximation is not valid. An important example is the case of motion of the telescope about a rotational axis. The drive servo will be stiff to low frequency disturbances, but transparent to high frequency disturbances — that is, disturbances which are above the characteristic servo bandwidth. In this case, it will be more accurate to assume a dynamic approximation in which the telescope responds with free motion and no restoring force.

3. WIND DISTURBANCES

The wind force will be proportional to the cross-section which the telescope presents. The wind force can be estimated from

$$f_{\mathrm{w}} = \frac{1}{2} C_D \sigma v(t)^2 A \quad , \tag{3}$$

where C_D is the drag coefficient, σ = air mass density $\approx 1.0\,\mathrm{kg/m^3}$ at 6500 ft elevation, and A is the area of the object normal to the wind. We can describe the wind torque about a chosen telescope mount axis with $T = f_{\mathrm{w}} r_{\mathrm{w}}$, where r_{w} is the moment arm at which the wind force effectively acts.

The wind contains buffeting power at all frequencies, but with a relatively cool spectrum. Above a characteristic frequency on the order of 0.1 Hz, the Kolmogorov tail of the wind velocity power spectrum $|V(f)|^2$ falls as $f^{-5/3}$.

4. THE COEFFICIENT OF DRAG

The coefficient of wind drag depends on the Reynolds number,

$$R = \frac{L v_o \rho}{\mu} \quad , \tag{4}$$

where v_o is wind speed, L is the characteristic size of the object (e.g. the diameter of a sphere or rod), ρ is the density of the fluid, and μ is the absolute viscosity.

4.1. Wind Induced Vibration in the Dynamic Limit

We would like to relate the power spectra of the optical path changes and angular tilts to the power spectrum of the wind velocity. This can be done through the following derivation.

Consider an angle $\gamma(t)$ which describes some angular motion of the mount or tube. The Fourier transform of $\gamma(t)$ will be denoted for the dynamic approximation by $G_{\mathrm{d}}(f)$, where f is the frequency. The angular acceleration is $d^2\gamma/dt^2$, and the Fourier transform of the angular acceleration will be called $A(f)$. By the Fourier derivative theorem

$$A(f) = -(2\pi f)^2 G_{\mathrm{d}}(f) \quad . \tag{5}$$

We would like to relate this to the wind velocity power spectrum. We note that the angular acceleration of the mount due to wind, in the absence of restoring force or friction, will be given by

$$\frac{d^2\gamma}{dt^2} = \frac{T(t)}{I} \tag{6}$$

where I is the torsional moment of inertia. The moment of inertia is related to the mass and the radius of gyration about the relevant axis by $I = mr_{\mathrm{g}}^2$.

Combining these terms,

$$\frac{d^2\gamma}{dt^2} = \frac{C_D \sigma v(t)^2 A r_{\mathrm{w}}}{m r_{\mathrm{g}}^2} \quad . \tag{7}$$

The Fourier transform of this equation will give

$$A(f) = \frac{C_D \sigma A r_{\mathrm{w}}}{m r_{\mathrm{g}}^2} \mathcal{F}[v^2] \tag{8}$$

where $\mathcal{F}[v^2]$ is the Fourier transform of the velocity squared. We would like to relate $\mathcal{F}[v^2]$ to $|\mathcal{F}[v]|^2$, which is known. To do this, consider a Fourier series representation of $v(t)$,

$$v(t) = v_o \left[1 + \sum_{n=1}^{n=\infty} b_n \cos(2\pi f_n t + \phi_n) \right] \quad . \tag{9}$$

In this representation, v_o is the "continuous" wind velocity, and the oscillations around this value are represented by the amplitudes b_n and the phases ϕ_n. Taking the square of $v(t)$ and keeping only the first order terms in the b_n,

$$v(t)^2 \approx v_o^2 \left[1 + 2 \sum_{n=1}^{n=\infty} b_n \cos(2\pi f_n t + \phi_n) \right] \quad . \tag{10}$$

This approximation will be good for analysis with frequency resolution no greater than about 1 Hz, under which conditions the low frequency peak of the wind buffeting spectrum will be effectively unresolved from the zero frequency "DC" wind velocity.

By analogy, the Fourier transform of $v(t)^2$ (except for the zero frequency term) can be written as

$$\mathcal{F}[v(t)^2] \approx 2 v_o \mathcal{F}[v(t)] = 2 v_o V(f) \quad . \tag{11}$$

Therefore,

$$A(f) \approx \frac{C_D \sigma A r_{\mathrm{w}}}{m r_{\mathrm{g}}^2} 2 v_o V(f) \quad . \tag{12}$$

We now have two expressions for $A(f)$ which must be approximately equal, so

$$-(2\pi f)^2 G_{\mathrm{d}}(f) \approx \frac{2 v_o C_D \sigma A r_{\mathrm{w}}}{m r_{\mathrm{g}}^2} V(f) \quad . \tag{13}$$

This lets us express $G_{\mathrm{d}}(f)$, the Fourier transform of the angular position, in terms of $V(f)$, the Fourier transform of the wind velocity. The power spectrum of the angle in terms of the power spectrum of the wind velocity is then

$$|G_{\mathrm{d}}(f)|^2 \approx \frac{4 v_o^2 C_D^2 \sigma^2 A^2 r_{\mathrm{w}}^2}{(2\pi f)^4 m^2 r_{\mathrm{g}}^4} |V(f)|^2 \quad . \tag{14}$$

This is useful because the power spectrum of the wind velocity, $|V(f)|^2$, is a function which can be estimated from studies of the wind and turbulence.

For scaling purposes, it is interesting to introduce the frequency dependence of $|V(f)|^2$,

$$|G_{\mathrm{d}}(f)|^2 \propto \frac{4v_o^2 C_D^2 \sigma^2 A^2 r_{\mathrm{w}}^2}{(2\pi)^4 m^2 r_{\mathrm{g}}^4} f^{-17/3} \quad , \tag{15}$$

showing an extremely rapid decline in angular rotation power with frequency.

4.2. Wind Induced Vibration in the Static Limit

The static approximation is much simpler. We can integrate under the wind velocity power spectrum, $|V(f)|^2$, to obtain the mean-square velocity fluctuation containing power only above a lower frequency bound f_1 and is given by

$$\overline{v_1^2} = \int_{f_1}^{\infty} |V(f)|^2 \, df \quad . \tag{16}$$

The maximum of a fluctuating variable is often approximated with $3\times$ the root-mean-square, so we can estimate the maximum wind velocity above f_1 as

$$v_{\mathrm{max}}(f_1) \approx 3\sqrt{\overline{v_1^2}} \quad . \tag{17}$$

4.3. Wind and Pointing

A natural standard for pointing (here, keeping the telescope accurately pointed at the source while tracking) is the random pointing variation introduced by atmospheric turbulence, normally called tilt. Above a characteristic frequency

$$f_0 \approx \frac{v}{\pi R} \tag{18}$$

where R is the telescope aperture radius and v the wind velocity, the atmospheric tilt power $|T(f)|^2$ is

$$|T(f)|^2 \propto f^{-11/3} \quad . \tag{19}$$

If we compare the telescope tilt power spectrum to the atmosphere tilt power spectrum (the ratio as a function of frequency) above the relevant characteristic frequencies for both, we find

$$\frac{\mathrm{Telescope-Wind}}{\mathrm{Atmosphere}} = \frac{|G_{\mathrm{d}}(f)|^2}{|T(f)|^2} \propto \frac{4v_o^2 C_D^2 \sigma^2 A^2 r_{\mathrm{w}}^2}{(2\pi)^4 m^2 r_{\mathrm{g}}^4} f^{-2} \quad . \tag{20}$$

This confirms that the telescope will not dominate the tilt error at the high frequency limit. Furthermore, the objective of controlling telescope tilt errors will be, by design, to reduce the telescope contribution sufficiently to push the crossover to low frequencies where the remaining errors can be removed by the closed loop guiding and tracking system (which includes a fast tilt correction servo).

4.4. Wind and Optical Path

The optical path error power, $|O(f)|^2$ associated with the atmosphere at high frequencies is predicted by Kolmogorov theory to be

$$|O(f)|^2 \propto f^{-8/3} \quad , \tag{21}$$

although there is not much experimental evidence available.

The optical path introduced by wind induced motion of the telescope can be derived from the expression above for angular motion, multiplied by a factor $r_{\rm opt}^2$ which accounts for the distance of the optical beam from the relevant rotational axis.

The telescope-wind contribution can be compared to the atmospheric optical path error in the ratio

$$\frac{\text{Telescope} - \text{Wind}}{\text{Atmosphere}} = \frac{|G_{\rm d}(f)|^2}{|O(f)|^2} \propto \frac{4v_o^2 C_D^2 \sigma^2 A^2 r_{\rm w}^2 r_{\rm opt}^2}{(2\pi)^4 m^2 r_{\rm g}^4} f^{-3} \quad . \tag{22}$$

For optical path errors, as for tilt errors, the telescope will make a negligible contribution in the high frequency limit, and again it will be desired to design the telescope to push the crossover to sufficiently low frequency that the fringe detection and optical path servo can remove the residual telescope induced optical path error.

4.5. Strategy for Limiting Coupling of Wind Buffeting into Optical Errors in the Dynamic Approximation

From the foregoing analysis, the wind power coupled into the tilt error will be,

$$|G_{\rm d}(f)|^2 \propto \frac{4v_o^2 C_D^2 \sigma^2 A^2 r_{\rm w}^2}{(2\pi)^4 m^2 r_{\rm g}^4} f^{-17/3} \quad . \tag{23}$$

For the case of optical path errors, multiply $|G_{\rm d}(f)|^2$ by $r_{\rm opt}^2$.

Several points can be made from this expression, most obvious and some not so obvious.

- The wind cross-section (A) should be low. Members should be selected appropriately.
- The drag coefficient (C_D) should be small. Some custom treatment of members may be beneficial.
- The wind moment arm ($r_{\rm w}$) should be small. The structure should be small, it should be relatively symmetric about axes, bulky components should be located near axes when possible.
- The moment of inertia ($mr_{\rm g}^2$) should be large. Both the mass and $r_{\rm g}$ should be maximized.
- The distance of the optical beam from the rotation axes ($r_{\rm opt}$) should be small.

Perhaps the most indicative observation is that the quantity $r_{\rm w}/r_{\rm g}^2$, (or $r_{\rm w} r_{\rm opt}/r_{\rm g}^2$ as appropriate) should be minimized. This clearly suggests the value of locating optional mass at the largest radial distance from the axes of rotation.

Note that this discussion specifically concerns the dynamic approximation, which is applicable to rotational axes above the drive servo bandwidth. Hence it does not apply to mechanical resonances, for which the static approximation is preferred.

4.6. Strategy for Limiting Coupling of Wind Buffeting into Optical Errors in the Static Approximation

In the static approximation, the angular deflection γ_s induced by a force $f_{\rm w}$ will be determined by

$$f_{\rm w}(t)r_{\rm w} = kg_s(t) \quad , \tag{24}$$

that is by the balance of wind force and restoring force for a spring constant k. Then the deflection will be

$$\gamma_s(t) = \frac{C_D \sigma A r_{\rm w} v(t)_2}{2k} \quad , \tag{25}$$

and the power spectrum will be

$$|G_s(f)|^2 \approx \frac{(C_D \sigma v_o A r_{\rm w})^2}{k^2} |V(f)|^2 \tag{26}$$

$$\propto \frac{(C_D \sigma v_o A r_{\rm w})^2}{k^2} f^{-5/3} \quad . \tag{27}$$

Again we see merit in minimizing $r_{\rm w}$, the moment arm of the wind force, and in maximizing the spring constant.

Although this static approximation cannot be used to estimate the coupling into a resonance, it can be used to estimate the power potentially available to couple into a resonance. For f, substitute an expression for the frequency of a resonance characterized by $\sqrt{\frac{k}{m}}$

$$|G_s(f)|^2 \approx \frac{(C_D \sigma v_o A r_{\rm w})^2 m^{5/6}}{k^{17/6}} \quad . \tag{28}$$

This again shows the approximately linear increase of wind buffeting power with mass. It also shows the very strong decrease with spring constant. This confirms the general observation that in eliminating a resonance by strengthening a member, the increased stiffness will tend to win over the increase in mass.

4.7. Strategy for Limiting Coupling of Wind Buffeting into Optical Errors in the Presence of Resonances

For the excitation of resonances, it is again necessary to consider a dynamic analysis. With the approach described above, this is not difficult. Consider a sinusoidal excitation of a damped system. From elementary texts, the solution for the motion is,

$$\gamma(t) = \frac{T \sin(2\pi f t - \phi)}{\sqrt{(2\pi)^4 I^2 (f^2 - f_o^2)^2 + b^2 f^2)}} \tag{29}$$

where T is the amplitude of the applied sinusoidal torque, I is the moment of Inertia, f_o is the natural (resonant) frequency of the system, and b is the damping constant.

The power transferred to the system is γ^2. Integrating γ^2 over frequency, we find that the integrated power is independent of the damping. This shows that the difference between a resonant and a non-resonant structure is not in the power transfer but in the frequency distribution of the resulting motion.

A more interesting quantity may be the maximum value of the amplitude *vs* frequency relation. For a poorly damped system, this will occur close to f_o.

$$\gamma_{\max}(f_o) \approx \frac{T}{b f_o} \quad . \tag{30}$$

For T we can substitute the value of the torque power spectrum at frequency f_o and find that

$$\gamma_{\max} = \frac{2 C_D \sigma v_o A r_{\mathrm{w}}}{b f_o} \left| V(f_o) \right|^2 \quad . \tag{31}$$

This result is independent of the mass and radius of gyration because the solution is for the steady state, after the driving torque has had time to build up power at the resonant frequency until the power input is in equilibrium with the dissipation of the damping mechanism.

Substituting the frequency dependence of $|V(f_o)|$, we find the amplitude peak is proportional to $f_o^{-8/3}$. Substituting for f_o the approximately proportional quantity $\sqrt{\frac{k}{m}}$, we find

$$\gamma_{\max} \propto \left(\frac{m}{k} \right)^{-4/3} \quad . \tag{32}$$

This shows that increasing the stiffness of the structure reduces the amplitude of the response. It also shows that the response will be limited by keeping the mass small. Again, increasing stiffness will often win over the associated increase in mass. However, with regard to resonances, it is desirable to avoid "unnecessary" mass such as counterweights, where it can be avoided.

Of course the new parameter available for adjustment of resonances is the damping coefficient, which is especially interesting because in the case of mechanical resonances, increases in damping do not have undesired side effects. (Driven axes remain a separate case, since mechanical damping of a driven axis will require higher drive motor power, slower slewing, or a decoupling mechanism.)

5. LOCATION OF THE ALTITUDE AXIS

All of this can actually be applied to make some real conclusions about the optical mount. Consider the design of the telescope tube and the location of the altitude axis along the length of the tube. The classical choice is near the primary. This will minimize the total moving mass. An extreme alternate is to locate the altitude axis near the center of the telescope tube. This will minimize the lever arm of wind forces on the top of the telescope, but will require counterweight mass at the top of the tube, and may result in some vignetting at the tertiary.

Consider a schematic telescope tube design. Call the length L (close to the telescope focal length). Assume that the mirror and cell and associated structure has a mass of M. Assume that the azimuth axis is located at a distance aL from the primary mirror, where $a > 0$ corresponds to the direction toward the secondary. The distance from the altitude axis to the top end is then approximately $(1-a)L$, and, in order to achieve balance, the mass at the top end must be $(\frac{a}{1-a})M$. If a is small, then, the required mass at the top end is also small. We suppose that the wind acts solely at the top end of the telescope, so the lever arm is $r_{\rm w} \approx (1-a)L$. The radius of gyration is the sum of the masses of the top and bottom end, each times its radial distance squared, given by

$$r_{\rm g} \approx (aL)^2 + \left(\frac{a}{1-a}L\right)^2 \quad . \tag{33}$$

From the analysis above, it is clear that in the dynamic approximation the amplitude of the tilt and path errors will be proportional to $r_{\rm w}/r_{\rm g}^2$. Table 1 shows the relative value of this ratio for different values of a. Note that $a = 0.5$ corresponds to an altitude axis at the midpoint of the tube, and $a = 0.0$ corresponds to the altitude axis in the mirror plane. (The "infinity" at $a = 0$ is an artifact of the assumption that the mirror end mass is located at a point.)

TABLE 1. Relative error in dynamical approximation.

a	$r_{\rm w}/r_{\rm g}^2$
0.5	0.4
0.4	1.0
0.3	2.6
0.2	7.8
0.1	40.

This shows that the tube with the "classical" location of the altitude axis near the primary is in this model approximately 100× more sensitive to wind buffeting than the mid-tube altitude axis design. Intuitively, this arises because the classical design places the wind torque at large radius, where it is effective in moving the telescope, and puts the mass at small radius of gyration where it is not effective at providing inertial resistance to wind force. The effect is obvious, but the amplitude is remarkable. In a detailed model the extreme values for small a will not be realized, but the strong trend will remain.

The dynamic approximation, above, is important for motion of the tube about the altitude axis. For motion of the tube orthogonal to the altitude axis the static approximation is most useful. In the static approximation the important ratio to consider is the ratio of wind force to stiffness. For a tube, the stiffness is proportional to l^{-3}, where l is the length. In this approximation, it is useful to consider the ratio of wind lever arm $r_{\rm w} \propto (1-a)$ to the stiffness of the longer section of the telescope tube $\propto (1-a)^{-3}$. The ratio is $(1-a)^4$, which has the values shown in Table 2. This shows that in the static approximation the mid-tube axis has an advantage of about 10×.

The remaining consideration in comparing altitude axis locations is the impact on resonances. We will estimate the value of $\sqrt{k/m}$ for the upper and lower sections of the tube. We will use l^{-3} with the appropriate tube section length, a and $1-a$, and the relevant mass estimates, M and $(\frac{a}{1-a})M$. Table 3 shows the relative resonances, in common units of $\frac{1}{\sqrt{M}}$.

This shows that as the altitude axis is moved towards the primary mirror, the lowest resonance of the mirror end is likely to increase significantly. The resonance of the top end first decreases (as the span increases) then the change in mass overtakes the length and the resonance begins to increase. The frequency shift is small and cannot be considered quantitatively meaningful, but the result should be taken as a strong suggestion that the lowest resonant frequency may not depend much on the axis location.

TABLE 2. Relative error in static approximation.

a	$(1-a)^4$
0.5	0.06
0.4	0.13
0.3	0.24
0.2	0.41
0.1	0.66

TABLE 3. Relative resonant frequencies.

a	Mirror End	Top End
0.5	2.8	2.8
0.4	3.9	2.6
0.3	6.1	2.6
0.2	11.	2.8
0.1	32.	3.5

In summary, the mid-tube axis has greatly reduced sensitivity to wind buffeting, except for a change (possible reduction) in lowest resonant frequency compared to a classical design. The recommended design strategy should be to look first at the mid-tube axis configuration, and confirm that the resonant frequencies are indeed reasonable. If so, this configuration should otherwise be dramatically superior in stability under wind buffeting.

6. OUTBOARD *VS* INBOARD MOUNTS

Among fork style mounts, the tube can be located symmetrically above the azimuth axis (called here "inboard") or offset (called here "outboard"). The radial offset of the optical axis constitutes a lever arm which comes into calculating the optical path error associated with rotation around the azimuth axis. We would like to contrast these mount concepts.

As a particular comparison, consider the observing situation in which the telescope is pointing towards a source at the horizon. Now consider the effect of small oscillations of the telescope about the azimuth axis. For a rotation of $\alpha/2$ the wavefront out of the telescope will rotate by α (assuming magnification equals unity, although this is of no importance in the final result).

The objective in this section is to estimate the significance of this effect. The conclusion depends on the assumptions about how the path difference will be measured.

6.1. Amplitude of Path Change

The simplest quantity to estimate is the amplitude of path change associated with an uncontrolled azimuth rotation α. For the inboard mount, the path change for the on-axis ray is zero, and for the ray at the outer edge of the pupil it is αR where R is the radius of the primary mirror. If the telescope is moved off axis by a distance aR, the optical path error will reach a maximum at the outer edge of the pupil of $(a+1)R$. For reasonable values of a such as $a = 1$, this is clearly a small number. However, it is considerably larger than the *mean* path error over the inboard pupil. One way of estimating the importance of this is the following.

6.2. Fringe Brightness Change with Azimuth Rotation

Consider the interference of a plane wavefront and a reference wavefront with a relative tilt of α. Assume that the optical path difference relative to an ideal reference wavefront is being stabilized to achieve maximum fringe brightness for the mean telescope position. However, suppose small fluctuations of the telescope pointing about the azimuth axis with resulting path differences. The optical path difference as a function of position across the wavefronts will be $\alpha x + aR + \delta$, where x is the distance distance from the origin of axes, a is the distance of the origin of axes from the intersection of the wavefronts, and δ is any additional phase shift between the wavefronts. We interpret these as follows. For $a = 0$ we have the on-axis fork, and for $a \neq 0$ we have the outboard fork with the center of the pupil at distance aR from the azimuth axis. The additional shift δ is due to OPD differences between telescopes. It will ultimately be set to zero here, but is carried in the calculation for future reference.

The fringe brightness can be written as

$$f = \frac{1}{2}\{1 + \cos[2\pi\alpha\nu(x + aR + \delta)]\} \quad . \tag{34}$$

We are interested in the average fringe brightness over any circular pupil of radius R. Consider first a pupil centered on the wavefront intersection. This might correspond to a horizon pointing on-axis mount oscillating slightly about the azimuth axis.

$$F = \frac{2}{\pi R}\int_0^R \left[1 + \cos\left[2\pi\alpha\nu(x + aR + \delta)\right]\left(1 - \frac{x^2}{R^2}\right)^{1/2}\right] dx \quad . \tag{35}$$

Following some rearrangement and substitutions, this can be written as

$$\begin{aligned} F \quad = \quad & \frac{2}{\pi}\left[\int_0^1 (1 - z^2)^{1/2} dz \right. \\ & + \cos[2\pi\alpha\nu(aR + \delta)] \int_0^1 \cos(2\pi\alpha\nu R z)(1 - z^2)^{1/2} dz \\ & \left. - \sin[2\pi\alpha\nu(aR + \delta)] \int_0^1 \sin(2\pi\alpha\nu R z)(1 - z^2)^{1/2} dz\right] \quad . \end{aligned} \tag{36}$$

After integration, the result is

$$F = \frac{1}{2} + \frac{J_1(2\pi\alpha\nu R)}{2\pi\alpha\nu R}\cos[2\pi\alpha\nu(aR + \delta)] - \frac{2}{\pi} S \sin[2\pi\alpha\nu(aR + \delta)] \quad . \tag{37}$$

In this expression, S is an infinite series

$$S = \sum_{k=0}^{\infty} \frac{(-i)^k (2\pi\alpha\nu R)^{2k+1}}{(2k+1)!!(2l+3)!!} \quad , \tag{38}$$

where the !! notation is defined for integer n by

$$(2n)!! = 2 \times 4..... \times 2n \tag{39}$$

and

$$(2n+1)!! = 1 \times 3..... \times 2n+1 \quad . \tag{40}$$

As a particular illustration, consider the case in which the fringe position is actively stabilized to maintain $\delta = 0$, that is maximum fringe brightness, for the mean telescope position, but suppose there are telescope azimuth oscillations at a frequency outside the bandwidth of the fringe stabilization. What will be the effect on the fringe brightness?

We can compare the change in the brightness, $1 - F$, for the outboard and inboard cases, Expanding the result for small α and keeping only the most significant terms gives

$$R = \frac{1 - F(a)}{1 - F(0)} \approx 1 + \frac{32a}{3\pi} + 4a^2 \quad . \tag{41}$$

Table 4 contains typical values for the ratio.

TABLE 4. Relative reduction in visibility.

a	R
0.0	1.0
0.2	1.8
0.4	3.0
0.6	4.5
0.8	6.3
1.0	8.4
1.2	11.
1.4	14.

This shows that a small offset of the pupil will have a small effect, but an offset comparable to the aperture radius is serious. For a typical outboard mount design, we could easily have $a > 1.4$, which would result in an instantaneous loss of fringe brightness more than $10\times$ greater than for the inboard, symmetric fork case. As noted earlier, the relevance of this calculation depends on the exact observational strategy for measuring fringe visibility. But it shows a large effect, and the fully outboard mount should probably be avoided unless the fringe detection technique is understood to be insensitive to such path errors.

7. NUMERICAL RESULTS

7.1. Parameter Values

Evaluating the Reynold's number from Equation 4, we will take $1\,\mathrm{kg/m^3}$ for the density of air, $0.1\,\mathrm{m}$ for the characteristic size of a telescope frame member, $v = 10\,\mathrm{m/sec}$ for the wind velocity, and for the viscosity of air $180\,\mu\mathrm{Poise} = 180{\times}10^{-7}$ kg/m-sec. This gives $R \approx 5 \times 10^4$.

From the *American Institute of Physics Handbook*, a table of drag coefficient *vs* R for cylinders gives a drag coefficient of $C_D = 1.2$. This probably should be increased to account for interaction between flow about various parts of the structure. We will take $C_D = 2.0$.

The area of the telescope structure is approximately $1.5\,\mathrm{m}^2$, and the wind acts primarily at the top end of the telescope with $r_\mathrm{w} \approx 1.5\,\mathrm{m}$. The moment of inertia of the tube about the altitude axis is approximately $700\,\mathrm{kg\,m^2}$.

We will take the wind velocity to be 10 m/s. This is a high value, meant to represent the highest wind in which there might be any chance of operating. The mean wind speed will be lower.

7.2. Wind Buffeting about the Altitude Axis

Evaluating Equation 14 for the case of wind buffeting induced motion of the tube about the altitude axis gives

$$|G_\mathrm{d}(f)|^2 \approx 6.6 \times 10^{-9} \frac{|V(f)|^2}{f^4} \frac{\mathrm{rad}^2}{\mathrm{sec}} \tag{42}$$

$$= 2.8 \times 10^{2} \frac{|V(f)|^2}{f^4} \frac{\mathrm{arcsec}^2}{\mathrm{sec}} \quad . \tag{43}$$

The spectrum of $|V(f)|^2$ for a wind velocity of 10 m/sec above a frequency f_o greater than about 0.01 Hz is reasonably represented by $|V(f)|^2 \approx 0.15 f^{-5/3}$. Making this substitution gives

$$|G_\mathrm{d}(f)|^2 \approx 0.42 f^{-17/3} \frac{\mathrm{arcsec}^2}{\mathrm{sec}} \quad . \tag{44}$$

The integral over the range $f_o < f < \infty$ is

$$G_\mathrm{d}(f_o)^2 = \int_{f_o}^{\infty} |G_\mathrm{d}(f)|^2 \, df \approx 9.7 f_o^{-13/3} \frac{\mathrm{arcsec}^2}{\mathrm{sec}} \quad , \tag{45}$$

and the square root, $G_\mathrm{d}(f_o)$, gives the expected RMS fluctuation of the angular pointing due to wind. Table 5 gives this result for several values of f_o.

Remember that this result is in the dynamic approximation with no restoring force. That is, drive stiffness is ignored. This result tells us that the drive will be relied upon to reduce the amplitude of buffeting in the vicinity of 1 Hz. It is probably unrealistic to expect the drive to significantly damp telescope motions at frequencies above 1 Hz with amplitudes of an arcsec or less. This implies that the tilt errors in the range 1-10+ Hz will have to be removed by the fast tilt correction system. It will be necessary for the drive servo to keep tilt errors within the dynamic range of the tilt correction system. If this is on the order of

TABLE 5. Predicted RMS Angular Buffeting in Altitude.

f_o (Hz)	$G_{\rm d}(f_o)$ (arcsec)
1.0	31.
10.	0.2

10 arcsec, the drive servo will have to operate at frequencies up to about 1 Hz. The very strong variation of amplitude with frequency also tells us that virtually no changes we can realistically make in the mount will change these boundaries very much. Thus making the telescope a factor of two more compact ($r_{\rm w}$ and $r_{\rm g}$ reduced) would make the amplitude of $G_{\rm d}$ about a factor of 2× smaller, but this would be the equivalent of a shift in frequency of only about 10%.

8. REFERENCES

American Institute of Physics Handbook, 1972, 3rd Edition, McGraw-Hill.